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Proposal for a nonlinear top-down toy model of the brain

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Abstract

Solutions to Newton's equations for particles in one-body potentials of form $V_1(x_i) = \sum_p A^{(i)}_{2p} x_i^{2p}$, where $p > 0$ and an integer, can be regarded as generators of infinite sequences of correlated frequencies $\{\Omega_m\}$. Simple nonlinear potentials can hence provide efficient ways of generating correlated information. Two-body potentials $V_2(x_i - x_j)$ can provide ways to communicate aspects of that information between the correlated frequency sequences. Temperature and noise can play a role in introducing a time scale across which the frequencies retain their identity. Introduction of explicit time dependence in the energy terms might be appropriate for constructing top down versions of toy models for the brain, something that is lacking at the present time. The richness of the nonlinear system along with the effects of heat baths, external noise and time dependence allows for the possibility of describing aging effects, processing of information "templates" in the brain and of the development of correlations between such "templates". In short, nonlinearity, interactions, noise effects and introduction of time-dependent energies might allow for the construction of "top-down" models of the brain with the eventual goal of possibly unifying the neurological, molecular biological, biochemical and psychiatric approaches toward studying the brain.

1. Introduction

Brain research has largely been a "bottom-up" process. Examples of areas in which significant progress has been made are (i) molecular biology of memory, (ii) biochemical approach to the functioning of the brain, (iii) neurobiology of the brain and (iv) sleep and memory studies from a psychiatric point of view. We discuss these developments briefly below.

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(i) *Molecular biology of memory*: One considers *implicit* and *explicit memory* as important components of memory when viewed from a molecular biology perspective. In *implicit memory* one deals with the memory for perceptual and motor skills, i.e., actions without conscious recall of past episodes. Significant progress has been made in the behavior of ion channels in a circuit of well-defined neuronal elements in the gill withdrawal reflex in a fish called Aplysia by Kandel et al. [1]. *Explicit memory* involves a specialized anatomical system in the medial temporal lobe and a structure deep into it called the hippocampus. Explicit memory can be classified into two categories—the short- and long-term phases. The short-term phase does not involve protein synthesis. The long-term phase remains to be well-explored [2]. It is known that the hippocampus contains a cellular representation of "extra-personal space", i.e., a cognitive map of space [3,4]. Lesions of the hippocampus interferes with spatial tasks [5]. There is also a special pathway in the hippocampus called the "perforant path". It has been demonstrated by Lømo and Bliss that chemically induced blockades in the perforant pathway blocks memory storage [6].

(ii) *Biochemical approach to the functioning of the brain*: Carlsson and coworkers have identified critically important aspects of chemical sensitivity that controls communication between the nerve cells [7]. These chemical signals can be carefully controlled and manipulated for therapeutic purposes (e.g., reserpine, zimeldine, prozac).

(iii) *Neurobiology of slow synaptic responses*: There are $\sim 10^{11}$ nerve cells in the brain. Each of these nerve cells communicate with $\sim 10^3$ cells at any instant. These cells communicate across time scales between 10^{-3} and 10^2 s. The nature of the chemical transmitters associated with fast and slow synaptic responses have been extensively probed by Greengard et al. [8].

(iv) *Role of sleep in learning and memory*: Hobson and others have studied the role of sleep in off-line memory reprocessing and have probed the process of dreaming in significant depth. Rapid eye movement (REM) sleep is believed to play a critical role in processing complex learning tasks as opposed to tasks associated with declarative memory [9].

In spite of rather significant progress in the above areas, as emphasized by Kandel and Crick and others, there is an acute lack of "top-down" approaches that might help develop much needed connections between the different approaches to studying the brain. One such effort has been pioneered by Crick. Crick, Koch, Edelman and others have tried to focus on conscious learning processes in the brain and have tried to connect such processes with that achieved by correlated dynamical behavior of neurons [10]. In the present work, I describe some ideas along with relevant justifications that may be of interest with regard to the construction of top-down models of learning processes.

2. Modeling learning processes

2.1. Edelman's scheme

Edelman defines the conscious brain as one that involves consolidation of learning between the hippocampus and the parietal cortex [11]. His scheme for describing higher-order consciousness can be summarized as follows. The *brain stem* and the *hypothalamus* are regarded as autonomic centers that are involved in current registration of internal states and values. The *primary* and *secondary cortex* are entities for processing world signals including proprioception and are involved in current perceptual categorization. The signals from the brain stem and the cortex are processed and presumably correlated in the *hippocampus*, the *amygdala* and the *septum*. Following the process of correlation between the current registration of internal states and values and current perceptual organization, a process of conceptual categorization transpires. The special value-category memory in the frontal, temporal, parietal cortex stores the conceptually categorized version of the internal states and values and perception-based categories. The frontal, temporal and parietal cortex is also accessed directly by the primary and secondary cortex for sight, hearing, touch, etc. via a semantic bootstrap through the Broca's and Wernicke's areas of the brain. Edelman also theorizes that there is direct connection between the primary and secondary cortex for sight and hearing and touch and the frontal, temporal and parietal cortex.

2.2. Incorporating nonlinearity

The goal of this work is to bring to focus the possible role of certain elementary dynamical models in describing learning processes. The models are of a generic nature, without direct reference to any specific brain. It may be assumed that any useful model should have the following properties, (a) numerically tractable, (b) exhibits slow relaxation or, roughly speaking, memory loss/aging where the degree of slowness can be tuned by some kind of "noise" effects and/or system parameters, (c) system dynamics can be highly susceptible to the effects of noise, (d) generates correlated frequency sets such that one can use frequency sets as tools to represent "gross" learnt information, (e) must be further parameterizable so as to incorporate the key role of neurons and perhaps other entities with regard to modeling the functioning of the brain [12].

In addition, I assume that the process of learning can be evolved in discrete steps, presumably between REM sleep cycles so as to incorporate and construct descriptions of learnt processes through significant spans of time. Such evolution of a model physical system may be associated with the evolution of memory templates in the brain and could conceivably unlock pathways to model processes such as memory loss due to lack of sleep, stroke and diseases such as Parkinson's disease and Alzheimer's disease.

Returning to physical systems, it is reasonable to start with a simple nonlinear system described by a particle of mass m in an on-site potential $V_1(x) = (a/4)x^4$, such that the total energy $E = (m/2)(\mathrm{d}x/\mathrm{d}t)^2 + V_1(x)$. We use x and $v = \mathrm{d}x/\mathrm{d}t$ to represent the position and velocity variables of the particle. The relevant equation of motion will be $m\,\mathrm{d}^2x/\mathrm{d}t^2 = -ax^3$. Such an equation of motion involves a solution with an infinite number of possible frequencies of the form $\Omega, 3\Omega, 5\Omega, \ldots, (2n+1)\Omega, \ldots,$ where Ω depends upon the total energy E of the system [13]. It is difficult to concoct a simpler system that offers an infinite set of correlated frequencies that are parameterizable. As in studies involving neural networks, if certain master frequencies are assumed to represent some elementary form of information, one can imagine that the infinite set of correlated frequencies represent the possibility of not only learning about something but the ability to acquire derived information about the same that may be representative of our naive ability to deduce obvious connections [14,15], which are all issues that can be perhaps represented via correlated frequencies to same base frequency, Ω. In the absence of a microscopic basis of a toy model, it is difficult to assign physical meaning to quantities such as mass, position, velocity and potential energy except to regard them as gross parameters with possible significance in the context of the brain. We shall return to briefly discuss these quantities again below. It is well acknowledged that learning processes are eventually controlled by neuronal activity and many other processes such as chemical and sensory processes [12]. If one must incorporate this aspect of learning within some toy model, it is reasonable to assume that the constants in the expression for E above be parameterized by neuronal frequencies. We shall briefly return to addressing this step below.

Our ability to learn information suffers changes due to factors such as aging, diseases, lack of sleep and more. One can model such processes that destroy or affect the set of correlated frequencies within the context of a trivial model (e.g., for the case of the dynamical system in a quartic potential well) by introducing a time-dependent mass or perhaps by introducing noise terms in the equation of motion or both. The frequencies associated with the system dynamics would hence be affected by the time-dependent mass and/or noise terms. Studies of the effects of noise on constant mass oscillators in single anharmonic potentials reveal that noise can have dramatic effects in predictably contaminating the base frequency of anharmonic oscillators, a result that bodes well for pursuing studies of simple nonlinear models for modeling learning processes [16].

3. Conceptual framework of a proposed toy model

We assume that elemental learned information can be characterized by correlated master frequency functionals, which we denote via the set $\{\Omega_p\}$, where p denotes the correlated set of master frequency functionals. Given that information learnt is associated with the details of neuronal functioning in the brain, it may be assumed that these frequencies are parameterized by "neuronal coordinates" that we denote via another set $\{v_q\}$, where the index q refers to some specific set of neurons that would orchestrate the frequency functionals described by $\{\Omega_p\}$. In addition, it is possible that there are chemical and sensory processes that transpire during learning. It is hence reasonable to assume that coordinates associated with chemical and sensory processes are also necessary to eventually parameterize the master frequencies. Thus, any model should be able to incorporate the various ways in which the master frequencies can be parameterized.

During a normal period when a subject is awake, the brain continuously learns new information. We impose no conditions on the amount of information that can be learnt. This unprocessed information is assumed to be stored in a memory area that is of limited capacity. It is presumed that the brain can only effectively learn across a finite time span. This time span would be comparable to the typical time a normal subject is awake. As ones ability to sleep deteriorates, one might argue that the ability

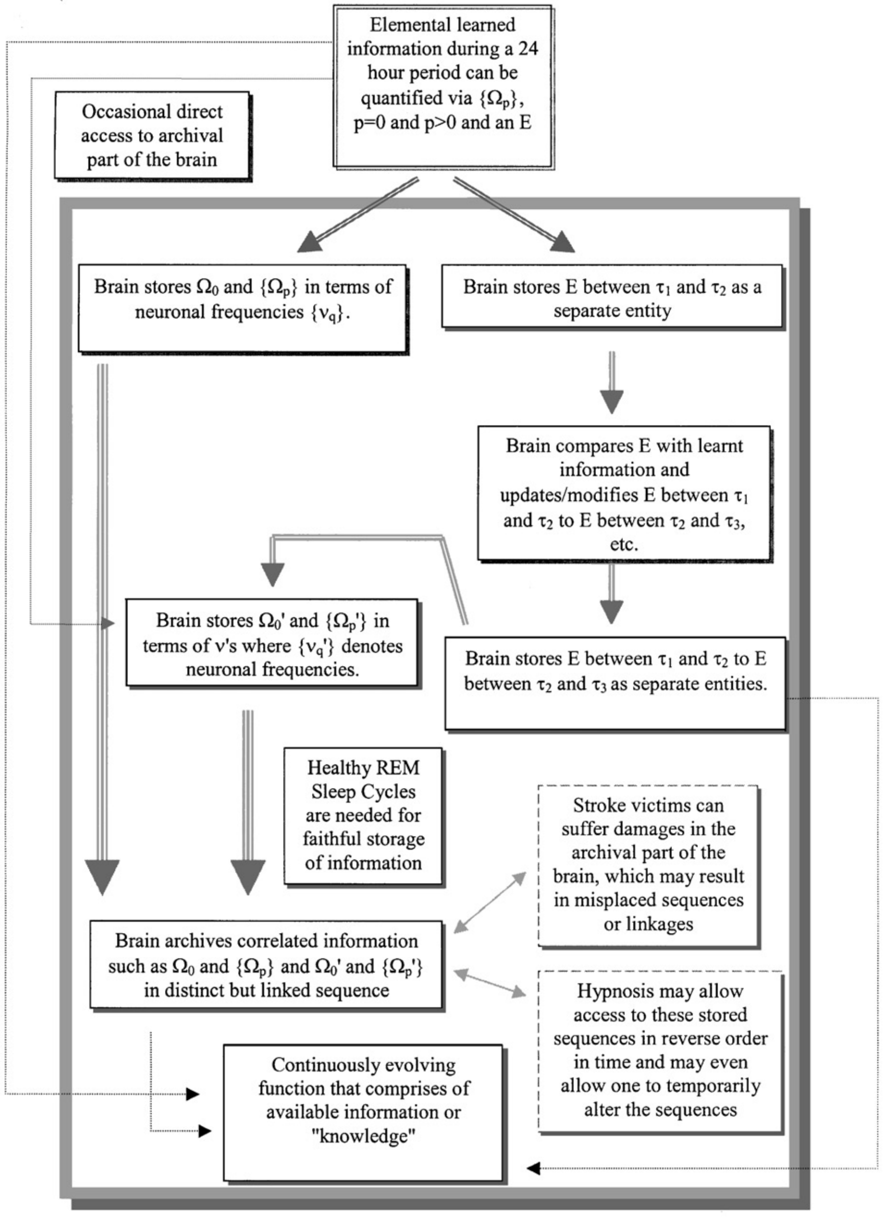


Fig. 1. Schematic of learning process during a period between and through the sleep phases.

to learn is adversely affected as well [17]. Fig. 1 provides a schematic of the processes of learning, archiving, updating and recollection that underlie the functioning of the model brain that is being perceived in this study.

We assume that the brain does not instantaneously archive the learnt information most of the time. That would seem like an inefficient way for a learning system to proceed, especially, if the need for such fast-time-scale archiving is not necessary for survival. Rather, the brain perhaps assimilates and structures learnt information in steps, first in the cache itself and then, during sleep cycles in the primary archive. Of course, the essential learnt information in the primary archive remains accessible to a healthy brain.

The only times when such direct information archival is conceivable would be when the information learnt is very closely related to previously learnt information that is archived and is readily accessible.[1] When one falls asleep, it is assumed that the brain correlates the elemental learnt information along with the details of the processes that generate the frequency functionals (in this case it is assumed that the equation(s) of motion are the generating equations for the frequency functionals and are characterized by mass and the inertial term, some anharmonic potential energy function $V_1(x)$ and coefficients associated with $V_1(x)$ that incorporate the roles played by the neurons and chemical or other sensory processes) with what is already archived. During this phase, it would be reasonable to assume in our model that the brain also rejects redundant information and consolidates new information with reference to related known information (Fig. 1).

It is possible that the brain stores the characteristic information about the equations that generate the master frequencies with their appropriate parameterization and the frequency sequences in separate regions of the brain. It seems apparent that we can generate our inferences based upon what we recall of an event even if we cannot directly retrieve the inferences directly. The process of comparing with learnt information and of updating information could be an integral part of the learning process itself. It is assumed that the process of updating learnt information and of archiving acquired information with related information transpires during REM sleep [9] and that the brain likely stores the generating equations for the frequency functionals with some periodic time sequence. Thus, healthy REM sleep cycles are essential for proper archival storage of information. As we age, our ability to get adequate REM sleep and hence our ability to properly archive information deteriorates (Fig. 1). In addition, with increasing age, it is also likely that our ability to retrieve information may change. Often stroke victims suffer from loss of memory of recent incidents while retaining some or significant memory of older events.

The storage of generating equations and of associated frequency functionals in time sequenced form would allow one to explain such observed behavior regarding memory loss of recent episodes or of those from early years or perhaps of some combination of both. It is conceivable that hypnosis may allow a subject to access stored sequences of information at earlier times under conditions of partial sleep (when bodily noise levels are low as is believed to be the case in REM sleep). One can imagine that it may even be possible by appropriate action (perhaps via hypnosis or via chemical

[1] One can hypothesize that the most efficient teaching techniques would be those that would allow a subject to archive as much of the learnt information as possible directly during learning or very shortly thereafter. As we know, most of the times, we absorb learnt information better after a night's sleep.

means) to temporarily or permanently alter the sequences and/or contents of the stored templates (Fig. 1).

4. A toy model for the brain

In our models below, we assume that frequency represents learnt information, potential energy landscape represents available information, the variable mass and the initial conditions control how much of the available information can be sampled and hence how much learning is possible and the concept of temperature is associated with "noise" type factors that are associated with the operational requirements of a living body, which can adversely affect our ability to learn under a variety of common circumstances.

Let us start with simple systems that describe particles, either interacting or otherwise, in on-site nonlinear potentials. For simplicity, we assume the system to be 1D. Analyzing the dynamical behavior of the system described by Eq. (1) below in the context of a *canonical ensemble* allows us to introduce relaxation and memory into the system. Alternately, we can also introduce some explicit time-dependent noise term and perhaps an explicit frictional damping term, with some appropriate form of noise, into the description of our system. The total energy for such a system can be written as

$$E = \sum_i p_i^2/2m_i + \sum_i V_i + \sum_{i>j} V_{ij} + B \,, \tag{1}$$

where the above system can be thought of as defined during the short-term learning process across a certain time period between say τ_1 and τ_2. In the above Hamiltonian, p_i and m_i denote the momentum and the mass of some hypothetical particle i, to be thought of as a parameter in our model, not necessarily an actual particle with a mass of course, V_i denotes the on-site potential in which this particle lives and V_{ij} specifies how each of these particles are linked. The term B denotes the presence of a bath as a source of noise. It should be recognized that the system described above is truly an exploratory toy model with the sole purpose of exploring whether models such as the one above can be exploited to describe some of the crucial features of the brain. One can imagine introducing explicit time-dependent driving terms into the system. Such driving can be, for instance, environment related. Spatial dimensionality can also be introduced and more involved interactions, such as three-body interactions, etc., can be invoked. The present goal, however, is to explore if the basic properties of our model make it appropriate for modeling aspects of brain functioning. In what follows, we analyze Eq. (1) for several cases.

4.1. Case I: Properties of one-body nonlinear potential

We let $V_{ij} = 0$, $B = 0$ and $V_i = x_i^2/2 + rx_i^4/4$ (Duffing potential). The solution to

$$x_i(t) = C \sum_{p=0}^{\infty} a_p \sin(2p+1)\Omega(r)t \,, \tag{2}$$

i.e., there is a specific correlated set of frequencies $(\Omega(E), 3\Omega(E), 5\Omega(E), \ldots)$ that characterize the frequencies of the particle in a Duffing potential (i.e., a potential with leading quartic anharmonicity) with

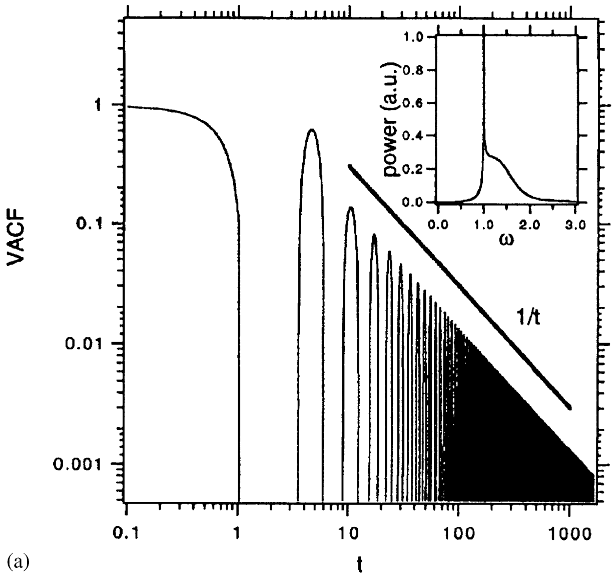
$$\Omega(E) = (1 + 3ra^2/4 + 3r^2a^4/128 - 57r^3a^6/4096 + \cdots)^{1/2} , \tag{3}$$

where $a \equiv Ca_0$, and where $\Omega(E) \sim 1 + \gamma(r)E$, $E \ll 1$ [13]. The amplitudes associated with each of these frequencies play a critical role in defining the importance of the frequencies in the dynamics of the particle and are of particular importance when the system is externally driven. The parameters E and m could be potentially related to age and physical health in some way. It is conceivable that a child may not be able to access much of the naive abilities of the brain. Similar conditions might apply to an aged person in failing physical health.

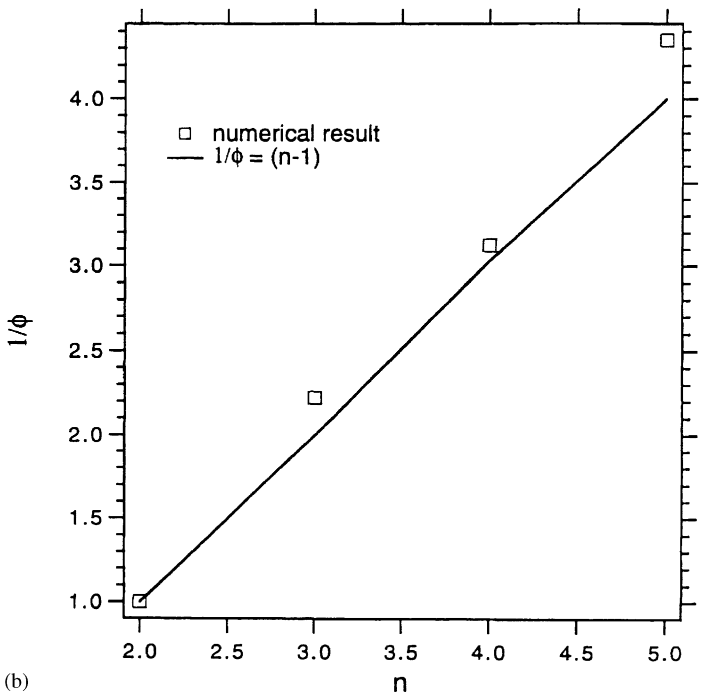
One can now pose the following "inverse" problem: Can one obtain a V_i and an E to generate a desired sequence of correlated frequencies with given amplitude distributions? If so, what are the limitations of such a scheme and how accurately can such an inverse problem be solved? How can such a scheme be generalized? All of these are issues that remain to be carefully explored. Ideally, if one assumes that information on some topic can be represented by a set of correlated frequency functionals, where each frequency is a function of neuronal coordinates, one may be able to determine highly efficient ways to store information. By altering neuronal coordinates (e.g., sensation of taste being one set of coordinates to sensation of smell being another) and keeping V_i invariant, one may be able to retrieve information about many independent properties associated with some learnt information, a feature that might be useful in the construction of broad-based top-down models of the brain that can incorporate some of the enormous complexity observed in brain functioning.

4.2. Role of temperature in case I

The relaxation behavior, as characterized for instance via velocity relaxation, i.e., $\langle v(t)v(0)\rangle/\langle v(0)\rangle^2$, where $\langle\ldots\rangle$ denotes a canonical ensemble average, i.e., $\langle\theta\rangle = \int g(E)\theta\exp(-\beta E)\,\mathrm{d}E/\int g(E)\exp(-\beta E)\,\mathrm{d}E$, where $g(E)$ denotes the density of states and $\beta \equiv 1/kT$, where k is the Boltzmann constant and T is the temperature and θ is some variable, can be computed for a particle in Case I. It turns out that for a particle in an anharmonic well where the lowest order anharmonic term is x^{2n}, behaves asymptotically as $1/t^{\phi}$, where $\phi = 1/(n-1)$ [18]. Fig. 2(a) provides a description of relaxation in a system with $V(x) = \frac{1}{2}x^2 + \frac{1}{2}x^4$ at $\beta = 1$. What such a relaxation process means is that a particle in an anharmonic well that is "weakly" coupled to some infinite heat bath such that the accessible energies of a particle due to its contact with the bath is described via a Boltzmann factor, loses memory of its initial state as a function of time. As n increases, $1/(n-1)$ decreases and hence relaxation becomes progressively slower (see Fig. 2(b)). Observe that for the harmonic oscillator case of $n=1$, there is no relaxation at all and when $n \to \infty$, there is no time evolution [18].



(a)



(b)

Fig. 2. (a) Absolute value of the $1/t$ velocity relaxation of a particle in well described by $V(x) = \frac{1}{2}x^2 + \frac{1}{2}x^4$ at $\beta = 1$. The Fourier transform of the relaxation function (i.e., the velocity power spectrum) is shown in the inset. (b) When $V(x) \sim x^{2n}$, the canonical ensemble relaxation function decays asymptotically as $1/t^{\phi}$, where $\phi = 1/(n-1)$. The figure shows that $1/\phi$ depends linearly upon $(n-1)$.

4.3. Case II: properties of connected anharmonic well systems

It is of interest to study the model of Case I with $V_{ij} \neq 0$ and $B = 0$. A simple case would be to let $V_{ij} = (\frac{1}{2})k_{ij}(x_i - x_j)^2$, i.e., a case where the anharmonic wells are connected via harmonic springs. The full nonlinear dynamical equations for such systems cannot be analytically solved at the present time. Mean field studies were carried out for quartic wells connected via harmonic springs by Krumhansl and Schrieffer [19]. Many special cases of such systems have been studied and may be germane to our discussions here. One such case would be when $V_i = 0$ and V_{ij} is an algebraic power law such as $V_{ij} = k(x_i - x_j)^q$, where $q > 2$. Such a case was originally studied by Fermi, Pasta and Ulam in 1954 [20] and led to the development of many of the modern concepts of solitary waves, which are known to exist in such systems [21]. Solitary waves are to be thought of as energy bundles. Thus, any perturbation imparted to such systems travels resides in the system as nondispersive energy bundles, which remain uncorrupted forever unless, of course, the conditions associated with their existence are altered [22].

Let us consider the dynamics of the above system in the context of a canonical ensemble. Thus, while $B = 0$, we can use the system Hamiltonian to construct a partition function for our system and then define time evolution of the system in the context of linear response with respect to equilibrium at some fixed temperature. Such a system was probed in the mean-field approximation as a model for structural phase transitions by Krumhansl and Schrieffer [19]. The time evolution of particles in such a system is rich and is not exactly solvable. However, it can be shown, via arguments based upon the continued fraction formalism [23] that the dynamics of any particle is strongly affected by the on-site anharmonic potential [24]. The inter-particle interactions largely affect the short-time dynamics of the system and the strengths of the inter-particle interactions compared to the degree of confinement of the particle to the on-site well [24].

4.4. Role of temperature in case II

The study of relaxation behavior of any particle in a system such as Case II is a problem that involves formidable difficulty. As far as I am aware of, no exact or even approximate solution of any relaxation process exists for systems such as these. However, it has been recently argued that the long-time dynamics of particles in such systems are controlled by one-body nonlinear potential wells. Inter-particle interactions play a role in determining the short-time behavior of these systems [24]. Thus, it can be inferred that interactions may play a role in controlling and manipulating short-time and high-frequency behavior in systems such as those described by Eq. (1) with $B = 0$.

4.5. Case III: properties of anharmonic wells with finite heat baths

Let us briefly discuss the case of $V_{ij} = 0$ and $B \neq 0$, i.e., when there is a bath that has the possibility of strongly interacting with the anharmonic oscillator. We consider

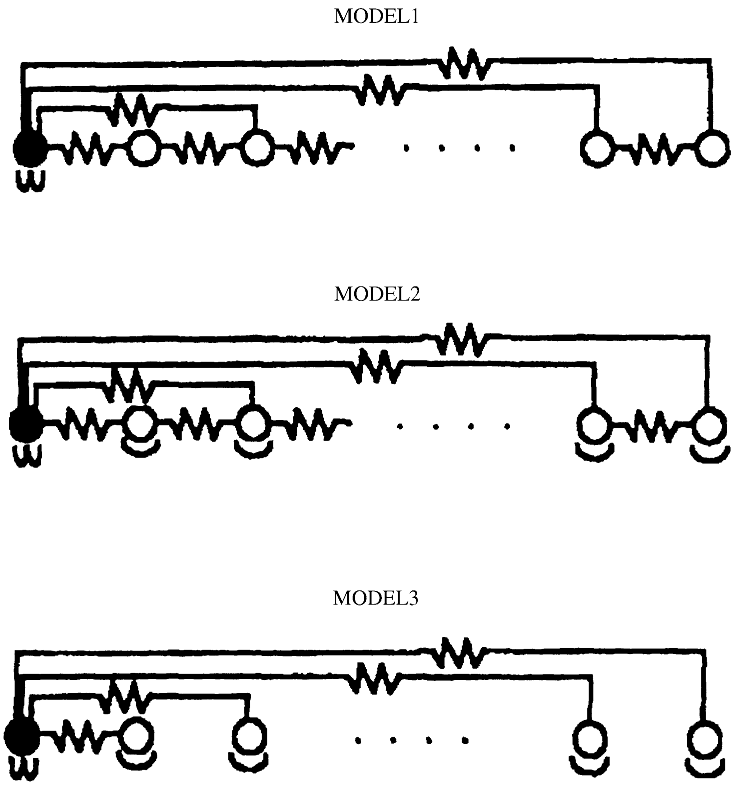


Fig. 3. Cartoons describing the models in Case III in the text. The zigzags denote harmonic springs, the "ω" denotes on-site quartic potential and the "$\cup$" denotes on-site harmonic potentials.

a quartic anharmonic oscillator in $V = (1/2)x^2 + (1/4)x^4$ and connect this oscillator to three distinct models of a heat bath (see Fig. 3). In each case, the heat bath will be regarded as a 1D system. In *model* 1, we let the anharmonic oscillator be connected via harmonic springs to every bath particle and we let every bath particle be connected with each other via harmonic springs. The system energy can be described as

$$E = p^2/2m + V + \sum_{i=1}^{N} p_i^2/2m + \sum_{i=1}^{N-1} (K_i/2)(x_i - x_{i+1} + l)^2$$

$$+ (K/2) \sum_{i=1}^{N} (x - x_i + il)^2 \,. \tag{4}$$

In *model* 2, we further confine each bath particle in a harmonic oscillator well but keep the expression for E otherwise unchanged. The system energy can be described as

$$E = p^2/2m + V + \sum_{i=1}^{N} p_i^2/2m + \sum_{i=1}^{N}(1/2)x_i^2 + \sum_{i=1}^{N-1}(K_i/2)(x_i - x_{i+1} + l)^2$$
$$+ (K/2)\sum_{i=1}^{N}(x - x_i + il)^2 . \quad (5)$$

In model 3, we set $K_i = 0$, and thus consider a case where the bath particles, while confined to harmonic potential wells, do not interact with each other [16]. In each case, we study the dynamics of the models by assigning the anharmonic oscillator a small velocity at $x = 0$. In models 1 and 2 we place the bath particles initially at rest but assign them appropriate positions. We know that in the absence of a bath, the dynamics of the anharmonic oscillator will be dominated by one base frequency at $\omega_0 = 1.0$ (given our choice of parameters in the expressions for E). It turns out that in model 1, the anharmonic oscillator dynamics is strongly affected by the bath and the frequency ω of the oscillator turns out to depend on N and K as follows for large N [16] (see Fig. 4):

$$\omega \propto \omega_0 + (NK)^{1/2} . \quad (6)$$

Thus, the collective effect of the bath oscillators on the dynamics of the anharmonic oscillator is that of a single one which oscillates with a harmonic coupling of NK. This effective "mega-oscillator" adds a "harmonic frequency" to the lowest frequency of the anharmonic oscillator. In model 2, there is a collective bath frequency that completely overwhelms the frequency of the anharmonic oscillator, which becomes a $1/N$ effect. In model 3, one obtains very similar frequency spectrum as in model 2 except that the peak at $\omega_0 = 1.0$ remains robust (is not a $1/N$ effect) but the collective peak also remains as before. In the limit $N \to \infty$, the collective peak upshifts in frequency to ∞ and hence can be disregarded in the sense that the net effect of the bath is to introduce high-frequency contaminants to the system dynamics and these effects can be calculated [16].

In short, finite baths can potentially destroy or "contaminate" learnt information [16]. It is conceivable that processes such as elementary psychosis, which can set in when a subject has not slept for more than 72 h or so, can be modeled as processes in which the lack of sleep raises the noise level associated with the operations of the body such that any learnt information during the later stages of a long period during which the subject is awake can become contaminated.

5. Summary and discussions

In this work, I have proposed that it may be reasonable to assume that instead of using Ising-type spin models to describe processes such as learning [14,15], one can use nonlinear potentials to do the same. The primary advantage of using nonlinear potentials lies in the fact that such potentials can be thought of as generators of infinite number of correlated frequencies. Correlated frequencies can be associated with

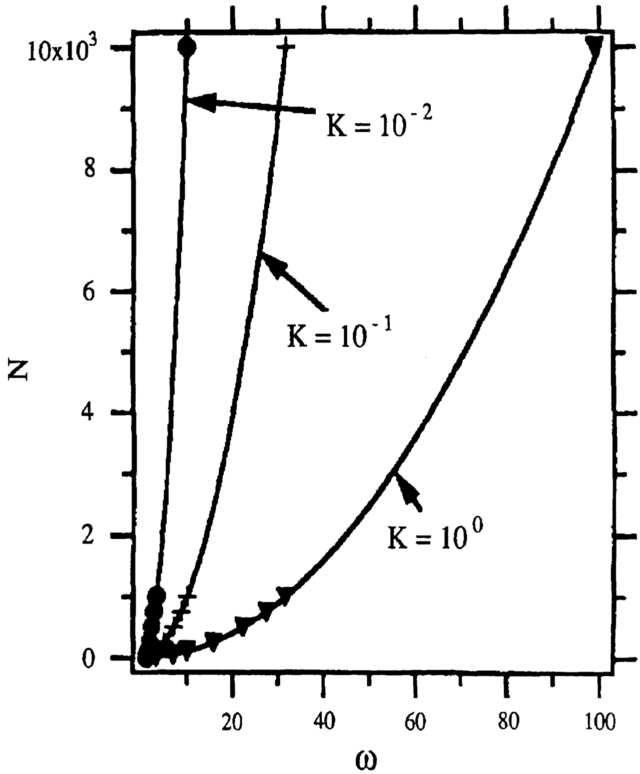


Fig. 4. Dominant frequency ω of the velocity power spectrum for the anharmonic oscillator connected to the bath as described by model 1. The symbols correspond to the simulations done at various sets of (N,K). The fitted curves have the empirical form $N(\omega,K) = -1-(1/K)+(1/10)K^{-0.693}\omega+(1/K)\omega^2$. Observe for large $N, \omega \propto (NK)^{1/2}$.

efficient learning processes. Interactions allow for high-frequency modifications to the frequencies generated by the nonlinear potentials. The effects of heat baths and of noise are to contaminate the correlated frequencies [24]. The analyses can be further parameterized in terms of neuronal frequencies and other variables. The model may have some relevance to information processing, retrieval and storage in the brain. Further, the model has the potential to address issues such as intuition, in which the brain can infer its own "new" correlations based upon categorization and comparison of stored correlated frequencies and based upon modifications of those frequencies due to any external driving factors.

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