Problem#1

A person standing on the top of a 10 m building throws a ball with a speed of 5 m/ s. The initial angle of the velocity vector with respect to y axis is 60^o.
After a while another person standing on the ground throws a ball upward with a speed of 7 m / s.
a) After what time(s) will the second ball reach the maximum height of the first ball?
b) What is the position of the first ball after 1.5 seconds ?

First we should know the maximum height reached by the first ball.

The equations for projectile motion are:

$$x = v_{0x} * t \qquad (1)$$

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g * t^2 \qquad (2)$$

$$v_x = v_{0x} \qquad (3)$$

$$v_y = v_{0y} - g * t \qquad (4)$$

$$v_y{}^2 = (v_{0y})^2 - 2 * g * (y - y_0) \qquad (5)$$

Where

$$v_{0x} = v * \cos \alpha_0 \qquad (6)$$

$$v_{0y} = v * \sin \alpha_0 \qquad (7)$$

In this problem

$$\alpha_0 = 90^o - 60^o = 30^o$$

$$v_{0x} = v * \cos(30) = 4.3 m/s$$

$$v_{0x} = v * \sin(30) = 2.5 m/s$$

At maximum height $v_y = 0$

Using eq(5) we get the maximum height for the first ball

$$y_{\max} = y_0 + \frac{(v_{0y})^2}{2*g}$$ $y_{max1}= h_{max1}$=10.6m

For the second ball

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g * t^2$$ where $y_0 = 0$, $y = h_{max1}$ $(v_{oy})_2 = 7$ m/ s

By solving the above equation we obtain

$$t = \frac{v_{0y} \pm \sqrt{(v_{0y})^2 + 2 * g * h_{\max}}}{g}$$

We get two values for t : 0.9 s and 2.34s

To find the position of the firs ball we use eq's (1) and (2)

$$x = v_{0x} * t$$

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g * t^2$$

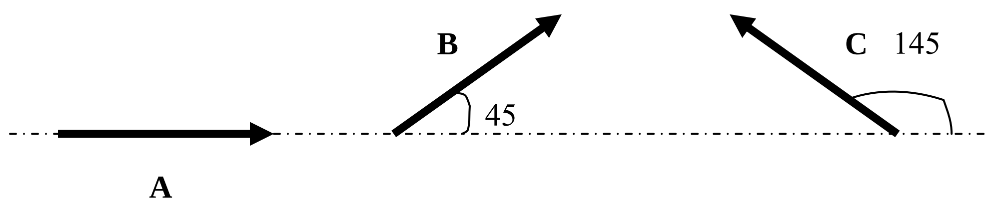
Where t = 1.5 s, v_{0x}=4.3 m/ s, v_{0y}=2.5 m/s, y_0=10m;

Plugging in the values we get

x = 6.5 m
y = 2.8 m

Problem#2

Three people A,B,C are moving with different speeds in different directions as shown below



The magnitudes of their velocities are v_A=3 m/s, v_B=2 m/s,v_C=4 m/s.

What is the speed of person C with respect to person A , $v_{C/A}$?

First we write the components of these velocities with respect to Earth.

$$(v_{A/E})_x = (v_{A/E})*\cos(0) = 3 \text{ m/s}$$
$$(v_{A/E})_y = (v_{A/E})*\sin(0) = 0$$
$$(v_{B/E})_x = (v_{B/E})*\cos(45) = 1.4 \text{ m/s}$$
$$(v_{B/E})_y = (v_{B/E})*\sin(45) = 1.4 \text{ m/s}$$
$$(v_{C/E})_x = (v_{C/E})*\cos(145) = -3.3 \text{ m/s}$$
$$(v_{C/E})_y = (v_{C/E})*\sin(145) = 2.3 \text{ m/s}$$

Let us write first the velocity of person C with respect to Earth

$\vec{v}_{C/E} = \vec{v}_{C/A} + \vec{v}_{A/E}$ and now $\vec{v}_{C/A} = \vec{v}_{C/E} - \vec{v}_{A/E}$

Now we can write on components:

$$(v_{C/A})_x = (v_{C/E})_x - (v_{A/E})_x = -6.3 \text{ m/s}$$
$$(v_{C/A})_y = (v_{C/E})_y - (v_{A/E})_y = 2.3 \text{ m/s}$$

The magnitude of $\vec{v}_{C/A}$ is given by

$$v_{C/A} = \sqrt{(v_{C/A})_x^{\ 2} + (v_{A/E})_y^{\ 2}}$$

$v_{C/A}$= 7.1 m/s

What is the speed of person B with respect to person C , $v_{B/C}$?
Again we write first the velocity of person B with respect to Earth

$\vec{v}_{B/E} = \vec{v}_{B/C} + \vec{v}_{C/E}$ and now $\vec{v}_{B/C} = \vec{v}_{B/E} - \vec{v}_{C/E}$

Again we we can write on components:

$$(v_{B/C})_x = (v_{B/E})_x - (v_{C/E})_x = 4.7 \text{ m/s}$$

$$(v_{B/C})_y = (v_{B/E})_y - (v_{C/E})_y = -0.9 \text{ m/s}$$

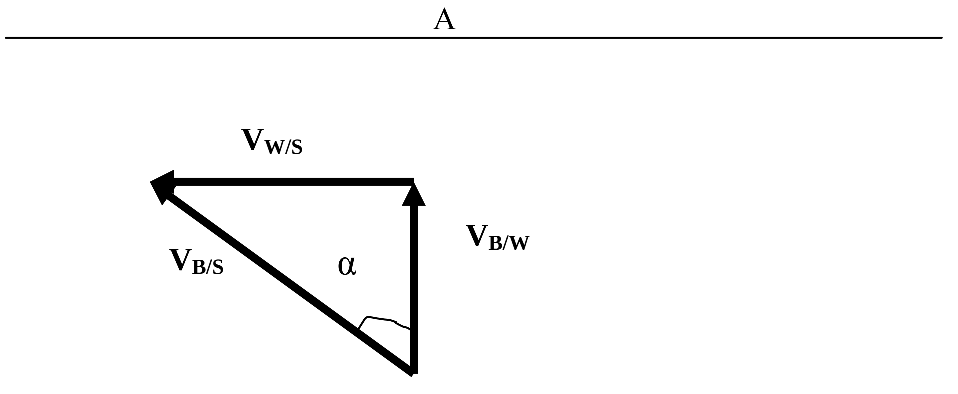
And the magnitude of $\vec{v}_{B/C}$ is

$$v_{B/C} = \sqrt{(v_{B/C})_x^{\ 2} + (v_{B/C})_y^{\ 2}}$$

$V_{B/C}$= 4.8 m/s

Problem#3

You are in a boat that can travel 3km/h in still water ($v_{B/W}$). You head your boat directly across a river whose current is 2km/h ($v_{W/S}$) with respect to the shore.
Find the velocity of your boat relative to the shore ($v_{B/S}$).



A vector is characterized by its magnitude and direction.

$$\vec{v}_{B/S} = \vec{v}_{B/W} + \vec{v}_{W/S}$$

We have a right triangle so

$$v_{B/S} = \sqrt{(v_{B/W})^2 + (v_{W/S})^2}$$

$$\tan\alpha = \frac{v_{W/S}}{v_{B/W}}$$

Where $v_{B/W} = 3\frac{km}{h} = 3*\frac{1000m}{3600s} = 0.83\frac{m}{s}$ and

$$v_{W/S} = 2\frac{km}{h} = 2*\frac{1000m}{3600s} = 0.55\frac{m}{s}$$

Plugging in the values we get

$v_{B/S}$= 0.99m/s

α = 33.4°

Problem #4

A projectile is launched at an angle α_0 = 45 with a velocity of 30 m/s.

a) Find the time of travel before it hits the ground.

The equations for projectile motion are:

$$x = v_{0x} * t \qquad (1)$$

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g * t^2 \qquad (2)$$

$$v_x = v_{0x} \qquad (3)$$

$$v_y = v_{0y} - g * t \qquad (4)$$

$$v_y^{\ 2} = (v_{0y})^2 - 2 * g * (y - y_0) \qquad (5)$$

First we find the components of the initial velocity:

$$v_{0x} = v * \cos \alpha_0 \ = 21.2 \text{ m/s}$$

$$v_{0y} = v * \sin \alpha_0 \ = \ 21.2 \text{ m/s}$$

We use

$$y = y_0 + v_{0y} * t - \frac{1}{2} * g * t^2$$

Where y=0 (it hits the ground) , y_0 = 0, v_{0y} = 21.2 m/s

$$v_{0y} = \frac{1}{2} * g * t \quad \text{so } t = 4.3 \text{ s}$$

b)What is the total distance traveled in the x direction?

$x = v_{0x} * t$ where v_{0x} =21.2.m/s t = 4.3 s

The maximum range is x = 91.2 m

c) What is the acceleration vector (magnitude and direction) at the the highest point ?

a = 9.8m/s^2 and points downward

Problem #5

The circular orbit of the moon about the earth has a diameter of D =770,000 km and the time to make one revolution is T =27.3 days .What is the acceleration of the moon in terms of g?

We use

$$a_{rad} = \frac{4 * \pi^2 * R}{T^2}$$

And we know that D= 2*R

Plugging in the values and converting the units 1 km = 1000m and

$$1 day = 24h * 3600 \frac{s}{h}$$

We find that

$$a_{rad} = 2.72 * 10^{-3} m/s^2$$

In order to express in terms of g we divide our result by 9.8 m/s^2 and we get

$$a_{rad} = 2.8 * 10^{-4} g$$