Absorption of Short Duration Pulses by Small, Scalable, Tapered Granular Chains

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Abstract

Making shock proof layers is an outstanding challenge. Elastic spheres are known to repel softer than springs when gently squeezed but develop strong repulsion upon compression and the forces between adjacent spheres lead to *ballistic-like* energy transfer between them. Here we demonstrate for the first time that a *small alignment* of progressively shrinking spheres of a strong, light-mass material, placed horizontally in an appropriate casing, can absorb ~ 80% (~90%) of the incident force (energy) pulse. The system can be scaled down in size. Effects of varying the size, radius shrinkage and restitutive losses are shown via computed "dynamical phase diagrams."

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Shocks concentrate significant amounts of mechanical energy into small regions of space across short times [1-5]. There is no accepted way to shock-proof a surface. A proposal involves the use of functionally graded materials (FGMs) [6] where the system density is varied to alter the elastic properties. Another idea is to arrange for an incident shock to travel along an alignment (e.g., horizontal) of elastic spheres, where the spheres progressively shrink in radius [7-8].

Let us take two spheres of radii R_1 and $R_2 = R_1(1-q)$, $0 < q < 1$, of some material of density ρ. Let the sphere with R_1, velocity v_1 collide with the sphere with R_2, which is at rest. Momentum conservation reveals $(1-q)^{-3} = v_2/(v_1-v_1')$, where v_2 is the velocity acquired post collision by R_2 and v_1' is the recoil velocity of R_1. As $q \to 1$, $v_2/(v_1-v_1')$ diverges. Thus, when a massive sphere hits a smaller sphere, only part of the kinetic energy (KE) is transferred and $v_2 >> (v_1 - v_1')$. Any progressive reduction in radii of successive spheres results in break down of propagating energy through collisions. Hence, a tapered chain (TC) with several hundred spheres should "absorb" a shock wave [7-8].

It would be desirable to explore much smaller alignments of elastic spheres to achieve desired shock absorption. One may then use many embedded TCs on a wall to protect the same from shocks. Here, *we discuss the physics of small TCs with shock absorption properties that are comparable to TCs with several hundred spheres* [7].

Using *hard spheres*, KE and momentum conservation laws [7-8], we find that in a TC of N spheres, the ratio of KE of the smallest grain (KE_{out}) to the same for the largest grain (KE_{in}) is

$$KE_{out}/KE_{in} = \{4(1-q)^3/[1+(1-q)^3]^2\}^{N-1}, \quad q << 1. \qquad (1)$$

Eq. (1) shows how shock absorption in a TC depends upon N. The nonlinear dynamics of grains with soft sphere potentials [9-11], held in horizontal alignment between fixed boundaries, is a challenge to solve.

We focus now on the ability of a TC with $N = 20$ to absorb a δ-function impulse incident at the large sphere and probe the force that will be experienced by the smallest grain as it collides with a *rigid* end wall. Such calculations require accurately solving the Newton's equations [12] for a dissipative [13], *strongly nonlinear* system *across many decades in time*. The force calculations could be a useful guide for experimental measurements of forces felt at a force sensor placed at the tapered end of the system.

Let the spheres be held within walls at zero external loading, that is the grains are not squeezed to begin with. The spheres interact via the repulsive Hertz potential [11]. The Hertz potential is defined in terms of the overlap between adjacent spheres with radii R_i and R_{i+1}, i.e., $\delta_{i,i+1} \equiv R_i + R_{i+1} - x_{i,i+1} \geq 0$, where $x_{i,i+1} \equiv x_{i+1} - x_i$ refers to the distance between the centers of the spheres. The potential energy between neighboring spheres in contact is given by [11]

$$V(\delta_{i,i+1}) = a_{i,i+1}\delta_{i,i+1}^{5/2}, \qquad (2)$$

and is zero otherwise. In Eq. (2), $a_{i,i+1} = (2/5D)(R_iR_{i+1}/[R_i+R_{i+1}])^{0.5}$, $D = 3(1- \sigma^2)/2Y$, σ is the Poisson ratio and Y is the Young's modulus. We assume that the spheres are made of Ti-6Al-4V (a hard material) with density $\rho = 4.42$ mg/mm^3, $\sigma = 0.34$ and $Y = 110$ Gpa. As we shall see, our results in Figures 1 (a,b), which report the system properties, are insensitive to the choice of materials. The radius of our largest sphere is 5.0 mm. We introduce dissipative effects [13] via the restitution coefficient which reduces the force between the grains during decompression ($F_{decompression}$) and compression ($F_{compression}$) as follows $F_{decompression}/F_{compression}$ = 1-w, where $w << 1$.

The equations of motion for each grain are solved via the Verlet Velocity algorithm [12]. The initial conditions are defined by assigning a velocity v_1=10m/s at $t = 0$ to the largest sphere, $v_i = 0$ ($i > 1$). Larger velocities, resulting in significant compressions between adjacent grains (~ 5% of grain radii) have also been employed and approximately the same results as in Figures 2 obtained. The time step of integration, $\Delta t = 10^{-5}$ μsec, with runs across ~ 5 x 10^3 μsec. Energy conservation holds up to eight decimal places when $w = 0$.

The force recorded at the tapered end is computed by calculating the momentum changes of the smallest grain during its collisions with the wall. When $w = 0$, the first collision with the wall typically does not record the maximum force felt by the smallest grain. Long time data reveals the existence of *collective effects* that lead to slightly larger forces being felt later by the smallest grain. However, when $w \neq 0$, the largest force is felt during the first end wall collision. Figure 1(a) presents a phase diagram to show how the maximum force, $f_{max} = F_{max}/F_{in}$ varies with q and w, where F_{max} is the maximum force recorded at the sensor at the tapered end and F_{in} is the maximum input force for the $w = 0$ case. We find that ~ 80% of F_{in} can be absorbed by TCs with $q = 0.09$, provided restitutive losses can be artificially controlled via appropriate cushioning.

The initial impulse is initiated into the system in the form of KE. As the impulse starts to propagate, approximately 55% of this energy is transmitted in kinetic energy form and the rest as potential energy. In Figure 1(b), we present calculations of KE_{out}/KE_{in} as functions of q and w. Our extensive analyses reveal that 90% of the input energy (which is all in KE_{in}) can be absorbed by TCs with q = 0.10. Further improvement in shock absorption of a TC can be effected by subjecting the system to external loading. Simulations show (Figure 1(c)) that an external loading force causing 0.3% compression of the largest sphere reduces KE_{out}/KE_{in} by a factor of 4.

When $w \to 0$ and $q = 0$, i.e., for monodispersed chains, energy loss in the chain is roughly constant everywhere (Figure 2(a)). However, increase in w, enhances loss where the forces are largest, i.e., at the end where the impulse is incident (Figure 2(g)). Tapering leads to high velocities at the tapered end. The presence of a force sensor at the tapered end leads to a large number of collisions with the sensor and hence large energy loss at the tapered end (Figure 2c)). When both q and w are large, the bulk of the energy loss is at the central part of the TC (Figures 2(f,i)). The use of nitinol spheres, in a phase where nitinol shows strong hysteresis or energy absorption [14], in the middle part of such TCs, can dramatically enhance shock absorption.

We have studied the shock absorption properties of TCs and have depicted the effects of radius shrinkage (q) and restitutive losses (w) in terms of "dynamical phase diagrams." Our studies demonstrate for the first time that small TCs with 20 spheres can absorb ~80% (~90%) of the incident force (energy) pulse (Figure 1(a-c)). Enhancement in energy absorption due to increasing N is well approximated by Equation (1). It is conceivable that arrays of TCs, embedded on a wall matrix, could help design impulse resistant walls.

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Figure Captions:

Figure 1: Phase diagram showing (a) f_{max} and (b) KE_{out}/KE_{in} felt at the tapered end vs restitution w and tapering q. Observe $KE_{out}/KE_{in} < 1$ for $q = 0$, $w = 0$ because part of the energy is carried as potential energy. In (c) KE_{out}/KE_{in} vs w and q is dramatically reduced when the system is under precompression and the precompression causes 0.3% reduction in the radius of the largest sphere.

Figure 2: The panels (a)-(i) show spatially resolved collisional energy loss between granular contacts in a 20 grain TC. For large w and q, the highest losses are at the central part of the TC (see discussion).

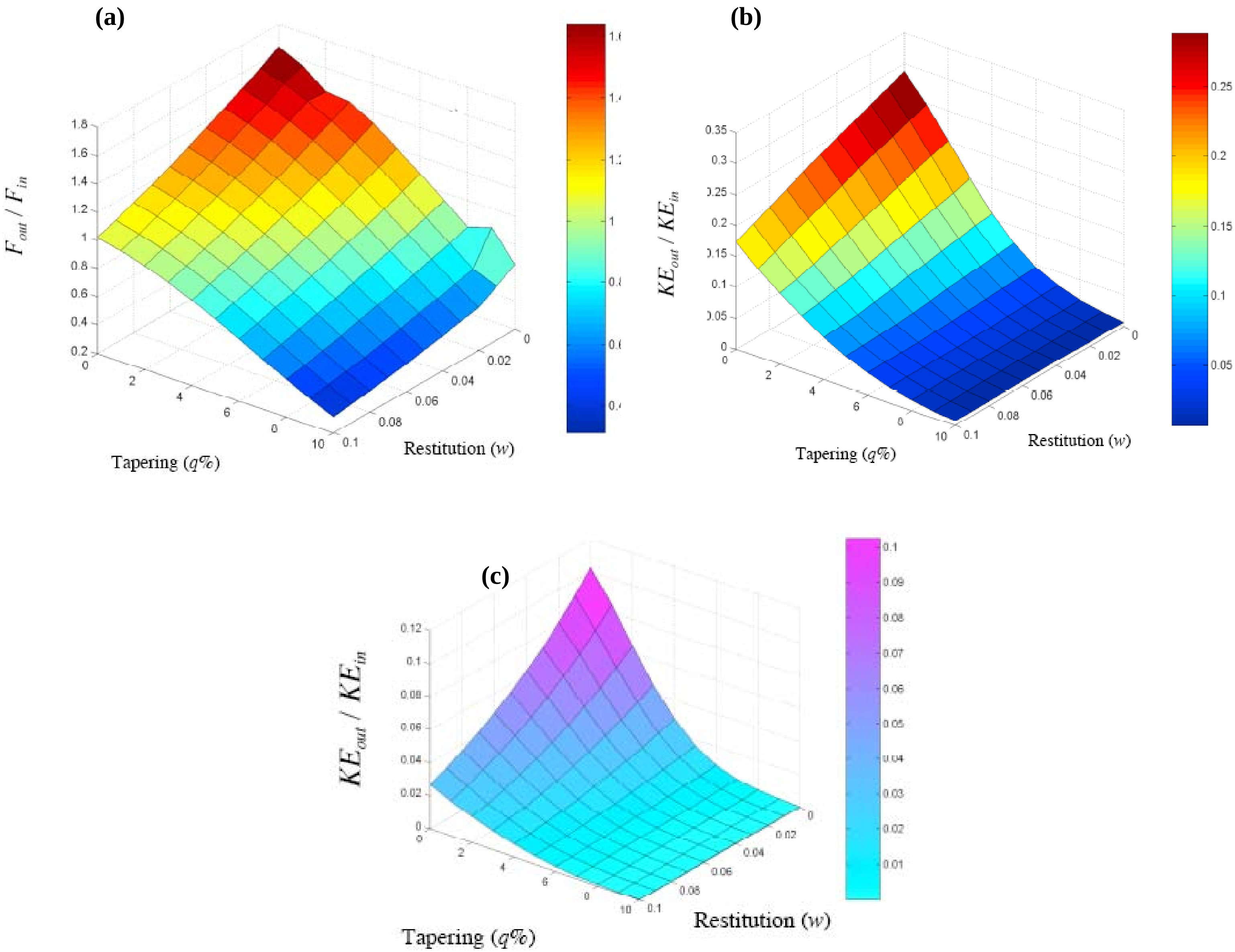

Figure 1
(Sokolow et al.)

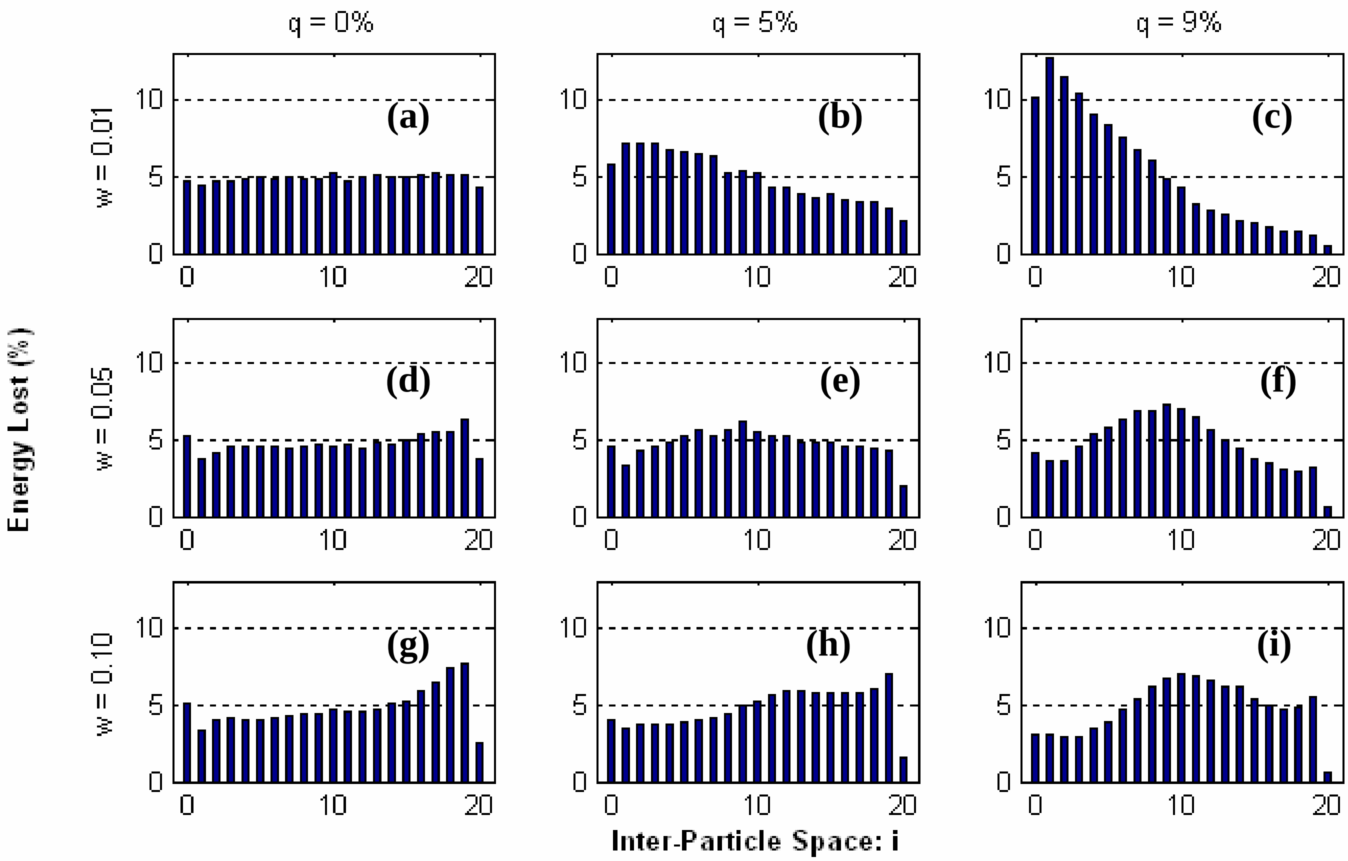

Figure 2
(Sokolow et al.)