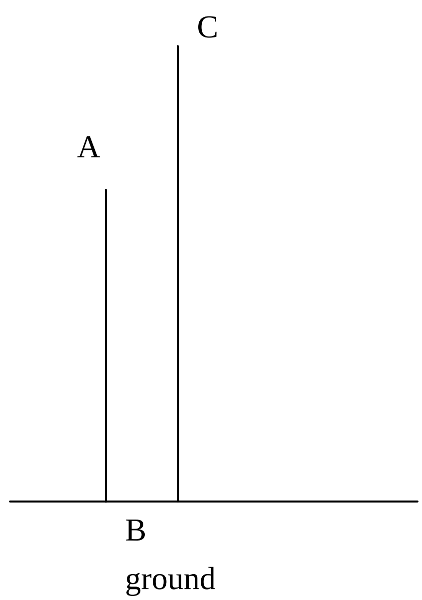
Problem#1

You throw a ball downward with an initial speed *v_{0}= 5 m/s* from the from the top of a building. The height of the building is 5 m. The ball strikes the ground bounces back and after a while it reaches its maximum height above the ground . (Assume an elastic collision and no friction with air)

a) Find the maximum height.



From A to B , we can write

$$v_1^{\ 2} = v_{01}^{\ 2} + 2ay_1$$

where v_{01} = v_0 =5 m/s,(the initial speed) a = -g = - 9.8 m/s^2 , and y_1 = -5 m

$$v_1^{\ 2} = (5\frac{m}{s})^2 + 2*(-9.8\frac{m}{s^2})*(-5m) = 123\frac{m^2}{s^2}$$
$$v_1 = 11.1\frac{m}{s}$$

From B to C we can write

$$v_2^{\ 2} = v_{02}^{\ 2} + 2ay_2$$

where v_{02} = v_1 (the initial speed along BC is the final speed along AB),

$a = -9.8\ m/s^2$ and y_2 is the maximum height the ball reaches so $v_2 = 0$

Combining these two equations we obtain

$$0 = v_1{}^2 + 2 * a * y_2 = 123(\frac{m}{s})^2 + 2 * (-9.8\frac{m}{s}) * y_2$$

$$y_2 = 6.3m$$

b) What will be the speed of the ball when it is 1 m above the top of the building?

If the ball is 1 m above the top of the building it means it is 6 m above the ground.

$$v_f{}^2 = v_1{}^2 + 2 * a * y_3$$

Where $v_1 = 11.9$ m/s, $a = -9.8\ m/s^2$, and $y_3 = 6$ m

Plugging in the values and taking the square root of v_f^2 we obtain

$v_f = 2.3$ m/s

Problem #2

Two cars A and B move at constant speeds in a straight line. The distance between the cars is constant. The speed of car A , $v_A = 7$ m/s.

a) If after 10 seconds the distance is still constant what the speed of car B will be.

If the the distance is constant both cars have the same speed

$$v_B = v_a = 7\frac{m}{s}$$

b) After $t_1 = 10$ seconds the car A is 2 m behind car B and both cars start accelerating. After $t_2 = 30$ seconds the car A is 5 m ahead car B. Find the distance that car B has traveled during this time interval if the car A accelerates at a constant acceleration $a_A = 0.4\ m/s^2$.

The total distance that car B has traveled during this time interval $t = t_2 - t_1$ is given by

The total distance that car A has traveled during t is given by:

$$d_A = v_A * t + \frac{1}{2} * a_A * t^2$$

But on the other hand

$$d_A = d_B + d_1 + d_2$$

From the last two equations we can write:

$$d_B + d_1 + d_2 = v_A * t + \frac{1}{2} * a_A * t^2$$

Which implies that $d_B = v_A * t + \frac{1}{2} * a_A * t^2 - d_1 - d_2$

Where t = 20 s, d_1= 2 m and d_2 = 5 m .,a_A = 0.4 m/s^2 and v_A = 7 m/s
Plugging in the values we get

d_B = 213 m

c) What is a_B?

$$d_B = v_B * t + \frac{1}{2} * a_B * t^2$$

$$a_B = \frac{2 * d_B - 2 * v_B * t}{t^2}$$

Plugging in the values we get

$$a_B = 0.37 m / s^2$$

Problem #3

You drive on the highway with 140 km/h when you spot a police car 100 m ahead.
a) At what constant acceleration should you brake your car such that when you pass the police car your speed is 80 km/h.

$$v_f^{\ 2} = v_i^{\ 2} + 2 * a * x$$

Where $v_f = 80\frac{km}{h} = 80 * \frac{1000m}{3600s} = 22.22\frac{m}{s}$, x = 100m,

$$v_i = 140\frac{km}{h} = 140 * \frac{1000m}{3600s} = 38.9\frac{m}{s}$$

Plugging in the values we get:

$$(22.22\frac{m}{s})^2 - (38.9\frac{m}{s})^2 = 2 * a * 100(m) \quad , \quad a = -5.1\frac{m}{s^2}$$

b) At what time after you pass the police car will your car stop?

$$v_f = v_0 + a * t$$

Where $v_f = 0$ (your car stops) , a = -5.1 m/s^2 .

$$t = \frac{v_0}{-a}$$

t= 7.6 s

Problem #4

You throw a ball upward with an initial speed v_0= 6 m/s . At what time does the ball reach ¾ of its maximum height?

We need two equations to find the time

$$\frac{3}{4} * y = v_0 * t + \frac{1}{2} * a * t^2$$

$$v_f^{\ 2} = v_0^{\ 2} + 2 * a * y$$ v_f =0 m/s, y = y_{max}, a = -9.8 m/s , v_0 =6 m/s

From the last equation

$$v_0^{\ 2} = 2gy_{\max},$$ $$y_{\max} = \frac{v_0^{\ 2}}{2 * g}$$ = 1.8 m

The first equation becomes

$$\frac{3}{4} * \frac{v_0^{\ 2}}{2 * g} = v_o * t - \frac{1}{2} * g * t^2$$ Rearranging we get

$$\frac{1}{2} * g * t^2 + (-v_0) * t + \frac{3}{8} * \frac{v_o}{2 * g} = 0$$ and t_1 =1.1 s

t_2 = 0.1 s

it reaches ¾ of the maximum height two times one on its way up and one on its way down.

Problem #5

Three motorcycles are moving in a straight line. Motorcycle B is 30 m ahead motorcycle C and motorcycle C is 20 m ahead motorcycle A. They move at constant speeds v_A = 7 m/s, v_C = 5 m/s, v_B= 4 m/s.
If the motorcycle A starts accelerating with a_A after 30 s he catches up with motorcycle B (which moves with constant speed). Find a_A.

The distance that motorcycle A will cover is:

$$d_A = d_{A-C} + d_{B-C} + d_B$$ where d_{A-C} =20 m , d_{B-C} = 30 m and d_B is the distance that motorcycle B has traveled

But

$d_B = v_B * t_1$ and the motion of motorcycle A is described by:

$$d_A = v_A * t_1 + \frac{1}{2} * a_A * t_1{}^2$$

Combining these three equations , we get

$$d_{A\_C} + d_{A-B} + v_B * t_1 = v_A * t_1 + \frac{1}{2} * a_A * t_1{}^2$$

$$a_A = 2 * \frac{d_{A-C} + d_{A-B} + (v_B - v_A) * t^2}{t^2}$$

Where $d_{A\text{-}C}$ =20 m, $d_{A\text{-}B}$=30m, v_B =4m/s, t_1 =30 s

Finally $a = 0.4m/s^2$

At what what time does motorcycle A catches up with Motorcycle C?

$$d_{A\_C} + d_C = v_A * t_2 + \frac{1}{2} * a_A * t_2{}^2$$

$d_c = v_c * t_2$ Replacing d_C in the equation above

We get

$d_{A\_C} + v_C * t_2 = v_A * t_2 + \frac{1}{2} * a_A * t_2{}^2$ Rearranging we get a quadratic equation in t_2

$$d_{A\_C} + v_C * t_2 - v_A * t_2 - \frac{1}{2} * a_A * t_2{}^2 = 0$$

Where $d_{A\text{-}C}$ = 20 m, v_C = 5 m/s, a_A = 0.4m/s^2

Neglecting the negative value , we get

$$t_2 = 39.2s$$