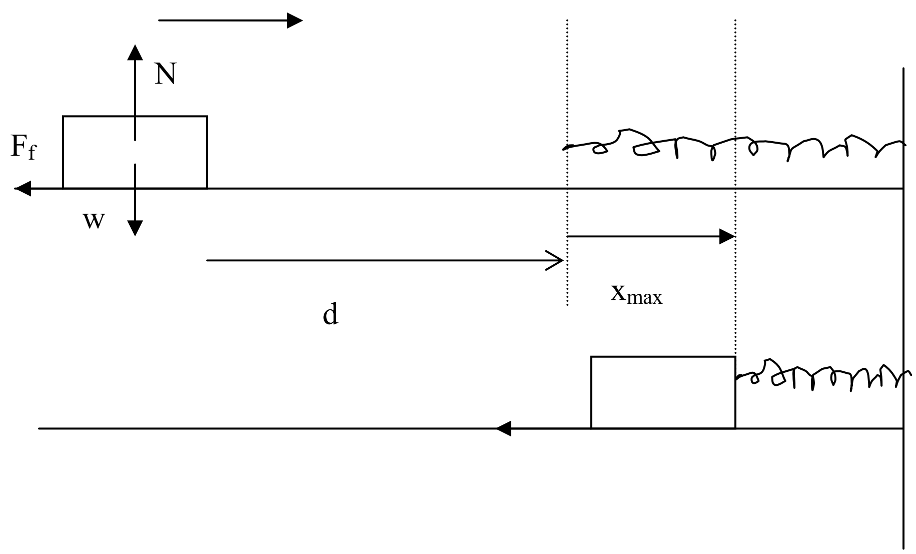
Problem #1

A 1 Kg box moving on a horizontal surface runs into a spring of force constant of 20 N/m. The box has a speed of 2 m/s when it is at a distance of 77 cm from the open end of the spring. The coefficient of kinetic friction is μ_k =0.2.
Find the maximum compression of the spring.

First we have to find the speed of the box just before it hits the spring. If the speed is zero we have no compression.



The work done by the friction force

$$W_{friction} = -F_f * d = -\mu_k * m * g * d = -1.96J$$

Work energy theorem:

$$W_{tot} = \Delta K$$

$$-\mu_k * m * g * d = \frac{1}{2} * m * v_f^2 - \frac{1}{2} * m * v_i^2$$

Where v_i = 2 m/s , d = .77m ,μ_k = 0.2
Solving we get v_f = 1 m/s

To find the maximum compression (which means the box will stop so its final speed will be zero) we apply again the Work-Energy theorem:
The work that the spring does on the box is

$$W_{s-on-b} = -\frac{1}{2} k * x_{max}^2$$

The work done by the friction force is

$W_{friction} = -F_f * x_{\max}$

The total work is the sum of the two previous terms

$W_{total} = W_{s-on-b} + W_{friction}$

So

$$W_{s-on-b} + W_{friction} = \frac{1}{2} * m * v_f^{\ 2} - \frac{1}{2} * m * v_i^{\ 2}$$

But $v_f = 0$ m/s,

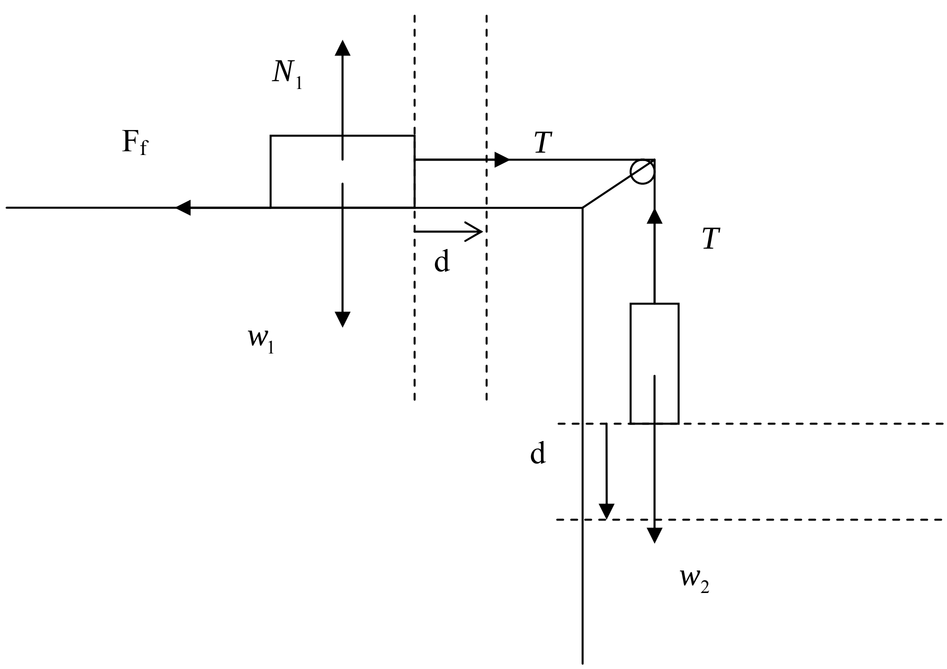
$$-F_f * x_{\max} - \frac{1}{2} k * x_{\max}^{\ 2} = -\frac{1}{2} * m * v_i^{\ 2}$$

Solving the quadratic equation we get x_{max} = 15 cm.

Problem #2

You are given the system below. The coefficient of kinetic friction is μ_k =0.18, m_1 = 4kg, m_2 = 6 Kg.

Find the speed of mass m_2 after it has moved 4 cm to the right starting from rest.(use work-energy theorem)



The speed of mass m_1 will be the same as the speed of mass m_2

The forces acting on mass m_1 are T, F_f , $w_1$and N_1.

The forces acting on mass m_2 are T, and w_2 .

The work done by the forces acting on m_2 will be

$$W_{2total} = (m_2 * g - T) * d$$

Where d = 4 cm, m_2 = 6 kg.

In order to find T we have to write the equations

$$m_2 * g - T = m_2 * a$$
$$T - \mu_k * m_1 * g = m_1 * a$$

Adding these equations we get the acceleration of the system

$$m_2 * g - \mu_k * m_1 * g = (m_1 + m_2) * a$$

So $a = \dfrac{m_2 * g - \mu_k * m_1 * g}{(m_1 + m_2)} = 5.17 m/s^2$

Now $W_{2total} = (m_2 * a) * d$

Work Energy theorem

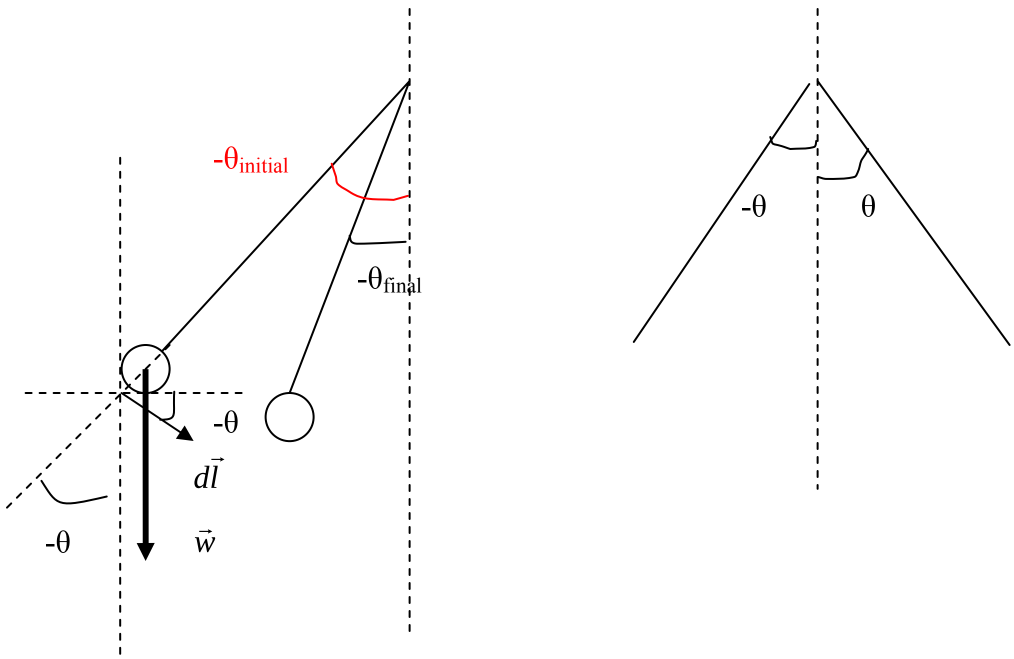
$$W_{2total} = \Delta K$$
$$(m_2 * a) * d = \frac{1}{2} * m_2 * v_f^{\ 2} - \frac{1}{2} * m_2 * v_i^{\ 2}$$

Where v_i = 0 m/s, a = 5.17 m/s^2

So $v_f^{\ 2} = \sqrt{2 * a * d} = 1 m/s$

Problem #3

Find the work done by the force of gravity on a pendulum that it is released from an initial position in which it makes an angle of 20^{o} with the vertical to a final position in which it makes an angle of 15^{o} with the vertical.The mass of the bob is 300 g and the length of the string is 1.5m.
Find the maximum speed (when the angle with the vertical is 0^{0}).



$$W = \int \vec{w} \cdot d\vec{l}$$

The infinitesimal vector displacement of magnitude ds can be written as :

$$d\vec{l} = \hat{i} * ds * \cos(-\theta) - \hat{j} * ds * \sin(-\theta) = \hat{i} * ds * \cos\theta + \hat{j} * ds * \sin\theta$$

Because sin(-θ) = -sin(θ) and cos(-θ) = cos(θ)

$$\vec{w} = -\hat{j} * w = \hat{j} * (-w)$$

$$\vec{w} \cdot d\vec{l} = -w * ds * \sin\theta$$

But $ds = L * d\theta$

So

$$W = \int \vec{w} \cdot d\vec{l} = \int_{-\theta_{initial}}^{-\theta_{final}} m * g * L * (-\sin\theta) = m * g * L * (\cos\theta_{final} - \cos\theta_{initial})$$

$W_{gravity}$ = 0.12J
The work done by gravity is positive because gravity pulls down and the pendulum moves down).

When v is maximum $\theta_{final} = 0$ so

$$W = m * g * L * (1 - \cos\theta_{initial})$$

And from Work Energy theorem

$$W = \Delta K$$

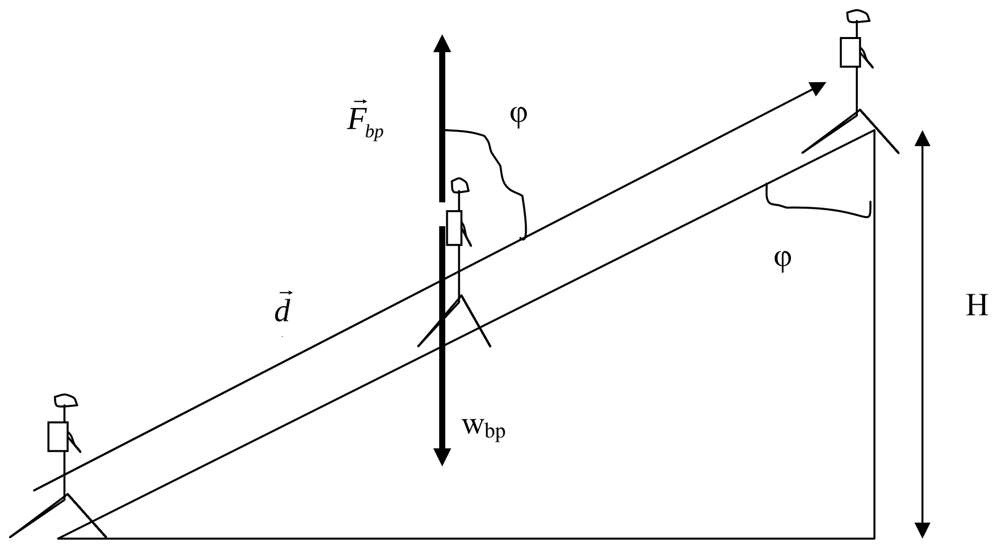
$$m * g * L * (1 - \cos\theta_{initial}) = \frac{1}{2} * m * v_f^{\ 2} - \frac{1}{2} * m * v_i^{\ 2}$$

Where v_i = 0 m/s

So $v_{max} = \sqrt{2 * g * L * (1 - \cos\theta_i)}$
=1.33 m/s

Problem #4

Determine that the work that you must do to carry your backpack (m_{bp} = 12kg) up a hill at a constant speed. The height of a the hill is H= 15m. Find the net work done on your backpack.



The speed is constant so we have no acceleration so the net force is zero.

$F_{bp} = m_{bp} * g$

$W = \vec{F}_{bp} * \vec{d} = F_{bp} * d * \cos\varphi$

But $d * \cos\varphi = H$

So $W = m * g * H = 1470J$

The net work is zero since the net force is zero(v = const so a=0)

$W_{total} = W_{bp} + W_{gravity} = 0$

So $W_{gravity} = -W_{bp} = -m * g * H = -1470J$

Problem #5

A 65 Kg person runs up a long flight of stairs of height 3 m in 7 seconds. Find the person's power output.

$$power = \frac{work}{time}$$

The work done is against the gravity.

$$P = \frac{m * g * h}{t}$$

Where t = 7 s, m=65 kg, and h = 3m
So
P = 273 W

Express the power in horsepower measurement units.

There are 750 W in 1 hp so $P = \frac{1hp}{750W} * 273W = 0.36hp$