

Problem #1

A deflated ball is dropped from an initial height of 1.4 m. The mass of the ball is 30 g. It bounces back to about 40 percent of the initial height. The time that the ball is in contact with the ground is around 0.1 seconds.

Find the force exerted on the ball by the ground.

When the ball hits the ground it has a speed v_1 after it bounces it has another speed

$v_2 < v_1$ otherwise it would have reached the initial height.

During the bounce the forces acting on the ball are W (down) and F_N (upward)

We take positive what is upward and to the right and negative what is downward and to the left.

$$x = x_0 + v_{0x} * t \quad (1)$$

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g * t^2 \quad (2)$$

$$v_x = v_{0x} \quad (3)$$

$$v_y = v_{0y} - g * t \quad (4)$$

$$v_y^2 = (v_{0y})^2 - 2 * g * (y - y_0) \quad (5)$$

We can use eq 5 to find the two speeds:

$$v_{1y}^2 = (v_{01y})^2 - 2 * g * (y_1 - y_{01})$$

Where $y_1=0$ (it hits the ground), $y_{01}=1.4$ m, $v_{01y}=0$

$$v_{1y} = \sqrt{2 * g * y_{01}}$$

$$v_{1y} = 5.2 \text{ m/s downward} \quad \text{or} \quad -5.2 \text{ (m/s)}$$

We use the same equation to find v_2 . We know that the ball reaches 40 percent of the initial height but this time it has an initial velocity

$$v_{2y}^2 = (v_{02y})^2 - 2 * g * (y_2 - y_{02})$$

Where $y_2=0.4*y_{01}$ (it reaches the maximum height)), $y_0= 0$ m, $v_{2y} =0$.

$$v_{02y} = \sqrt{2 * g * y_{02}}$$

$$v_{02y} = 3.3 \text{ m/s} \quad \text{upward}$$

The equation we need on y –direction is given by

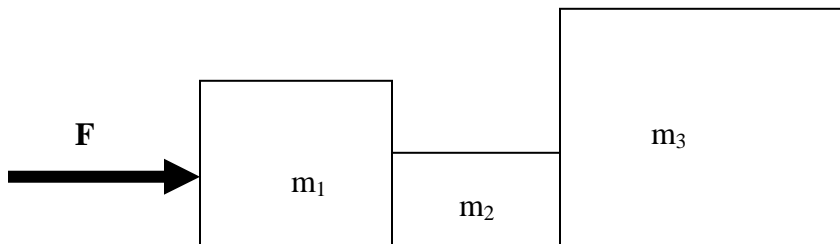
$$F_N - W = m \frac{\Delta v}{\Delta t}$$

Where $W=mg = 0.3 \text{ N}$, $m = 30 \text{ grams} = 0.03\text{kg}$, and $\Delta v = v_2-v_1$ and $\Delta t = 0.2 \text{ s}$, $\Delta v = 3.3- (-5.2) =8.5 \text{ m/s}$ so

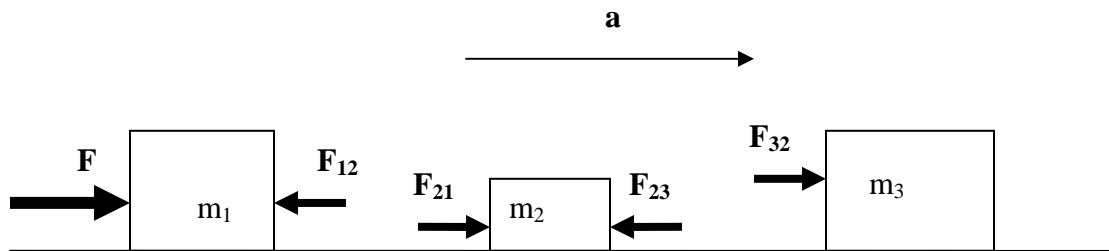
$$F_N = W + m \frac{\Delta v}{\Delta t} \quad \text{and} \quad F_N=2.9\text{N} \quad \text{upward}$$

Problem #2

You push horizontally with a force equals to the weight of a 1.17 kg ball on a set of crates on a frictionless surface(see figure below). The masses of the crates are $m_1 =4 \text{ kg}$, $m_2 = 2.5 \text{ kg}$, and $m_3= 5 \text{ kg}$. Find all the forces that act on each crate.



Let us draw the forces that act on each crate



Where

F_{12} is the force with which the crate 2 acts on crate 1

F_{21} the force with which the crate 1 acts on crate 2 ,

F_{23} is the force with which the crate 3 acts on crate 2

F_{32} is the force with which the crate 2 acts on crate 3

Newton's Second Law

$$\sum F_x = ma_x$$

All three crates are moving with the same acceleration, so for crate 3 we can write

$$\vec{F}_{32} = m_3 * \vec{a}$$

We have motion just on x axis so we can write

$$F_{32} = m_3 * a$$

For the second crate we have:

$$F_{21} - F_{23} = m_2 * a \quad \text{where } F_{23} = F_{32} \text{ (their magnitudes are equal) and}$$

$$\vec{F}_{32} = -\vec{F}_{23} \text{ (they are opposite as vectors) } \quad \text{Newton's Third Law}$$

For the first crate we have

$F - F_{12} = m_1 * a$ where $F_{12}=F_{21}$ (their magnitudes are equal) and $\vec{F}_{12} = -\vec{F}_{21}$ (they are opposite as vectors)

For the whole system we have

$$F = (m_1 + m_2 + m_3) * a$$

Where $F = m * g$

From the last equation we get $a = \frac{F}{m_1 + m_2 + m_3}$ where $a = 1 \text{ m/s}^2$

From the equation $F - F_{12} = m_1 * a$ we find $F_{12}=F_{21}= 7.5 \text{ N}$

The net force on crate 1 is $F_{1\text{net}} = m_1 * a = 4 \text{ N}$

From the equation $F_{21} - F_{23} = m_2 * a$ we get $F_{23}=F_{32}= 5 \text{ N}$

The net force on crate 2 is $F_{2\text{net}} = m_2 * a = 2.5 \text{ N}$

And finally from the equation $F_{32} = m_3 * a$ we get $F_{32} = 5 \text{ N}$

The net force on crate 3 is $F_{3\text{net}} = m_3 * a = 2.5 \text{ N}$

Problem # 3

You are on an unknown planet and throw a ball from 1.5 m above the ground with an initial speed of 20 m/s , with an angle of 30° with respect to the horizontal. The ball lands at a distance of 30 m. The mass of the ball is 0.2kg.

What is the weight of the ball?

The weigh of the ball on that planet will be $W = m * g_{\text{planet}}$

The equations we need are:

$$x = v_{0x} * t$$

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g_{planet} * t^2$$

We get the time of flight from the first equation $t_{flight} = \frac{x}{v_{0x}}$ and

And we use the second equation to find g_{planet}

$$y = y_0 + v_{0y} * \left(\frac{x}{v_{0x}}\right) - \frac{1}{2} * g_{planet} * \left(\frac{x}{v_{0x}}\right)^2$$

Where $y = 0$, $y_0 = 1.5\text{m}$, $x = 30\text{ m}$

$$v_{0x} = v * \cos \alpha_0 = 17.3 \text{ m / s}$$

$$v_{0y} = v * \sin \alpha_0 = 10 \text{ m / s}$$

Finally we get

$$y_0 * \frac{2 * v_{0x}^2}{x^2} + 2 * v_{0y} * \left(\frac{v_{0x}}{x}\right) = g_{planet}$$

$$g_{planet} = 12 \text{ m / s}^2$$

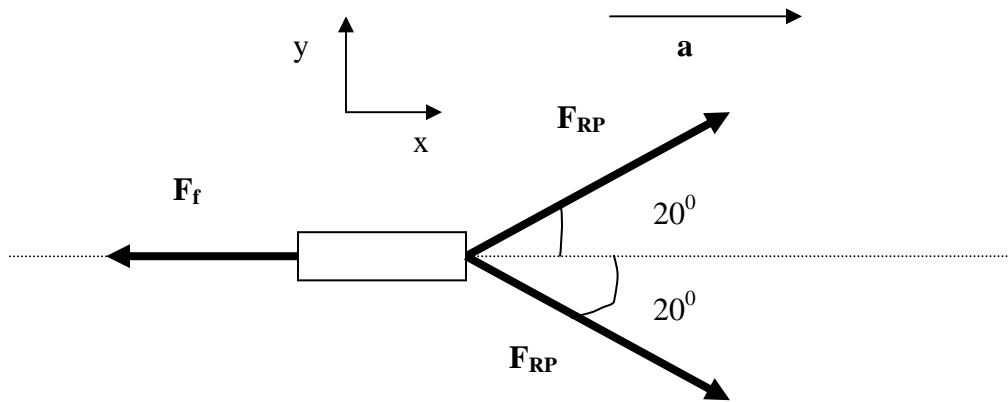
so the weight of the ball on that planet will be $W = m * g_{planet} = 24\text{N}$

Problem # 4

Two people are pulling a crate by means of the ropes. Each rope makes an angle of 20° with respect to the horizontal (see figure below) . The mass of the crate is 80 kg.

The crate is moving at a constant acceleration of 2 m / s^2 to the right. The force exerted on each person by its rope is 100 N .

Find the friction force F_f .



$$\sum F_x = ma_x \quad \text{where } a_x = 0$$

$$\sum F_y = ma_y \quad \text{where } a_y = a$$

The friction force has just x-component because the resultant of the two forces is along x axis.

Writing on components along x axis:

$$F_{RP} * \cos 20^\circ + F_{RP} * \cos 20^\circ - F_f = m * a$$

$$F_f = 2 * F_{RP} * \cos 20^\circ - m * a$$

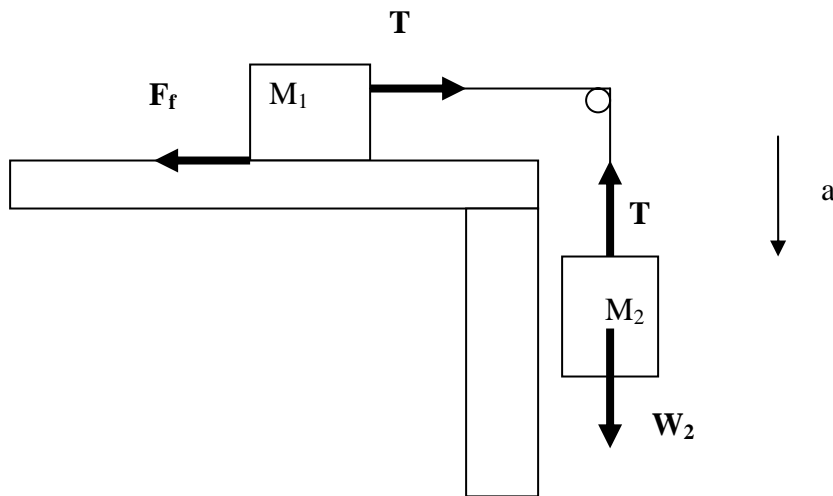
Where $F_{RP} = 100 \text{ N}$, $m = 80 \text{ Kg}$

$$F_f = 8 \text{ N}$$

Problem #5

Two boxes are connected by a rope over a pulley. The friction force F_f is 20 N . The masses of the two boxes are $M_1 = 3$ kg and $M_2 = 4$ kg.

Find the acceleration of the system (assume that the rope does not stretch).



The acceleration of the system will be the same for the both boxes.

We can write

Newton's second Law:

$$\sum F = ma$$

On the horizontal we have:

$$T - F_f = M_1 * a$$

And on the vertical we can write:

$$W_2 - T = M_2 * a$$

Adding both equations we get:

$$W_2 - F_f = (M_1 + M_2) * a$$

$$a = \frac{W_2 - F_f}{M_1 + M_2} \quad \text{where } W_2 = M_2 * g = 39.2 \text{ N, } F_f = 20 \text{ N, } M_1 = 3 \text{ kg and}$$

$$\text{and finally} \quad a_{\text{system}} = 2.74 \text{ m/s}^2 \quad M_2 = 4 \text{ kg}$$