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Physica A 299 (2001) 551–558

PHYSICA A

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Thermalizing an impulse

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Received 19 March 2001

Abstract

We study the propagation of an impulse through a finite chain of N elastic beads in which the grain diameters progressively shrink by a factor q. We show that it should be possible to construct "tapered" chains that can effectively thermalize shock waves. © 2001 Elsevier Science B.V. All rights reserved.

PACS: 46.40.Cd; 45.70.−n; 43.25.+y

The propagation of an impulse through a chain of monodisperse elastic grains exhibits interesting physics [1–7]. When the grains in the chain barely touch one another, most of the energy of an impulse that has been initiated at one end of the chain propagates as a solitary wave [1–14]. For a chain of spherical elastic beads, these solitary waves are about three grain diameters wide [13,14]. When the grains in the chain are precompressed, the solitary wave becomes dispersive [8–10,15–17]. When dissipative effects such as restitution and friction are taken into account in a chain of grains in contact at zero external loading, the propagating solitary waves attenuate in amplitude [18,19] although the width remains invariant. Presence of loading allows dispersion of the solitary wave but it does not allow the possibility of distributing the energy of the solitary wave fairly uniformly throughout the system [8–10].

In this communication, we ask the following question: Is it possible to convert an impulse into thermal energy? If such a thing would be possible, one may be able to design structures that will successfully absorb shock waves and convert the energy of the shock wave to heat energy. Such shock absorbing structures can perhaps be

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PII: S0378-4371(01)00340-5

retrofitted to protect bridges, dams, ships, large buildings, roads, automobile bumpers and the like.

To explore the above question, we consider a chain of spherical elastic grains where the grains barely touch each other. We assume that the grain radii progressively shrink by some factor q as one travels along the chain from one end (say, the left end) to the other (say, the right end). The grains interact via the Hertz potential [20]. We describe the energy associated with the repulsive interaction between any two compressed spheres labelled i and $i+1$ of radii R_i and R_{i+1} (while they are uncompressed) as follows: we define the "overlap" between the two adjacent grains by $\delta_{i,i+1}$, which can be described as $\delta_{i,i+1} \equiv R_i + R_{i+1} - z_{i,i+1}$, where we let $z_{i,i+1} \equiv z_{i+1} - z_i$ represent the distance between the centers of the two adjacent spheres. The interaction energy between the granular spheres is expressed as a function of this overlap as follows:

$$V(\delta_{i,i+1}) = \frac{2}{5D}\sqrt{\frac{R_i R_{i+1}}{R_i + R_{i+1}}}\,\delta_{i,i+1}^{5/2} \equiv a\delta_{i,i+1}^{5/2}\,, \tag{1}$$

where the constant

$$a \equiv \frac{2}{5D}\sqrt{\frac{R_i R_{i+1}}{R_i + R_{i+1}}}$$

and where

$$D = \frac{3}{4}\left(\frac{1-\sigma_i^2}{E_i} + \frac{1-\sigma_{i+1}^2}{E_{i+1}}\right)\,, \tag{2}$$

in which σ_i, σ_{i+1} and E_i, E_{i+1} are the Poisson's ratios and the Young's moduli of the two bodies, respectively. The potential energy $V(\delta_{i,i+1}) = 0$ when $z_{i,i+1} > R_i + R_{i+1}$. The $V(\delta_{i,i+1}) \sim \delta_{i,i+1}^{5/2}$ law in Eq. (1) above arises due to purely geometrical effects and is an exact result for perfectly spherical grains (for a detailed derivation of Eq. (1), see Ref. [20]).

We focus on the propagation of an instantaneous perturbation that is initially impacted onto the left most (i.e., the largest) grain of the chain. The equation of motion of a grain of mass m_i at location z_i is given by

$$m_i\ddot{z}_i = a_{i,i-1}[\{\Delta_{i-1,i} - (z_i - z_{i-1})\}^{3/2}] - a_{i,i+1}[\{\Delta_{i,i+1} - (z_{i+1} - z_i)\}^{3/2}]\,. \tag{3}$$

In Eq. (3), we define $\Delta_{i,i+1} \equiv R_i + R_{i+1}$. Given $\Delta_{i,i+1}$, the initial positions of all the grains in the chain are specified. The velocity of the first grain is specified at time $t = 0$. The velocities of all the other grains are initially set to zero. The velocity of the first grain therefore defines the physical properties of the solitary wave that propagates down the chain.

When $m_i = m$, it is well known that an impulse initiates a traveling solitary wave [11,12,21,22]. When the radii of the spherical elastic beads shrink by a factor q, the traveling solitary wave (which is three grains wide) must get squeezed into a smaller size. Given that energy must be conserved, the breakdown of translation symmetry for $q \neq 1$ implies that the solitary wave loses its reflection symmetry and is destroyed. It turns out that the leading edge of the original solitary wave, which sits on progressively lighter grains, accelerates with respect to the trailing edge. The wave therefore progressively broadens in width and modifies in shape. The leading edge of the pulse, which sits on progressively lighter masses, while accelerating, carries progressively less kinetic energy. Thus, the amplitude of the leading edge of the original signal attenuates rapidly in space and time. The trailing edge stretches out farther and farther back. In the following paragraphs, we present results that support the arguments made above and support our observation that "tapered" granular chains can act as shock absorbing structures.

In our simulations, we use the following numerical values to calculate our results. We assume that the spheres are made of lead and we ignore restitution and frictional dissipation. The reason for ignoring restitution, which is quite high for lead, is to make the most conservative estimate of impulse attenuation. In an actual physical system, the attenuation will therefore be more pronounced than that predicted in the calculations here. We assume that Young's modulus E for lead is $16.1\,\text{GPa} = 1.61 \times 10^{10}\,\text{Pa} = 1.61 \times 10^{10}\,\text{N/m}^2$, the Poisson's ratio $\sigma = 0.44$ and the density of lead to be $11\,343\,\text{kg/m}^3$. We assume that the radius of the first lead sphere is 0.15 m and the initial velocity imparted to the largest and leftmost grain to be 0.1 m/s. This impulse leads to a compression of about 10^{-3} in the grain diameters at the left end of the chain. In the dynamical simulations, we set $dt = 5 \times 10^{-7}$ s. The total number of steps across which the calculations have been performed is typically about 4×10^6. Our energy conservation is accurate to 0.04%.

Fig. 1 describes the velocities and the kinetic energies of grains as a function of time for a tapered chain with $q = 0.045$. The small value of q is chosen to properly represent the dynamical evolution of the propagating impulse via a few panels. The tapered chain is 100 grains long. The impulse is initiated in the left end of a 100 grain monodisperse chain, which precedes the 100 grain tapered chain. Fig. 1(a) shows the velocities of grains in the tapered chain (e.g., grain number 100 is the first grain in the tapered chain). The data demonstrates that the leading edge of the wave travels progressively faster than the trailing edge which progressively slows down. Fig. 1(b) shows the dynamics of grains numbered 97, 107, 117 and 127 from left to right as functions of time. The plot for grain number 97 clearly shows a propagating solitary wave [11,12]. The data for grain number 107, 117 and 127 show that the grains initially speed up and then move forward at some steady speed. We terminate the time domain plot at about the time when the leading edge of the pulse traveling through the chain hits the right wall. Acceleration data for grains 107, 117 and 127 (not shown here) indicate that these grains possess zero acceleration during the time periods when the data in Fig. 1(b) for these grains "flatline", thus suggesting that the grains lose contact with one another as the pulse propagates. This is a reasonable result because the lighter masses are expected to move faster than their more massive neighbors to the left. The kinetic energy data shown in Figs. 1(c) and (d) show that progressively less energy is carried by the leading edge. The maximum attenuation achievable clearly depends on q and on the number of grains N in the tapered chain.

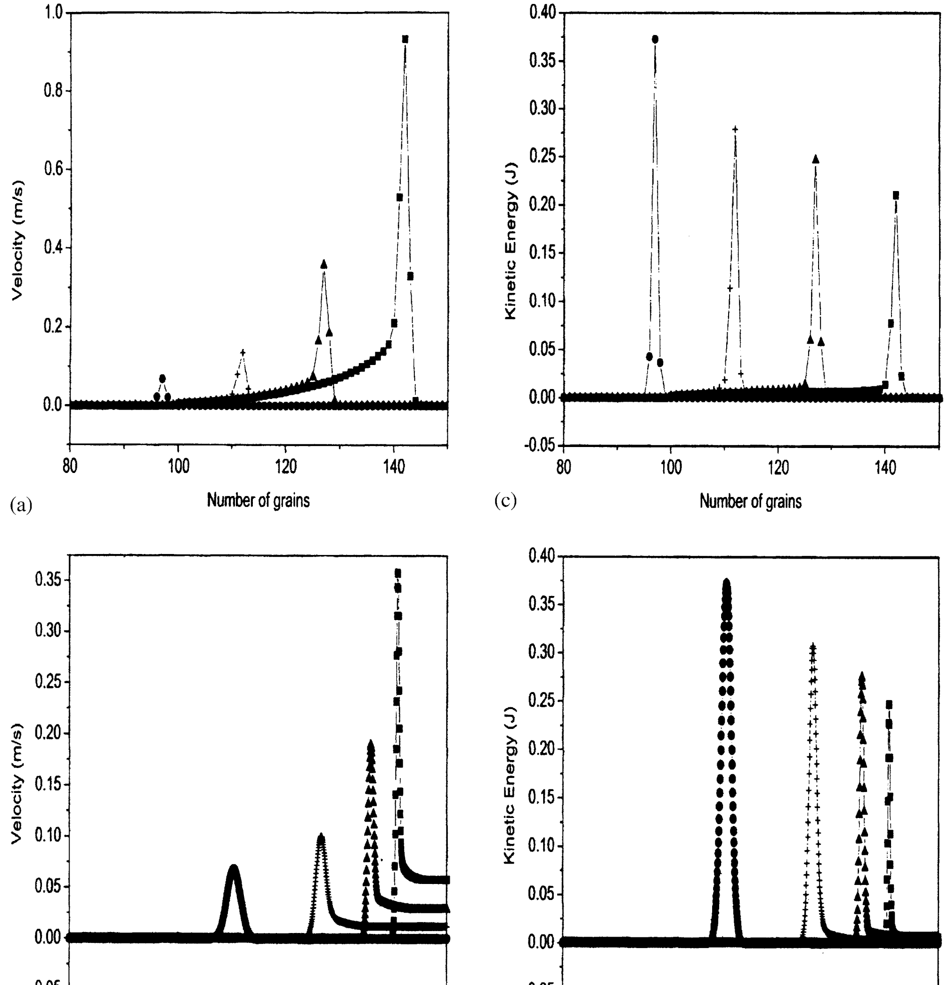

Fig. 1. The data show results for a chain of 200 grains, where the first 100 are monodisperse and the next 100 shrink in diameter by a factor $q = 0.045$. The monodisperse part of the chain coverts the impulse into a well defined solitary wave. (a) Plot of velocity versus grain number at 4 different times, (b) plot of velocity for grain numbers 97, 107, 117 and 127 (from left to right) as functions of time, (c) plot of kinetic energy of the entire system at 4 different time instants showing a decay in the leading edge of the energy and the development of a trailing tail, (d) plot of kinetic energy versus time for grain numbers 97, 107, 117 and 127 (from left to right).

Fig. 2 shows data from calculations carried out for tapered chains of various lengths and of various magnitudes of q. When an impulse is generated at one end of a monodisperse chain, a solitary wave is generated, which propagates down the chain at uniform velocity. For the system with the above mentioned parameters, if the magnitude of

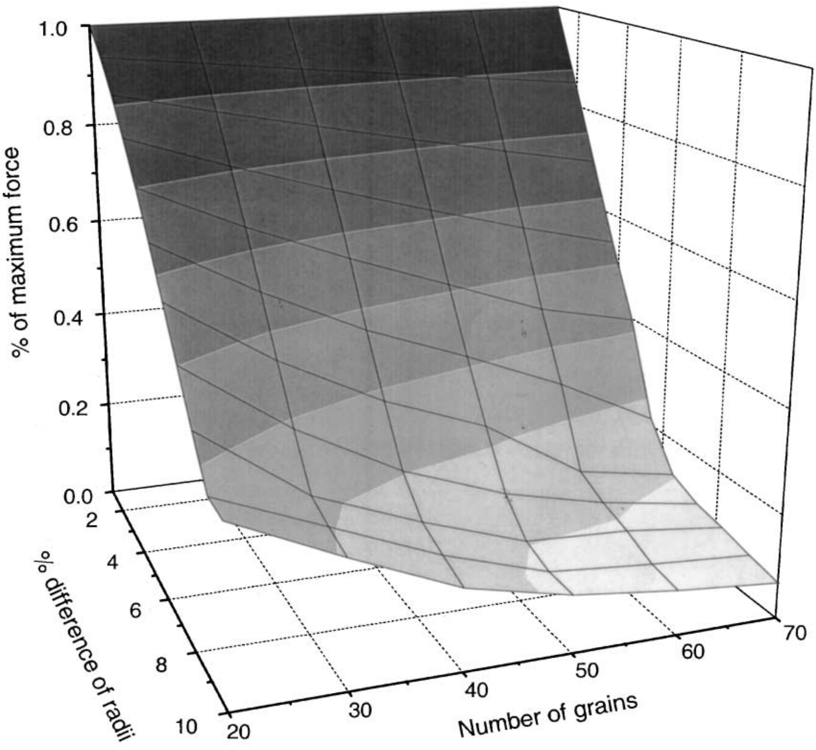

Fig. 2. Plot of fraction of the maximum force recorded at the wall, which is at the end of the tapered chain, for various magnitudes of q and N. Thermalization is achieved when the reduction in the maximum force recorded at the wall is about 90%.

the force is recorded at the other end of the chain, it turns out to have an amplitude of 7640.5 N. If instead of having monodisperse chain one considers a tapered chain, the magnitude of the peak force recorded at the other wall is significantly less. The percentage by which the force at the other end is reduced is shown along the z-axis of Fig. 2. We show that the force recorded at the right (or wall) end is reduced by about 90% when $q = 0.1$ and $N = 70$ (a rather small chain). The behavior of the force versus time recorded at the end wall for such large reductions in amplitude reveal that one finds a steady, noisy pattern at the wall. The pattern suggests that the impulse initiated at the left end of the tapered chain is approximately "thermalized" by the time it reaches the right end in the sense that the energy bundle transmitted into the system by an impulse is reduced to a long-lived, approximately continuous set of hits onto the wall.

To further illustrate the nature of the force recorded at the wall as a function of time, we carried out some lengthy dynamical simulations for various q values using chains of 500 grains. The data are shown in Fig. 3(a)–(f). In Fig. 3(a), we present the force recorded at the end wall as a function of time when the chain is monodisperse. It is

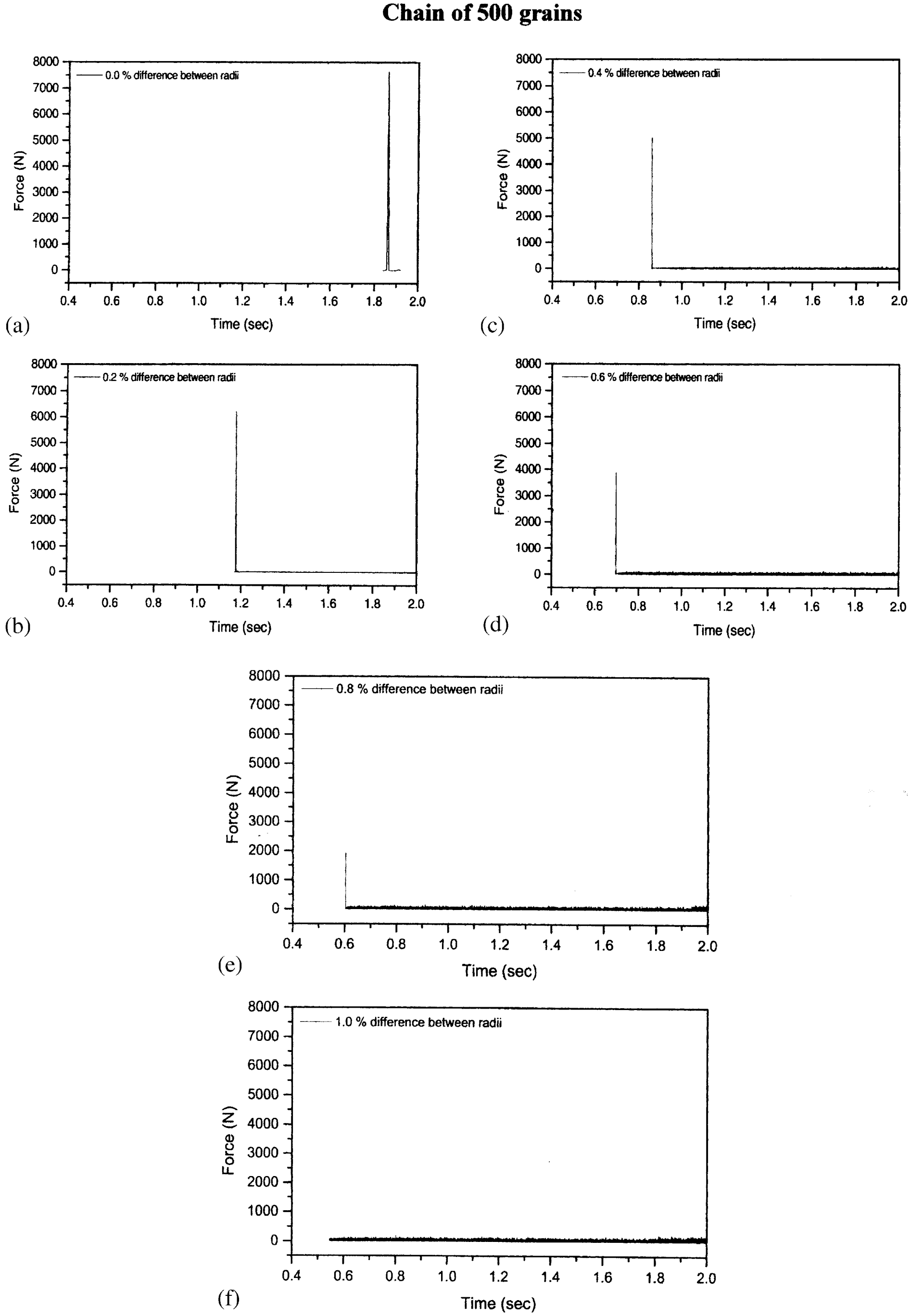

Fig. 3. Data shows the force recorded at the wall, which is at the end of tapered chains of 500 grains. The percentage difference between radii are shown in each case starting from (a) 0%, to (f) 1%. The maximum force recorded at the wall for the monodisperse chain (case (a)) is 7640.5 N while that recorded for the case of the chain in (f) is 196.4 N.

clear that there is no thermalization in this case and a pure solitary wave is transmitted through the system. The amplitude of the first peak recorded at the wall, which is 7640.5 N in Fig. 3(a) shrinks steadily as q is made progressively larger. For the case when $q = 0.01$ in a 500 grain chain (Fig. 3(f)), the first peak is of magnitude 196.4 N and is no longer distinguishable from the rest of the features in the force versus time plot. We contend that the impulse is completely thermalized in the case shown in Fig. 3(f).

In summary, we recall that an impulse generated at the boundary of a monodisperse chain of elastic spheres propagates as a purely solitary wave [1–7,11–14]. When the grain diameters progressively shrink in radius by some factor $q < 1$, the spatial symmetry of the solitary wave is destroyed. The leading edge of the wave travels progressively faster whereas the trailing part of the wave travels progressively slower. For appropriate combinations of q and N, one may attain thermalization of the impulse. Our studies ignore restitution and friction related losses [18,19]. Therefore, our estimate of reducing the amplitude of a shock wave by 90% or more (see for example Figs. 2 and 3(f)) in a purely conservative system study is pessimistic. Larger amplitude reductions should be expected in experimental studies of tapered chains. Further, one should be able to design $3D$ structures with geometrically arranged tapered chains in such a way that most of the energy of a typical shock wave can be absorbed and thermalized. These issues will be discussed in a more elaborate analysis [23].

The research reported here has been partially supported by the National Science Foundation Grant NSF-CMS-0070055 and by the Sandia National Labs of the Department of Energy under Contract No. DE-AC-04-94AL-85000. We thank Dr. Alan J. Hurd for his interest in this research.

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