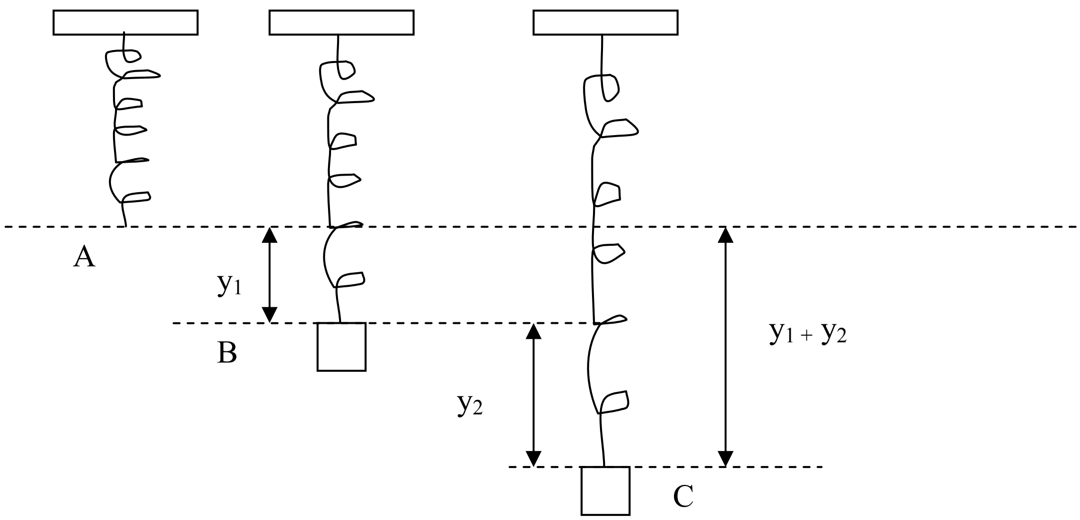
Problem #1

A spring hangs vertically with no mass at its end. If you attach mass to its bottom, the new equilibrium position will be 7 cm lower. Now you pull the mass 10 cm down and release it.
Find the speed of the mass when it passes the original equilibrium position.



From the stretch to the new equilibrium position , we can find the force constant k

$m * g = k * y_1$ so $k = \frac{m * g}{y_1}$

From C to A we use energy conservation

$$E_{initial} = E_{final}$$

$$K_{initial} + (U_{spring})_{initial} + (U_{gravity})_{initial} = K_{final} + (U_{spring})_{final} + (U_{gravity})_{final}$$

Where

$$K_{initial} = 0 (v_{initial} = 0)$$

$$(U_{spring})_{initial} = \frac{1}{2} * k * (y_1 + y_2)^2$$

$$(U_{gravity})_{initial} = -m * g * (y_1 + y_2)$$

And

$$K_{final} = \frac{1}{2} * m * v_{final}{}^2$$

$$(U_{spring})_{final} = 0$$

$$(U_{gravity})_{final} = 0$$

$$\frac{1}{2} * m * v_{final}{}^2 = \frac{1}{2} * \frac{m * g}{y_1} * (y_1 + y_2)^2 - m * g * (y_1 + y_2)$$

$$v = \sqrt{\frac{g}{y_1}(y_1 + y_2)^2 - 2 * g * (y_1 + y_2)}$$

Solving for v we get v = 20 m/s

Problem #2

A marble of mass 100 g is released from rest from a height of 12 cm above the ground on a frictionless loop (see the picture below). The diameter of the circular loop is 8 cm. After it leaves the circular loop it goes up on an frictionless inclined plane.

What will the speeds of the marble be at points 2,3. What is the speed of the marble at point 4 if the h_4 = 12.1 cm.

Find the maximum distance that will travel on the incline.

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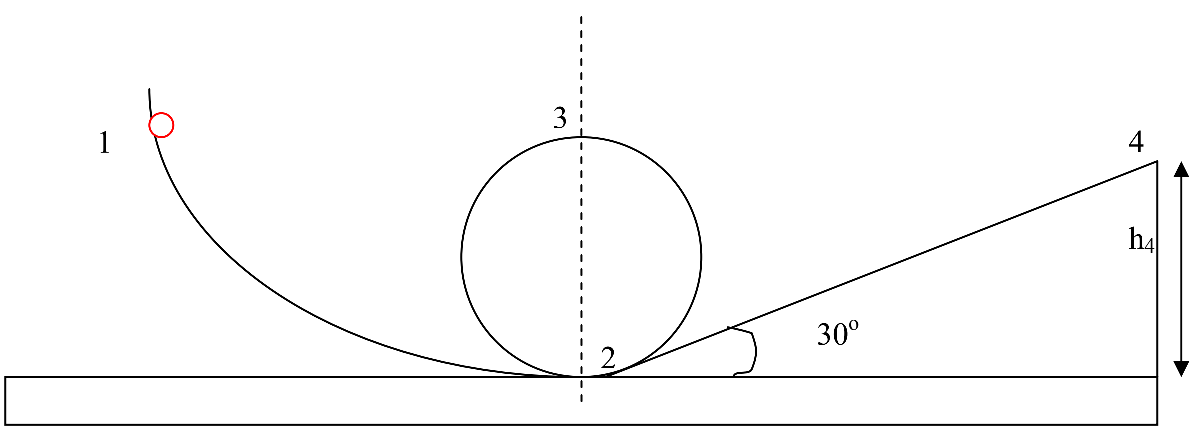


We have no friction so the energy is conserved

$$E_{initial} = E_{final}$$

$$K_{initial} + (U_{gravity})_{initial} = K_{final} + (U_{gravity})_{final}$$

$E_1 = \frac{1}{2} * m * v_1^2 + m * g * h_1$ where $v_1 = 0$ so $E_1 = m * g * h_1$ so

$E_1 = 0.18$ J

$$E_1 = E_2$$

$E_2 = \frac{1}{2} * m * v_2^2 + m * g * h_2$ where $h_2 = 0$ so $E_2 = \frac{1}{2} * m * v_2^2$

$v_2 = 6$ m/s

$$E_1 = E_3$$

$E_3 = \frac{1}{2} * m * v_3^2 + m * g * h_3$ where $h_3 = 8$ cm

$v_3 = 0.9$ m/s

$$E_1 = E_4$$

$$E_4 = \frac{1}{2} * m * v_4^2 + m * g * h_4$$

So $v_4^2 = 2 * (\frac{E_1}{m} - g * h_4)$

If we plug in the value we get $v_4^2 < 0$ which means the marble will never reach the height of 12.1 cm.

At the highest point the marble has no kinetic energy so

$$E = m * g * h_{max}$$

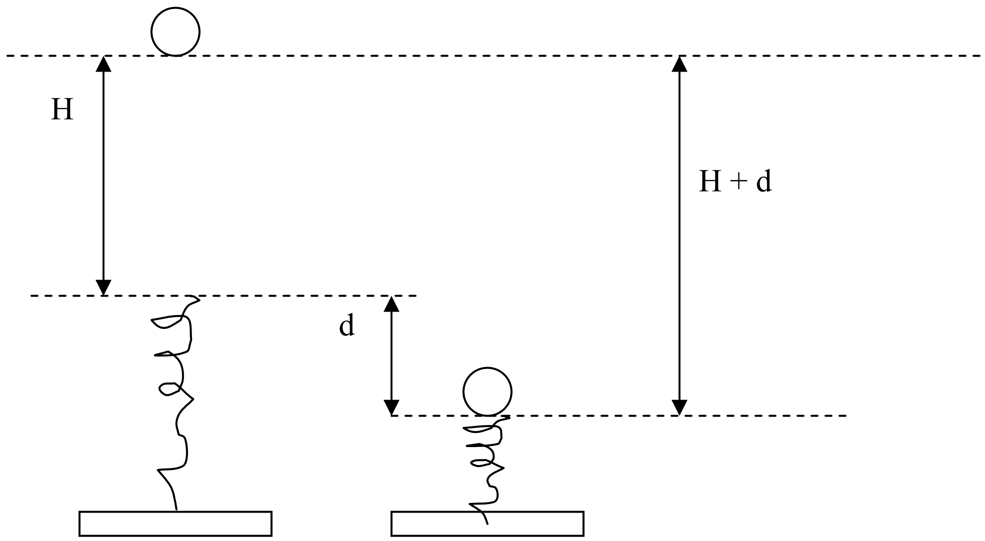
$\sin(30^o) = \frac{h_{max}}{d_{max}}$ so $d_{max} = \frac{h_{max}}{\sin(30)} = \frac{E_1}{m * g * \sin(30)}$

Plugging in the values we get

$d_{max} = .37$ m $= 37$ cm

Problem #3

You drop a 2 kg ball form a height of 1 m above the top of a spring that is placed vertically. The force constant is 1800 N/m .
Find the maximum compression of the spring.
The ball rebounds back. Find the speed of the ball when it is at a height of 1/3 of the initial height.
(Neglect the air friction).



a)

Energy is conserved

$$m*g*(H+d)=\frac{1}{2}*k*d^2$$

$$\frac{1}{2}*k*d^2-m*g*d-m*g*H=0$$

Solving the quadratic equation we get

$$d=\frac{2*m*g\pm\sqrt{(2*m*g)^2+4*k*2*m*g*H}}{2*k}$$

d = .16 m = 16 cm
b)
Again we apply the principle of conservation of energy

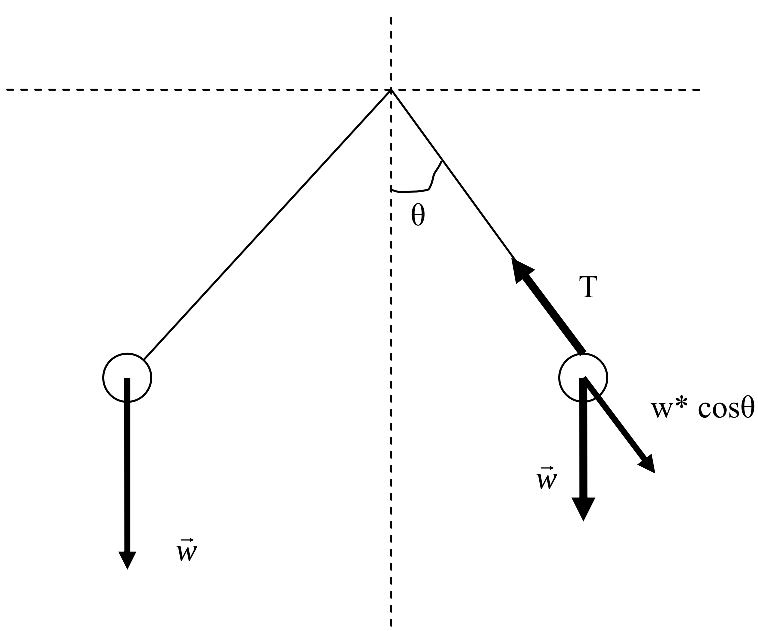
$$\frac{1}{2}*k*d^2 = m*g*(\frac{1}{3}H+d)+\frac{1}{2}*m*v^2$$

So $v = \pm\sqrt{\frac{k*d^2-\frac{2}{3}(H+3*d)*m*g}{m}}$

v = 3.65 m/ s

Problem #4

A simple pendulum of length 1.5 m is released from the horizontal position. The mass of the bob is .25 kg
a) Find the maximum speed of of the mass.
b) Find the speed when the string makes an angle of 30 with the vertical
c) Find the tension in the string in both cases.



a)
The bob will have a maximum speed at the lowest point

From the release point to the lowest point we have

$$K_{initial} + (U_{gravity})_{initial} = K_{final} + (U_{gravity})_{final}$$

If we choose y = 0 at the initial position we get

$$0 = \frac{1}{2} * m * v_{\max}{}^{2} + m * g * (-L) \quad \text{so} \quad v_{\max}{}^{2} = 2 * g * L$$

Plugging in the values we get

v_{max} = 5.42 m/s

b) From the release point to the point where the angle is θ = 30^0 we can write

$$K_{initial} + (U_{gravity})_{initial} = K_{final} + (U_{gravity})_{final}$$

Where

$K_{initial} = 0$, $(U_{gravity})_{initial} = 0$

$K_{final} = \frac{1}{2} * m * v_{final}{}^{2}$, and $(U_{gravity})_{final} = m * g * (-L * \cos\theta)$

$v^{2} = 2 * g * L * \cos\theta$

So v =5.04 m / s

c)
The net force will provide the centripetal acceleration

$$T - m * g * \cos\theta = \frac{m * v^2}{L}$$

But $v^{2} = 2 * g * L * \cos\theta$

So $T = 3 * m * g * \cos\theta$

When v is maximum θ = 0 so T = 7.35 N
When θ = 30 T = 6.37N

Problem #5

A 4 kg projectile is launched straight up with an initial speed of 25 m/s, and reaches a height that is 95 percent of the height that it would have been reached if it had been no air resistance.
Find the work done on the projectile by the force of the air resistance.

$$K_{initial} + (U_{gravity})_{initial} + W_{other} = K_{final} + (U_{gravity})_{final}$$

$$K_{initial} = \frac{1}{2} * m * v_{initial}{}^{2}$$

$$(U_{gravity})_{initial} = 0$$

$$K_{final} = 0$$

$$(U_{gravity})_{final} = m * g * (\frac{85}{100} * h)$$

With no air resistance

$(U_{gravity})_{final} = m * g * h$ so $m * g * h = \frac{1}{2} * m * v^{2}$

So

$$W_{air} = m * g * (\frac{85}{100} * h) - \frac{1}{2} * m * v_{initial}{}^{2} = (\frac{95}{100} - 1) * \frac{1}{2} * m * v_{initial}{}^{2}$$

Plugging in the values we get

W_{air}= - 62.5 J