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MECHANICAL ENERGY PROPAGATION IN GRANULAR ALIGNMENTS: BASIC PHYSICS AND POTENTIAL APPLICATIONS

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ABSTRACT
We consider an alignment of elastic grains where the individual grains repel upon contact according to the nonlinear Hertz potential. We further assume that the alignment is under zero loading, i.e. the grains barely touch one another when any perturbation is initiated. We show that energy transport in these systems is unique, supports a new kind of solitary waves, and potentially affords a new way of looking at energy exchange between particles in strongly nonlinear systems. The properties of these solitary waves and the consequences of interactions between the solitary waves are presented. Few potential applications of the physics are mentioned.

INTRODUCTION
Solids, liquids and gases can often be found in the equilibrium state. Consider water in a bottle, for instance. If we slightly perturb the bottle, the water quickly settles at its original level, no clue of the perturbation can be seen after the water level has settled. The temperature of the system can be accurately measured to great accuracy by an ordinary thermometer. An equilibrium state is one that does not depend upon the initial conditions, one in which the particles both individually and collectively show a Gaussian distribution of velocities and in which the kinetic energy of the system is approximately equally partitioned among the constituent molecules (the equipartition theorem), which in turn sets the temperature of the system [1].

What allows the equipartitioning of energy? Typically, a perturbation initiated at some initial time $t = 0$ spreads throughout the system. This spreading happens in a way such that energy sharing is readily possible between the particles in the system. Thus, eventually, all particles end up sharing the system's energy more or less equally over time [2-4].

What if the interactions in a system are such that the exchange of energy between the individual particles is somehow not easily possible? What if the energy exchange must happen between small groups of particles? Each particle will then not end up having more or less the same energy over time. Thus, the temperature of such a system may not be well defined. Yet conceivably, the particles may still have Gaussian distribution of velocities and reach a state that could be independent of initial conditions. Such a system would then exhibit an equilibrium-like state that would have large temperature fluctuations and would be different than the equilibrium state that we typically encounter in conventional solids, liquids and gases [5-6].

Indeed there are systems in which energy exchange happens between groups of particles. A small impulse propagating through an alignment of elastic spheres is one such system. There may be other examples, such as chains with algebraically nonlinear inter-particle interactions, e.g., with purely quartic interactions [7]. There may even be biologically relevant systems that realize such nonlinear chains. In what follows, we briefly discuss one of these systems.

INTERACTIONS IN STRONGLY NONLINEAR SYSTEMS
Let us consider two adjacent elastic spheres i and $i+1$ made of the same material (neither the shape of the grains nor the identity of the materials associated with them are necessary assumptions) in mutual contact. When these spheres with radii R_i and R_{i+1} are squeezed

against each other such that they are $x_{i,i+1}$ apart, they obviously repel. This repulsive potential $V(\delta_{i,i+1})$, where the overlap $\delta_{i,i+1} \equiv R_i + R_{i+1} - x_{i,i+1}$ is described by what may be called the generalized Hertz law (Hertz law typically refers to the case of spheres) [8], which states that

$$V(\delta_{i,i+1}) = \frac{2}{5D}\sqrt{\frac{R_i R_{i+1}}{R_i + R_{i+1}}}\delta_{i,i+1}{}^{n} \qquad \text{(Eq. 1)}$$

where for spherical grains one finds $D = (3/2)(1-\sigma^2)/Y$, σ and Y are the Poisson ratio and the Young's modulus of the material, respectively. The potential vanishes when the spheres are not in contact. For spheres, $n = 5/2$. The Hertz potential is softer than the harmonic potential for sufficiently small overlaps and is steeper than the harmonic potential for sufficiently large overlaps. It is difficult to solve the Newton's equations of motion for any granular alignment with the generalized Hertz interaction. Nesterenko solved the dynamical equations in the long wavelength approximation where he assumed that the length scale of any propagating excitation far exceeds the grain diameter [9-12] and reported that an impulse propagates as a solitary wave in these systems. Later, Sen and Manciu constructed an improved solution of the solitary wave that removes Nesterenko's long wavelength assumption [13]. However, both the solutions are only capable of describing the system dynamics infinitely far from boundaries, i.e. they are stationary state solutions.

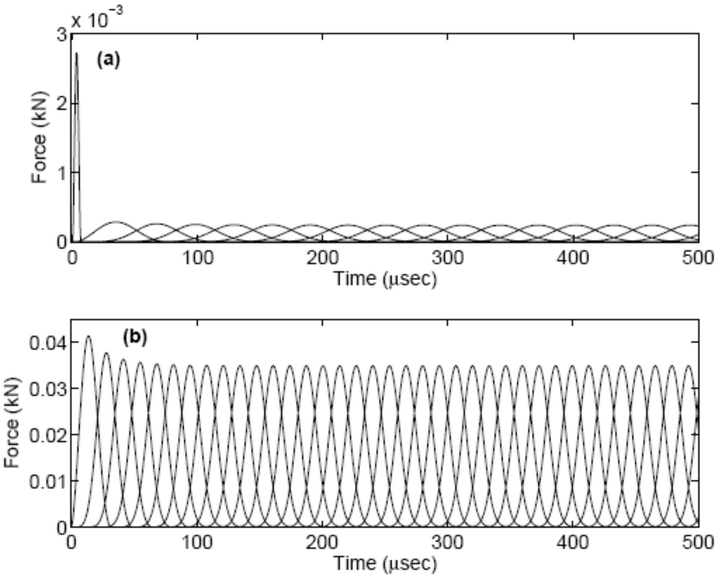

Figure 1.- Panels (a) and (b) show solitary wave formation in a chain of stainless steel beads as a function of time. The first line at the left represents force felt by the edge grain, where the impulse has been initiated, the second line is for the second grain and so on. Solitary wave formation time is independent of the amplitude of the initial perturbation.

The upshot of the existing analyses is that energy transport from one grain to another inevitably starts off slowly (due to the softness at short range) in time and then speeds up (due to the

Table 1.- Grain displacements and velocities in a solitary wave for $n = 5/2$

Grain No. i	u_i	v_i
1	1.00000	8.82354×10^{-6}
2	0.99906	8.85046×10^{-3}
3	0.093457	0.32838
4	0.049829	1.00000
5	0.0064320	0.32414
6	9.10110×10^{-5}	8.59780×10^{-3}
7	5.29234×10^{-8}	8.38470×10^{-6}

steepening of the potential) and ends abruptly. Detailed simulations for all $n > 2$ reveal that energy transport is in bundled form in these systems. In other words, these systems necessarily transport energy via solitary waves. The width of the solitary waves depend upon the magnitude of n. For $n = 5/2$, the width is about 7 grain diameters wide [14-15] as shown in Table 1. As $n \to 2$, the width diverges. As $n \to \infty$, the width tends to one grain diameter. The width of the waves depend only upon n. Further, solitary waves that carry more energy move faster (see Figure 1). There are no sustained oscillations exhibited by any grain in the course of energy

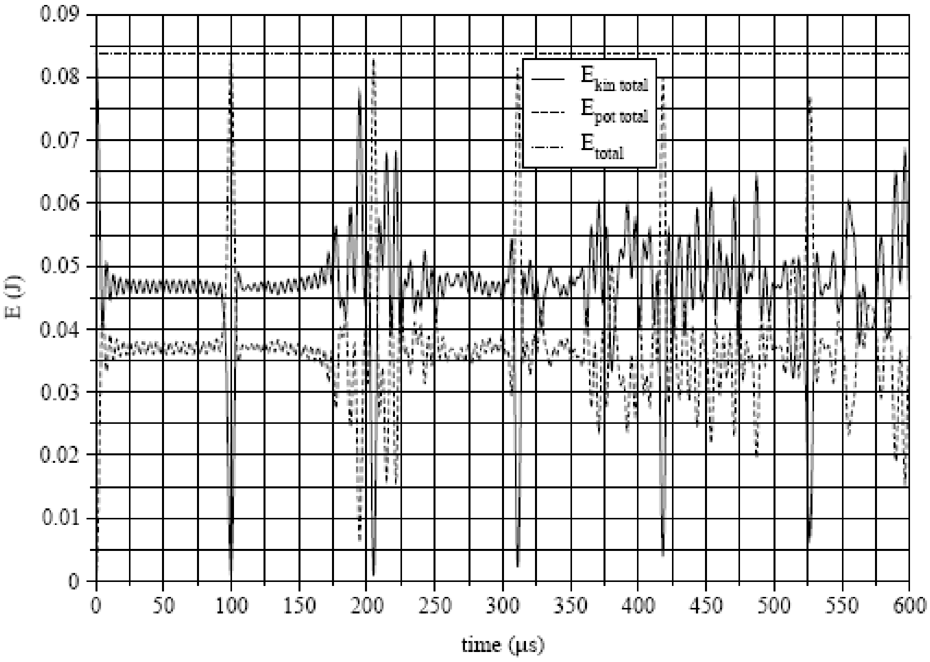

Figure 2.- Total kinetic and potential energies of a system of elastic steel spheres between rigid walls are shown. Observe that a solitary wave forms by the time $t \approx 25\mu s$, wall collision happens at $t \approx 100\mu s$. Repeated collisions between solitary waves and between solitary waves and walls leads to quasi-equilbrium phase at $t >> 600\mu s$.

transport from one grain to the next (Figure 1). This means that the grain displacement is not sinusoidal but is better represented by a $1 - \tanh[f(x_i)]$ function [13] and therefore sound propagation is not possible in these systems. Nesterenko has aptly described granular alignments as examples of sonic vacua [9-12].

SOLITARY WAVE COLLISIONS

One typically does not deal with infinitely long alignments. Thus, boundaries are important. Let us now consider a chain placed between two rigid walls. If a pulse excitation is started at one end of the alignment, one would expect that it takes the system some space and time to form a solitary wave [14-15]. Our extensive dynamical analyses confirm this expectation. In a system of spherical grains it typically takes a length scale of 10-15 grain diameters to form solitary waves. If we assume that initially all energy is kinetic and is carried by a single grain, subsequent grain-grain compressions allow the conversion of part of this kinetic energy into potential energy. Eventually the system reaches the preferred energy distribution between kinetic and potential energies that is dictated by Eq. (1) and the Virial theorem [16], which states that average kinetic energy of the system is $nE/(n+2)$ where E is the system energy. The length scale of 10-15 grain diameters is the range within which the preferred energy distribution sets in and the solitary wave forms. When $n < 5/2$, the solitary waves take a larger distance to form. On the other hand when $n > 5/2$ the solitary waves form across shorter length scales. Once formed, the grains that carry the solitary wave at any given time instant carry fixed portions of the total kinetic and potential energies carried by the wave. It is needless to say that the wave is a symmetric object about its geometric center. Given the discrete nature of the underlying alignment, the solitary waves in granular alignments must always have a length which is an odd multiple of a grain diameter.

What happens when a solitary wave collides with a boundary or with each other? In continuum media, solitary waves pass through each other. Solitary waves in granular alignments are unique in the sense that when a solitary wave collides against a rigid wall, the grain at the very "front" of the wave turns around and moves in the opposite direction. However, the second to the very first grain would still be moving towards the wall when the leading grain has just turned around. Thus, during the process of a wall collision, a solitary wave must break down and subsequently reform. Given the imperfect collision that any solitary wave must suffer against a wall, the reformed waves carry less energy than the original wave that hit the wall. Further, since we are talking only about energy conserved systems, the remaining energy is used to make solitary waves of smaller amplitude that carry less energy and subsequently move slower compared to its parent wave [17-18]. We call these waves secondary solitary waves (see Figure 2). One can now see that the above arguments remain valid whether a solitary wave collides with a wall or with one another. Thus, secondary solitary waves must form when a solitary wave bearing system is placed within fixed boundaries. The softness of the boundaries, the detailed geometrical properties of the alignment (e.g., number of grains in the alignment, whether there are even or odd numbers of grains), grain shapes, and material properties of the grains play roles in determining the energies carried by the secondary solitary waves. There is no question, however, that once the original waves are lost, the system never can return to that original state. The formation of secondary solitary waves as a by-product of solitary wave-wall collision has now been experimentally observed by Job et al. [19]

THE QUASI-EQUILIBRIUM STATE

Thus, the presence of boundaries leads to the formation of solitary waves with various energies, moving at a distribution of velocities. Typically, we find that the grains in a confined granular alignment exhibit a Gaussian distribution of grain speeds, a result that is consistent with the

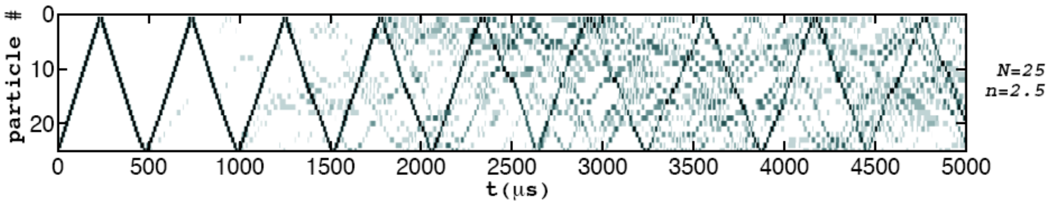

Figure 3.- Onset of quasi-equilibrium is shown in a system with 20 steel spheres held between rigid walls. The grey scale provides a visual representation of the kinetic energy fluctuations. If equipartition is reached, all fluctuations would vanish and the background of the screen will appear completely grey.

Central Limit Theorem in elementary statistics. In addition to the breaking down of the solitary waves, it should be noted that two solitary waves that are traveling in the same direction with comparable velocities can reinforce one another and lead to the formation of a solitary wave of larger amplitude. Eventually therefore a steady state must be reached by the system, where at any given instant of time, one finds a large number of solitary waves, of a large variety of sizes, propagating through one or the other direction in the chain. In this steady state we find that the energy of the system fluctuates significantly against what could have been an average kinetic energy per grain [20]. While there are a variety of ways to characterize such fluctuations, the simplest and the most effective way is visual and this is shown in Figure 3.

NONLINEAR WAVE ENGINEERING

We have focused above on the basic physics associated with mechanical energy propagation in granular alignments. It turns out that granular systems afford us with a wide range of fascinating applications and we mention two in closing.

Nanoscale Inkjet Printers: A particular kind of ferrofluid consists of 10-15 nanometer sized grains of γ-Fe_2O_3 suspended in water. When this ferrofluid is placed in certain porous materials such as the commercially available Anapores, which are structures with vertical bores with bore diameters that can be as small as 15-20 nanometers or so in a vertical magnetic field, the grains align to form vertical chains in the pores. If energy pulses are delivered at the base of the system, an impulse can be initiated at the base of each chain in each Anapore. These pulses would travel up the chains as solitary waves and can be made to carry enough energy to overcome the surface tension of the water and hence can be ejected out of the ferrofluid filled Anapores. The magnetic nanoparticles which carry a coating of the liquid layer as they leave can then be used to make nanoscale droplets of ink that can subsequently be embedded onto any surface. Calculations have been carried out to demonstrate the feasibility of the nanoscale inkjet printer [21]. The concept can be used in a variety of different kinds of ferrofluids and colloids with related properties. However, to our knowledge, experiments remain to be performed.

Tapered granular chains: In these systems, the grain radii shrink progressively in some uniform way along any direction of the chain. Regardless of which direction the radii shrink in, impulse propagation through tapered chains involve energy transport through size mismatched grains, which in effect break down the propagating energy pulse. The system hence turns out to be an efficient and scalable shock absorber. A great deal of work has been performed to characterize the properties of granular shock absorbers [22-25]. Recent experiments show that these scalable shock absorbers can be useful in a variety of daily life applications [26].

CONCLUSIONS

In summary, we have shown that impulses travel through granular alignments as solitary waves. The widths of these waves can be manipulated by controlling a variety of factors such as the system size, grain shapes and material properties, nature of boundaries and so on. Granular alignments placed within confining boundaries possess the potential to exhibit a novel equilibrium like phase where initial condition dependence is lost, the grains possess a Gaussian distribution of speeds but equipartition theorem is violated.

The nonlinear mechanical energy propagation properties of granular alignments open up the possibility of many engineering applications. We have mentioned the ferrofluid nanoscale inkjet printer and the tapered chain shock absorbers as two realizations of such engineering applications.

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References: [1] C. Thompson: Equilibrium Statistical Mechanics. Pergamon, Oxford (1987)

[2] S. Sen: A tutorial on solving the Liouville equation. Physica A **360** (2006) 304-324

[3] S. Dattagupta: Dynamical Correlations. Academic, New York (1986)

[4] S.W. Lovesey: Condensed Matter Physics. Benjamin-Cummins, Reading (1980)

[5] S. Sen, J.M.M. Pfannes, T.R. Krishna Mohan: The quasi-equilibrium state: a tale of certain soundless systems. Journal of the Korean Physical Society. **46** (2005) 571-573

[6] S. Sen, M. Manciu, J.D. Wright: Soliton-like pulses in perturbed and driven Hertzian chains and their possible applications in detecting buried impurities. Physical Review E **57** (1998) 2386-2397

[7] T.R. Krishna Mohan, S. Sen: A new equilibrium phase in discrete nonlinear chains. Pramana – Journal of Physics 64 (2005) 423-431

[8] H. Hertz: Über die berührung fester elstischer körper. J. für reine und angewandte Mathematik **92** (1881) 156-171

[9] V. Nesterenko: Propagation of nonlinear compression pulses in granular media. Journal for Applied Mechanics and Technical Physics **5** (1983) 733-743

[10] A.N. Lazaridi, V.F. Nesterenko: Observation of new types of solitary waves in one dimensional granular medium. Journal of Applied Mechanics and Technical Physics **26** (1985) 405-408

[11] V. Nesterenko: Solitary waves in discrete media with anomalous compressibility and similar to “sonic vacuum.” Journal de Physique IV, Colloquium C8, supplement au Journal de Physique III, **4** (1994) C8-729-C8-734

[12] V.F.Nesterenko, A.N. Lazaridi, E.B. Sibiryakov: The decay of solitary at the contact of two "Acoustic Vacuums." Journal of Applied Mechanics and Technical Physics, **36** (no.2) (1995) 166-168

[13] S. Sen, M. Manciu: Solitary wave dynamics in generalized Hertz chains: an improved solution of the equation of motion. Physical Review E **64** (2001) 056605
[14] A. Sokolow, E. Bittle, S. Sen: Formation of solitary wave trains in granular alignments. Europhysics Letters **77** (2007) 24002
[15] S. Job, F. Melo, A. Sokolow, S. Sen: Solitary wave trains in granular chains: experiments, theory and simulations. Granular Matter (2007) in press
[16] R. Doney: Nonlinear dynamics in tapered granular chains. PhD Thesis. SUNY-Buffalo (2007)
[17] M. Manciu, S. Sen, A. Hurd: Crossing of identical solitary waves in a chain of elastic beads: Physical Review E **63** (2001) 0116614
[18] F. Manciu, S. Sen: Secondary solitary wave formation in Hertz chains. Physical Review E **66** (2002) 016616
[19] S. Job, F. Melo, A. Sokolow, S. Sen: How Hertzian solitary waves interact with boundaries in a 1D granular medium. Physical Review Letters **94** (2005) 178002
[20] E. Avalos, R. Doney, S. Sen: Interaction of solitary waves in confined granular alignments and the quasi-equilibrium state. Chinese Journal of Physics (2007) in press
[21] S. Sen, M. Manciu, F. Manciu: Ejection of ferrofluid grains using nonlinear acoustic impulses: Applied Physics Letters **74** (1999) 1479-1481
[22] S. Sen, F. Manciu, M. Manciu, Thermalizing an impulse. Physica A **299** (2001) 551-558
[23] R. Doney, S. Sen: Impulse absorption by tapered horizontal alignments of elastic spheres. Physical Review E **72** (2005) 041304
[24] A. Sokolow, J. Pfannes, R. Doney, M. Nakagawa, J. Agui, S. Sen" Impulse absorption in small tapered chains. Applied Physics Letters **87** (2005) 254104
[25] R. Doney, S. Sen: The decorated tapered, tapered, highly nonlinear granular chain. Physical Review Letters **97** (2006) 155502
[26] F. melo, S. Job, F. Santibanez, F. Tapia: Experimental evidence of shock mitigation in a Hertzian tapered chain. Physical Review E **73** (2006) 041305