PHY 513, F06 EX. 2A-1

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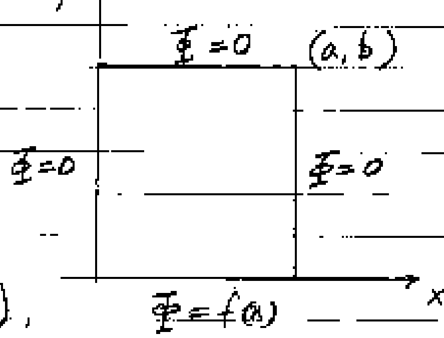
Set 2A: Electrostatics I.

1. Consider a two-dimensional interior Dirichlet problem for the rectangular boundary shown in the figure below. The boundary conditions we shall adopt are that Φ be zero on the three sides of the rectangle shown, and that it reduce to a known function $f(x)$ on the lower side, i.e., for $y=0$ $(0 < x < a)$. Assume first that



(a) $\Phi(x,y) = (A\sin\lambda x + B\cos\lambda x)\cdot(C\sinh\lambda y + D\cosh\lambda y)$, and show that to satisfy the boundary conditions.

$$\Phi(x,y) = \sum_{n=1}^{\infty} F_n \sin\frac{n\pi x}{a}\sinh\frac{n\pi}{a}(b-y),$$

where F_n is some arbitrary constant.

(b) Next prove that $F_n \sinh\frac{n\pi b}{a} = \frac{2}{a}\int_0^a \Phi(x,0)\sin\left(\frac{n\pi x}{a}\right)dx$ and write down the final expression for $\Phi(x,y)$ in terms of $\Phi(x,0)$, $\sin\frac{n\pi x}{a}$ and $\sinh\frac{n\pi}{a}(b-y)$.

2. Use Gauss' theorem to prove the following: (a) Excess charge placed on a conductor must lie entirely at the surface; (b) A closed hollow conductor shields its interior from fields due to charges outside, but does not shield its exterior from the fields due to charges placed inside it; (c) $\vec{E}$ at the surface of a conductor is perpendicular to the surface and $|\vec{E}| = \sigma/\epsilon_0$ where σ is the surface charge density.

3. Each of three charged spheres of radius a, one conducting, one having a uniform charge density within its volume, and one having a spherically symmetric charge density that varies radially as r^n $(n > -3)$, has a total charge Q. Use Gauss' theorem to obtain the electric fields both inside and outside each sphere. Sketch the behavior of the fields as a function of radius for the first two spheres, and for the third with $n = -2, +2$.

4. The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1 + \frac{\alpha r}{2}\right),$$

where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

5. Two long, cylindrical conductors of radii a_1 and a_2 are parallel and separated by a distance d, which is large compared with either radius. Show that the capacitance per unit length is given approximately by

$$C \cong \pi\epsilon_0 \left(\ln \frac{d}{a}\right)^{-1}, \quad a = \sqrt{a_1 a_2}$$