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**WINNING A BATTLE:  
THE IMPORTANCE OF KNOWING THE “NEIGHBORHOOD”**

LINDA SHANAHAN

*Department of Physics  
State University of New York at Buffalo,  
Buffalo, New York 14260, U.S.A.  
lls2@buffalo.edu*

SURAJIT SEN

*Department of Physics, State University of New York at Buffalo,  
Buffalo, New York 14260-1500, U.S.A.  
sen@dynamics.physics.buffalo.edu*

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We propose that a battle between two conflicting entities can be modeled as a kinematic problem on a lattice. We examine the parameters relevant to such simulations, and describe global patterns emerging from a lattice model of a conflict with a few simple rules. We conclude that models of this type display interesting and relevant properties and warrant more in-depth analysis. The focus in this work is on the role of knowledge of the “neighborhood” in winning a battle.

## 1. Introduction

Can battles between attacking and defending forces be modeled as a simple kinematic problem on a lattice? The short answer is “yes.” Here we seek to determine the parameters relevant to simulating the start, progression, and outcome of such battles. We examine in particular the impact of knowledge regarding the neighborhood on the outcome of the battle and the global patterns emerging from models of struggle, of win and loss on simple lattices. These studies may be germane to the study of processes such as land warfare<sup>1,2</sup>, evolution of diseases in a living entity<sup>3</sup>, or spread of social and/or political and health crises<sup>4</sup>. Can these patterns provide insight into the general progression of diverse processes, such as short-term victory versus long-term loss, the speed with which disease may spread through a body or a population, whether a disease spreads or goes into remission, the role of available intelligence (technology) in determining outcomes, and the effects of differing strategies?

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## 2. Model

### 2.1. General description

We construct a simple physical model consisting of a two-dimensional square lattice or matrix and simple deployment rules. The work derives its inspiration from the pioneering studies of Bak, Dhar and others<sup>5,6</sup> The lattice represents the battlefield, spatial or otherwise, on which conflict takes place. Each opponent begins with a quantitative allotment representing forces and/or resources. Individual attackers and defenders are assigned numeric strength values and deployment strategies which result in occupation of the array sites by the combatants. Casualties result when opposing forces occupy the same site. Combatants do not move on the battlefield, but rather occupy their given sites, engage intruders, and provide field information which may be used for determining further deployments. Battles continue until either the attackers or the defenders have exhausted their reserve forces. We define the outcome as a win for the adversary still retaining forces in reserve (that is, not on the battlefield) after the battle ends. If both sides have essentially deployed all forces to the conflict, the battle is a draw. While this win/loss definition is somewhat arbitrary, it allows for general discussion of results.

### 2.2. Simulation

We use a  $50 \times 50$  square lattice,  $\mathbf{K}$ . Each challenger is allotted an initial reserve of  $S = 500,000$  resource points. The defense begins by occupying the lattice at random sites with resources having a mix of strength values of  $d = +1$  and  $d = +2$  points, at occupation density,  $\mathbf{D}$  (the percentage of occupied sites), so that each  $\mathbf{K}(\mathbf{i}, \mathbf{j}) \rightarrow \mathbf{0}, +1, \text{ or } +2$ . The battle begins with the attackers deploying at random locations on the battlefield with an attack density,  $\mathbf{A}$ , and strength values of  $a = -1$  and  $a = -2$ . The site values are summed with each iteration, resulting in a one-to-one casualty ratio. Sites with values of zero are neutral.

Following the initial attack, some fraction of the sites will have negative values, indicating they are now held by the aggressor. On subsequent iterations, the aggressor and the defender deploy additional resources from their reserves according to the following algorithm.

### 2.3. The Attack model

In taking the aggressive approach, the attacker tries to maximize value by always deploying resources in the direction with the greatest density of defenders pursuant to the available information. A neighbor list is defined by an  $n \times m$  matrix about each attack site,  $\mathbf{K}_A(\mathbf{i}, \mathbf{j})$ . The size of the neighbor list is dictated by the range variable  $\mathbf{r}$ , the percent of the field size in any direction for which information is available at site  $K_A$ . For an  $L \times L$  matrix, the range is  $|\mathbf{r}| = (\mathbf{r}/\mathbf{L})(100)\%$ . The attacker receives information from a neighbor list with maximum size  $(2\mathbf{r} + 1) \times (2\mathbf{r} + 1)$ . The occupation or resource values of all sites in each quadrant of the neighbor list

at  $\mathbf{K}_A(\mathbf{i}, \mathbf{j})$  are summed. If the computed sum is largest in, say, the first quadrant, then site

$$\mathbf{K}(\mathbf{i} - \mathbf{1}, \mathbf{j} + \mathbf{1}) \rightarrow \mathbf{K}(\mathbf{i} - \mathbf{1}, \mathbf{j} + \mathbf{1}) + \mathbf{a}_{\max} \quad (1)$$

(if this site is still within the battlefield) and so on. If adjacent quadrants, for example quadrants I and II, have equal values,

$$\mathbf{K}(\mathbf{i}, \mathbf{j} + \mathbf{1}) \rightarrow \mathbf{K}(\mathbf{i}, \mathbf{j} + \mathbf{1}) + \mathbf{a}_{\max} \quad (2)$$

If opposing quadrants (i.e. I and III or II and IV) or if three or all four quadrants of the neighbor list have the same value, there is no new deployment from that site during that particular iteration (however, this case is rarely realized). The rationale for using maximum strength values for the attackers is their strategic need to maximize effectiveness. The battlefield is not updated until all such decisions have been made.

The defense then deploys reinforcements to the battlefield, again randomly with values  $d = +1$  and  $d = +2$  at the occupation density  $\mathbf{D}$ , such that for any reinforced site

$$\mathbf{K}(\mathbf{i}, \mathbf{j}) \rightarrow \mathbf{K}(\mathbf{i}, \mathbf{j}) + \mathbf{d} \quad (3)$$

### 3. Results

We present herein some preliminary results from the aggressive attack model, to illustrate the emergent patterns and stability issues. Of particular interest were the nature and patterns of sites held by defense and attack forces during and after the battle for differing range or neighbor list sizes. We also considered the stability of the win/loss outcome for different values of these variables in terms of dependence on the random number generator of the software.

In a medium range battle, the aggressor deploys resources based on a neighbor list of nearby, but not too-distant, sites. The intelligence information allows for coordination of the attack from differing directions. An information-driven pattern of attack site clusters emerges with cluster dimensions on the scale  $\mathbf{r}$ , indicating that most newly deployed forces remain within a localized area securing a region of sites within that area. Sites held by the attackers which share the same information act so as to reinforce each other. Figure (1) shows the results of a battle with  $\mathbf{D} = 50\%$ ,  $\mathbf{A} = 15\%$ ,  $\mathbf{r} = 26\%$ , which resulted in a draw.

At large range (we used up to  $\mathbf{r} = 50\%$ ) the attack deployment is influenced by information from far distant sites, resulting in a mix of clusters and strings occupied by the aggressor. Overlapping and competing information results in more random, less efficient, deployments. Since the neighbor list includes more sites towards the center of the field, the aggressors will tend to move away from the edges of the field, which will be dominated by the defense. The aggressors spread resources far from the original attack sites, which may (being less strongly guarded) subsequently be recaptured by the defense. Despite having an apparent lack of strategy, the

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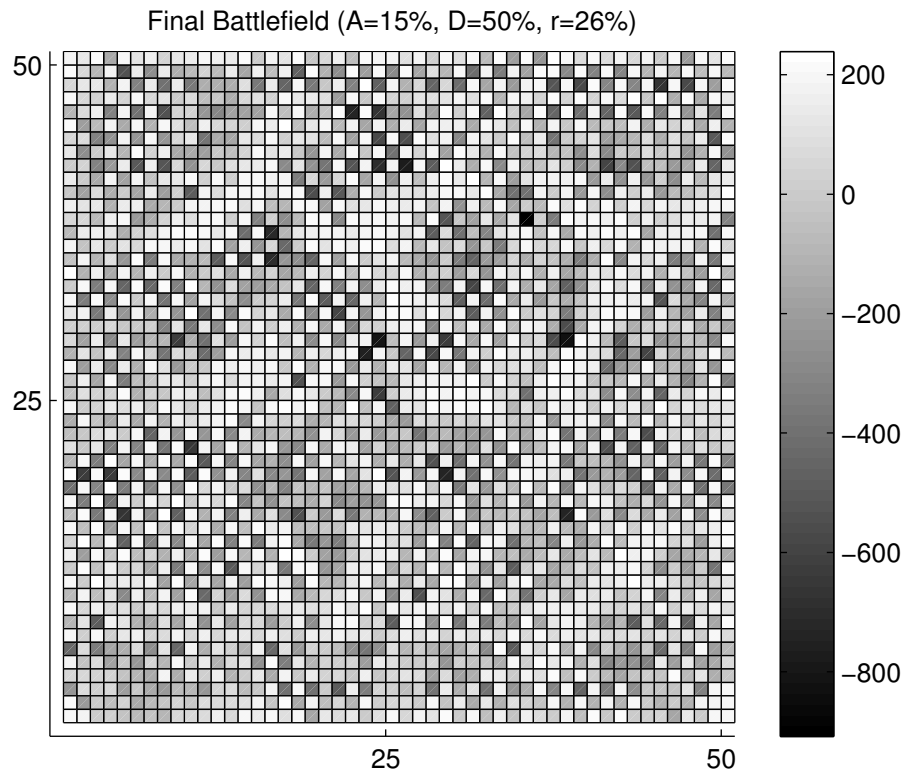


Fig. 1. Clusters formed in medium range aggressive battle, quantity of resources at each site

defense may gain the advantage and actually win the battle, even at very low attack densities. Notably, the sophisticated technology implied by larger communication distances has limited value if too-distant data is used in the decision making. Figure (2) illustrates another draw with  $\mathbf{D} = 50\%$ ,  $\mathbf{A} = 15\%$ ,  $\mathbf{r} = 50\%$ .

The stability of the outcomes in terms of defense win, attack win, or draw given the same initial parameters decreases with increasing range,  $\mathbf{r}$ , and with increasing attack density,  $\mathbf{A}$ . Figure (3) shows stability diagrams for the aggressive model, with several different values of  $\mathbf{D}$ . Each diagram represents the results of testing the model once for each input data set, with  $\mathbf{A}$  varying from 5% to 100% of the field in increments of 5%, and  $\mathbf{r}$  varying from 2% to 48% in 2% increments, for a total of 240 battles. In these figures, the three lightest shades represent values for which the outcome varies with the random generator between defender, attacker, and draw. Darker regions represent values for which the win goes consistently to

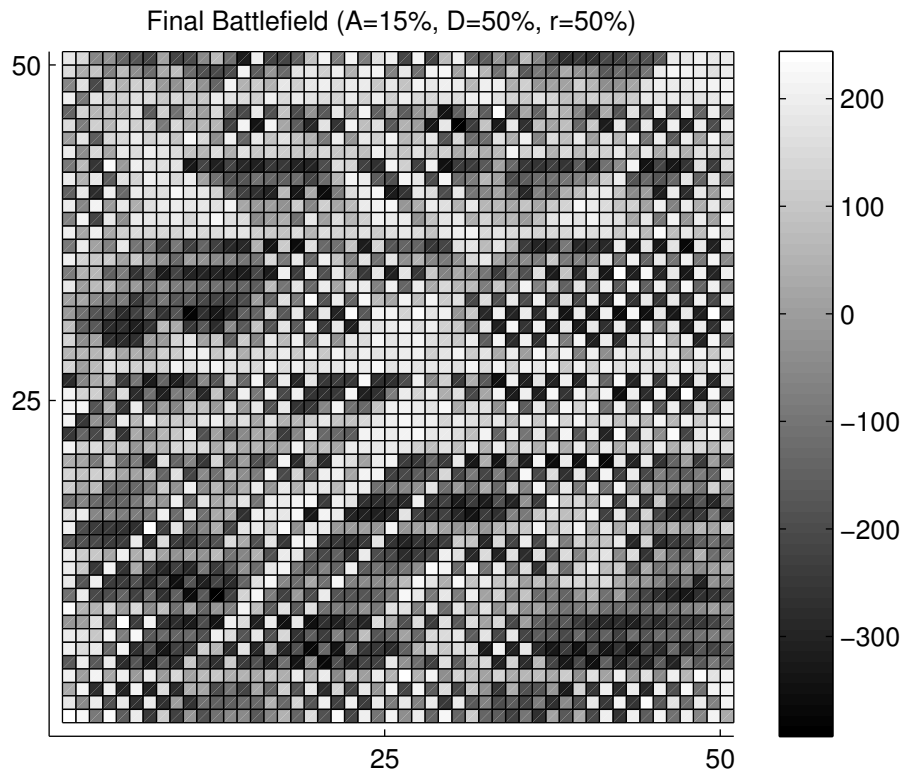


Fig. 2. Long range aggressive battle, quantity of resources at each site

the attackers.

#### 4. Conclusions

We examined the time evolution of battles for a variety of input values and found interesting complex adaptive behavior at many levels. In general, the initial attack and defense positions were important in determining final positions and outcomes. The value of an attack plan versus a random approach was also clear. Small but cohesive (connected) groups of attackers were able to make inroads against large deployments of defenders. The availability and use of information from neighboring sites was a key parameter in determining outcomes. Defense wins or draws were more likely when the attacker becomes too dispersed attempting to respond to conflicting or competing information.

Our initial findings suggest that such lattice models exhibit significant richness

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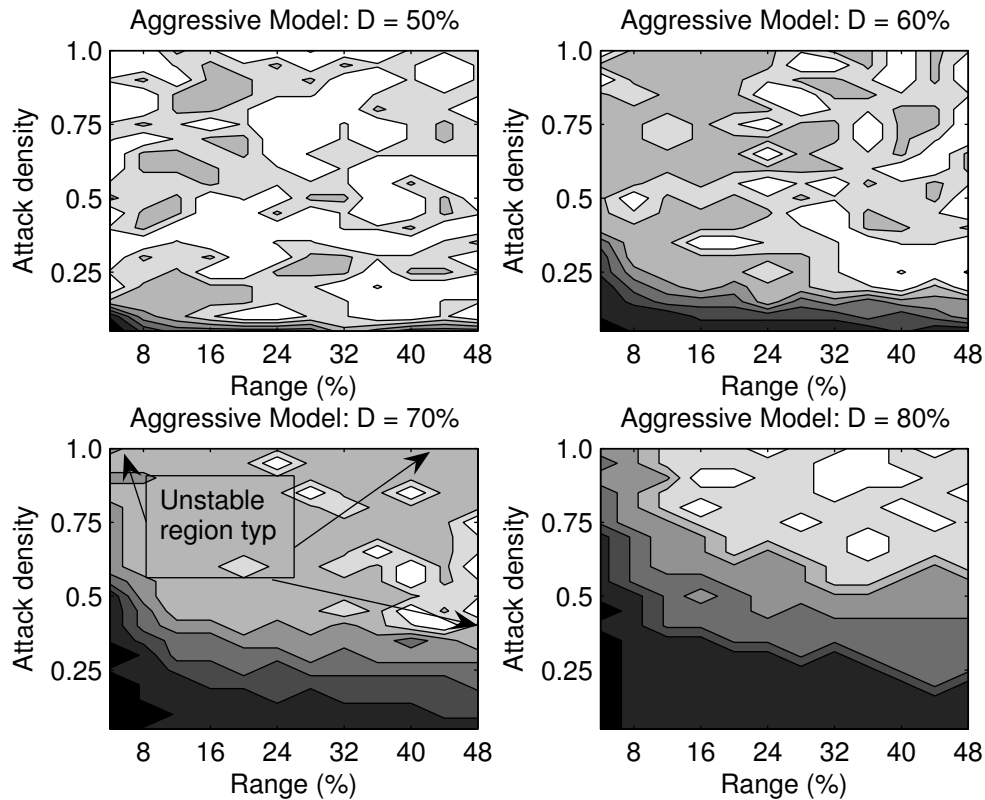


Fig. 3. Outcome stability contour plots for aggressive model, varying defense densities

meriting further investigation and development of this type of battle model.

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