Problem #1

Imagine a head-on collision between two toy-cars (a truck and a car). The mass of the truck is 5 kg and the mass of the car is 3 kg. Before the collision the truck has a speed of 3 m/s to the right and the car has a speed of 4 m/s and is moving to the left. What are the final velocities of the car and the truck after they collide if the ratio of final total kinetic energy to initial total kinetic energy is 0.1?
(Treat each car as a particle)

We have to a completely inelastic collision.

Momentum conservation

$$p_{initial} = p_{final}$$

Where

$p_{initial} = m_1 * v_1 + m_2 * v_2$ and

$$p_{final} = m_1 * v_1^{'} + m_2 * v_2^{'}$$

$$m_1 * v_1 + m_2 * v_2 = m_1 * v_1^{'} + m_2 * v_2^{'} \qquad (1)$$

Where v_1=4 m/s , v_2 = - 3 m/s , $v_1^{'}$ is the speed of the truck after collision and $v_2^{'}$ is the speed of the car after collision.
The initial total kinetic energy is

$$K_{initial} = \frac{1}{2} m_1 * v_1^2 + \frac{1}{2} m_2 * v_2^2$$

The final total kinetic energy is given by

$$K_{final} = \frac{1}{2} m_1 * v_1^{'2} + \frac{1}{2} m_2 * v_2^{'2}$$

From the problem we know that

$$\frac{K_{final}}{K_{initial}} = 0.1 \qquad (2)$$

From eq (1) we can find the expression for , $v_1^{'}$

$$v_1^{'} = \frac{p_{initial} - m_2 * v_2^{'}}{m_1}$$ so now

$$K_{final} = \frac{1}{2} m_1 * \left(\frac{p_{initial} - m_2 * v_2^{'}}{m_1} \right)^2 + \frac{1}{2} m_2 * v_2^{'2}$$

But $K_{final} = 0.1 * K_{initial}$

So

$$\frac{1}{2} m_1 * \left(\frac{p_{initial} - m_2 * v_2^{'}}{m_1} \right)^2 + \frac{1}{2} m_2 * v_2^{'2} = 0.1 * K_{initial}$$

By expanding the terms we get

$$\frac{1}{2} \frac{p_{initial}^2}{m_1} - \frac{1}{2} \frac{2 * p_{initial} * m_2 * v_2^{'}}{m_1} + \frac{1}{2} \frac{(m_2 * v_2^{'})^2}{m_1} + \frac{1}{2} m_2 * v_2^{'2} - 0.1 * K_{initial} = 0$$

$$(m_2{}^2 + m_1 * m_2) * v_2^{'2} - 2 * m_2 * v_2 * p_{initial} + p_{initial}{}^2 - 2 * 0.1 * K_{inial} = 0$$

By solving this quadratic equation in $v_2^{'}$ we get

$$v_2^{'} = \frac{2m_2 * p_{initial} \pm \sqrt{(2m_2 * p_{initial})^2 - 4(m_2^2 + m_1 * m_2)(p_{initial}^2 - 2 * 0.1 * m_1 * K_{initial})}}{2(m_2^2 + m_1 * m_2)}$$

Finally by plugging in the values we get

$v_2' = 2.55 m / s$

We neglect the negative value because the car has a mass smaller than the truck and it cannot move to the left since there is a head-on collision.

$$v_1' = \frac{p_{initial} - m_2 * v_2'}{m_1}$$ so $v_1' = .67$ m/ s

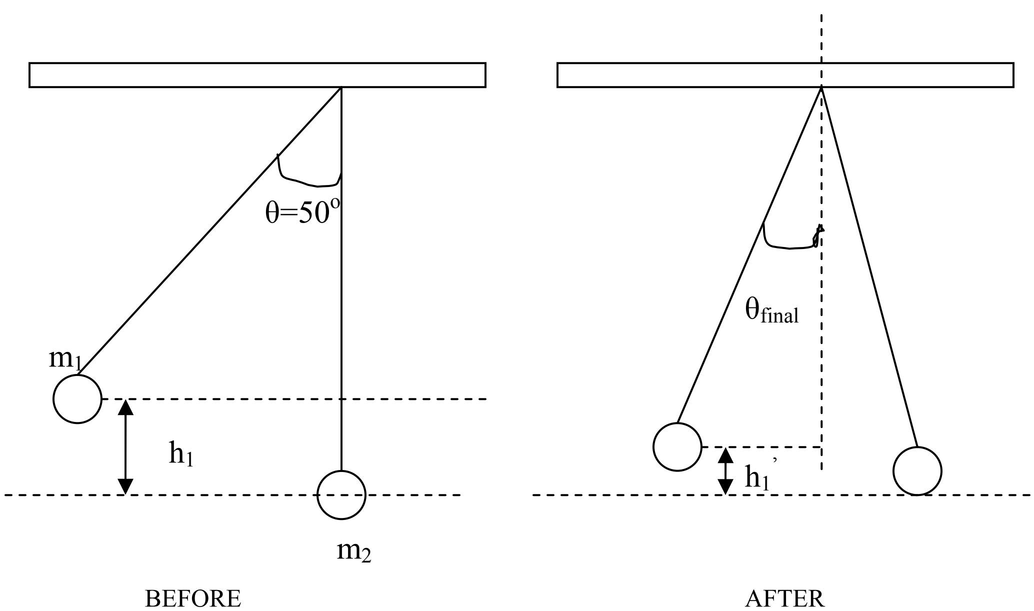
The truck continues to move to the right and the car has reversed its direction .

Problem #2

Two strings that are both 1.8 m long are attached to the same point on a ceiling. Two spheres with masses $m_1 = 1$ kg and $m_2 = 2$ kg hang at the ends of strings(see the picture below)

After the sphere 1 is released they collide elastically.

What is the maximum angle that the the string holding sphere 1 makes with respect to the vertical?



Conservation of energy gives us the speed of sphere1 before collision

$$m_1 g h_1 = \frac{1}{2} m_1 v_1^2$$

And $h_1 = L(1-\cos\theta)$

Where L = 1.8 m and θ = 50^{o}

So

$v_1 = \sqrt{2g * L * (1-\cos\theta)}$ so v_1 = 3.55 m/ s

Momentum conservation

$$p_{initial} = p_{final}$$

Where

$p_{initial} = m_1 * v_1$ and

$$p_{final} = m_1 * v_1^{'} + m_2 * v_2^{'}$$

So $m_1 * v_1 = m_1 * v_1^{'} + m_2 * v_2^{'}$ (1)

The relative velocity relation for an elastic collision

$$(v_{rel})_{final} = -(v_{rel})_{initial}$$

Where

$(v_{rel})_{initial} = v_1$ and $(v_{rel})_{initial} = v_1^{'} - v_2^{'}$ so we have

$$v_1^{'} - v_2^{'} = -v_1 \qquad (2)$$

Equations (1) and (2) are simultaneous equations and can be solved for $v_1^{'}$ and $v_2^{'}$.

We are interested in $v_1^{'}$ so by replacing $v_2^{'} = v_1^{'} + v_1$ from eq (2) into eq (1) we get

$$m_1 * v_1 = m_1 * v_1^{'} + m_2 * (v_1^{'} + v_1)$$

And

$v_1^{'} = \frac{(m_1 - m_2)}{(m_1 + m_2)} * v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} * \sqrt{2g * L * (1-\cos\theta)}$ so $v_1^{'}$ = - 1.18 m /s

If $m_2 > m_1$, then $v_1^{'} < 0$ so the ball 1 recoils back to the left

In order to find the maximum angle for sphere1 we apply again the conservation of energy to find the height to which m_1 recoils and express that height in terms of L and θ_{final} .

$m_1 g h_1^{'} = \frac{1}{2} m_1 v_1^{'2}$ and $h_1^{'} = L(1 - \cos\theta_{final})$

So

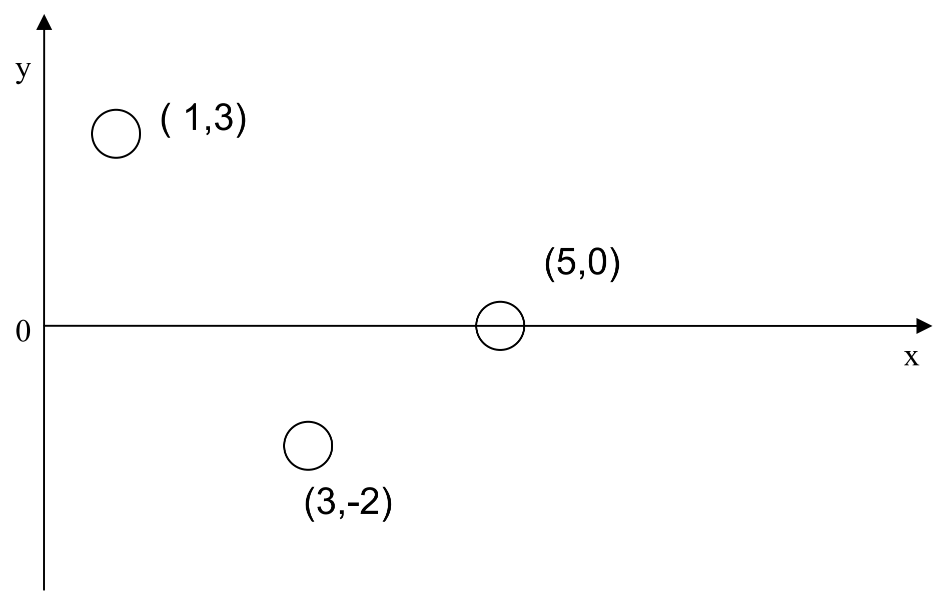
$$\cos\theta_{final} = 1 - \frac{v_1^{'2}}{2g * L}$$

So $\theta_{final} = 16.26^{o}$

Problem #3

Three pointlike masses are placed as shown below .Their coordinates are (1,4) , (3,-2) and (5,0). Their weights are 9.8 N,19.6 N and 14.7N

Find the distance from the origin to the center of the mass.



The position vector of the center of mass can be expressed as:

$$\vec{r}_{cm} = \frac{m_1 * \vec{r}_1 + m_2 * \vec{r}_2 + m_3 * \vec{r}_3}{m_1 + m_2 + m_3}$$

By resolving into its components we get

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

Where $m_1 = \frac{w_1}{g} = 1kg$,

$m_2 = \frac{w_2}{g} = 2kg$

and $m_3 = \frac{w_3}{g} = 1.5kg$

$x_1 = 1$, $y_1 = 3$, $x_2 = 3$, $y_2 = -2$, $x_3 = 5$, $y_3 = 0$,

By plugging in the values we find

$x_{cm} = 3.22$

$y_{cm} = -0.22$

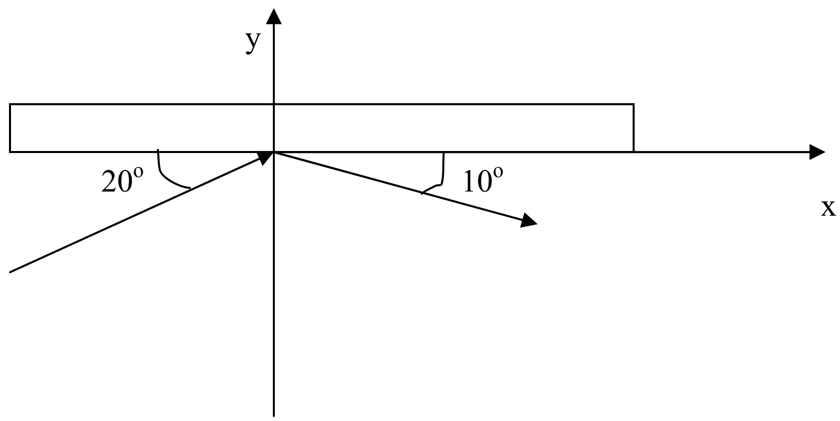
The distance from the origin to the center of mass is given by:

$$d = \sqrt{x_{cm}^2 + y_{cm}^2}$$

Ater we plug in the values we get d =3.23

Problem #4

A race-car with a mass of 100 kg collides with a racetrack wall. Before the collision the car has a speed of 50 m/ s along a straight line at 20^o from the wall. After the collision the car has a speed of 30 m/s at 10^o from the wall (see the picture below).
a) Find the impulse (magnitude and direction) on the car .
(Treat the car as a particle).
b)If the collision lasts for 0.012 s what is the magnitude of the average acceleration during the collision?



Definition of impulse

$$\vec{J} = \vec{p}_{final} - \vec{p}_{initial}$$

$\vec{p}_{initial} = m * \vec{v}_{initial}$ and $\vec{p}_{final} = m * \vec{v}_{final}$

So $\vec{J} = m * \vec{v}_{final} - m * \vec{v}_{initial}$

Resolving into its components

$$J_x = m(v_{final,x} - v_{initial,x}) = m[v_{final} * \cos(350) - v_{initial} * \cos(20)]$$

So J_x = - 1744.04 kg*m/s

$$J_y = m(v_{final,y} - v_{initial,y}) = m[v_{final} * \sin(350) - v_{initial} * \sin(20)]$$

J_y = - 2231.05 kg*m/s

The impulse magnitude is given by

$J = \sqrt{(J_x^2 + J_y^2}$, J = 2831.83 kg*m/s

The direction is given by the angle that the vector J makes with the x-axis

$\tan\theta = \frac{J_y}{J_x}$ so $\theta = \tan^{-1}\frac{J_y}{J_x}$

The physically correct result might be displayed as the answer + 180^o

So θ =52 + 180 = 232^o

b) $J = F_{average} * \Delta t$

$F_{average} = m * a_{average}$

From the above equations we get

$a_{average} = \frac{J}{m * \Delta t}$

Where Δt = 0.012 s , m = 100 kg, and J = 2831.83 kg*m/s

So $a_{average}$= 2359.86 m/s^2

Problem #5

The final velocity of a rocket when all the fuel is burned away is called the *burnout velocity*.

Find the burnout velocity of a rocket that moves vertically if the ratio m_{fuel} / m_0 is 80 percent, t = 100s and v_{ex} is 2500 m/s.

When a rocket moves vertically in the presence of gravity the motion is different than when gravity is not present.

The force of gravity will act on the rocket so

$$P_2 - P_1 = -F_g * dt$$

$$P_2 = (m + dm) * (v + dv) + (-dm)(v - v_{ex})$$

$$P_1 = mv$$

Eq (1) becomes

$$m * dv = -dm * v_{ex} - dm * dv + mg * dt$$

We neglect the term dm*dv .
By dividing by dt we get

$$m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} + mg$$

Remember that dm/ dt is negative because it is the rate of change of the rocket's mass.
Reversing the sign in front of dm/dt we get

$$-v_{ex}\frac{dm}{dt} = m\frac{dv}{dt} + mg$$

Dividing by m we get

$$\frac{-v_{ext}}{m}\frac{dm}{dt} - \frac{dv}{dt} - g = 0$$

and $\frac{1}{m}\frac{dm}{dt} = \frac{d(\ln m)}{dt}$ so

$$-v_{ex}\frac{d(\ln m)}{dt} - \frac{dv}{dt} - g = 0$$

Integrating with respect to t we get

$$-v_{ex}\ln m - v - gt = const$$

And to find the constant we set t = 0, when v=0 and m=m_0

$$const = -v_{ex}\ln m_0$$

Such that we can finally write

$$v = v_{ex} * \ln\frac{m_0}{m} - gt$$

The burnout velocity is the final velocity of a rocket when all the fuel is burned away

$$v_{burnout} = v_{ex} * \ln\frac{m_0}{m_0 - m_{fuel}} - gt$$

Where v_{ex} = 2500 m/s , m_0 is the mass at t=0 and m_{fuel} is the mass of the fuel

$$v_{burnout} = v_{ex} * \ln\frac{m_0}{m_0 - 0.8m_0} - gt$$

After we plug in the values we get

$v_{burnout}$ = 979.99 m/s