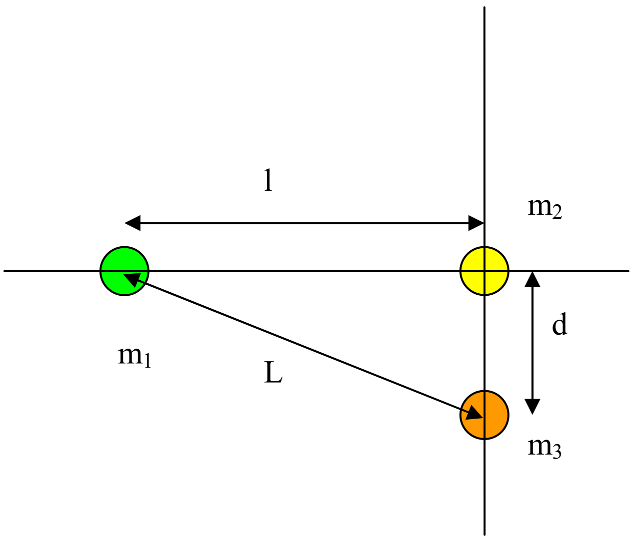
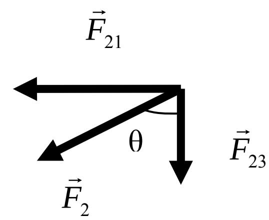
Problem # 1

Find the net force (direction and magnitude) acting on particle 2 (see figure below).The mass of ball1 is m_1= 2 kg, mass of ball 2 is 2*m_1 and the mass of ball 3 is m_1/2. The distance between ball1 and ball2 is l =4 cm and the distance between ball 1 and 3 is L=5 cm.





$$F_{21} = \frac{G * m_1 * m_2}{l^2}$$ where G= 6.67*10^{-11} N*m/kg^2 , m_1 = 2 kg, m_2= 4 kg and l=0.04m so $$\vec{F}_{21} = (-\hat{i}) * F_{21} = (-\hat{i}) * 3.33 * 10^{-7} N$$

$$F_{23} = \frac{G * m_2 * m_3}{d^2} \qquad F_{23} = \frac{G * m_2 * m_3}{(L^2 - l^2)}$$

where G= 6.67*10^{-11} N*m/kg^2 , m_2 = 4 kg, m_3= 1 kg and d=0.03m

so $$\vec{F}_{23} = (-\hat{j}) * F_{23} = (-\hat{j}) * 2.96 * 10^{-7} N$$

$$\vec{F}_2 = (-\hat{i}) * F_{21} + (-\hat{j}) * F_{23}$$

$$F_{2,net} = \sqrt{(F_{21})^2 + (F_{23})^2} = 4.46 * 10^{-7} N$$

$$\tan\theta = \frac{F_{21}}{F_{23}}, \quad \text{so } \theta = \tan^{-1}(\frac{F_{21}}{F_{23}}) = 48.36^{o}$$

Problem #2

A satellite with a mass of m=600 kg is orbiting around the Earth in a circular orbit at a distance of h=300 km from Earth's surface.
Find its angular momentum.
Find the time it takes the satellite to complete one revolution.
Find its total mechanical energy .
If you replace the satellite with another satellite that is four times heavier what will be the time to complete two revolutions?

a) $L = m*v*r$ where $r = R_E + h$

Newton's second law

$$\frac{G*M_E*m}{r^2} = \frac{m*v^2}{r} \quad \text{so } v = \sqrt{\frac{G*M_E}{r}}$$

$$L = m*\sqrt{\frac{G*M_E}{R_E+h}}*(R_E+h) = m*\sqrt{G*M_E*(R_E+h)}$$

Where

$G= 6.67*10^{-11}$ $N*m/kg^2$, $R_E= 6.37*10^6$ m, $h=0.3*10^6$ m, m= 600 kg,
$M_E = 5.98*10^{24}$ kg

So

$L=30.9*10^{12}$ kgm^2/s

b) $v = \frac{2*\pi*r}{T}$

$$T = \frac{2*\pi*r^{\frac{3}{2}}}{\sqrt{G*M_E}} = \frac{2*\pi*(R_E+h)^{\frac{3}{2}}}{\sqrt{G*M_E}} \quad \text{so } T = 54*10^2 \text{ s}$$

c) $E = K + U$

$$K = \frac{1}{2}m*v^2 = \frac{1}{2}\frac{G*m*M_E}{(R_E+h)}$$

$$U = -\frac{G*M_E*m}{(R_E+h)}$$

So $E = -\frac{G*M_E*m}{2*(R_E+h)} = 17.94*10^{-9}\,J$

d) the mass of the satellite does not appear in formula for T so

$t = 2*T = 108*10^2\,s$

Problem#3

An asteroid is moving directly toward the earth. When it hits the earth its speed is 14.14km/s. At what distance above the Earth's surface had the satellite a speed of 10 km/s? Neglect air resistance.
Find the ratio between the gravitational forces.

a)

The mechanical energy is conserved

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m*v_i^2 - \frac{G*M_E*m}{r} = \frac{1}{2}m*v_f^2 - \frac{G*M_E*m}{R_E}$$

So $\frac{1}{r} = \frac{1}{G*M_E*m}*[\frac{1}{2}*m*(v_i^2 - v_f^2) + \frac{G*M_E*m}{R_E}$

Where
G= $6.67*10^{-11}$ N*m/kg^2 , R_E= $6.37*10^6$ m, M_E = $5.98*10^{24}$ kg, $v_{initial}$=10^4m/s and v_{final}=$14.14*10^3$ m/s

So r = $0.8*10^7$ m

$h = r - R_E$

So h= $0.125*R_E$

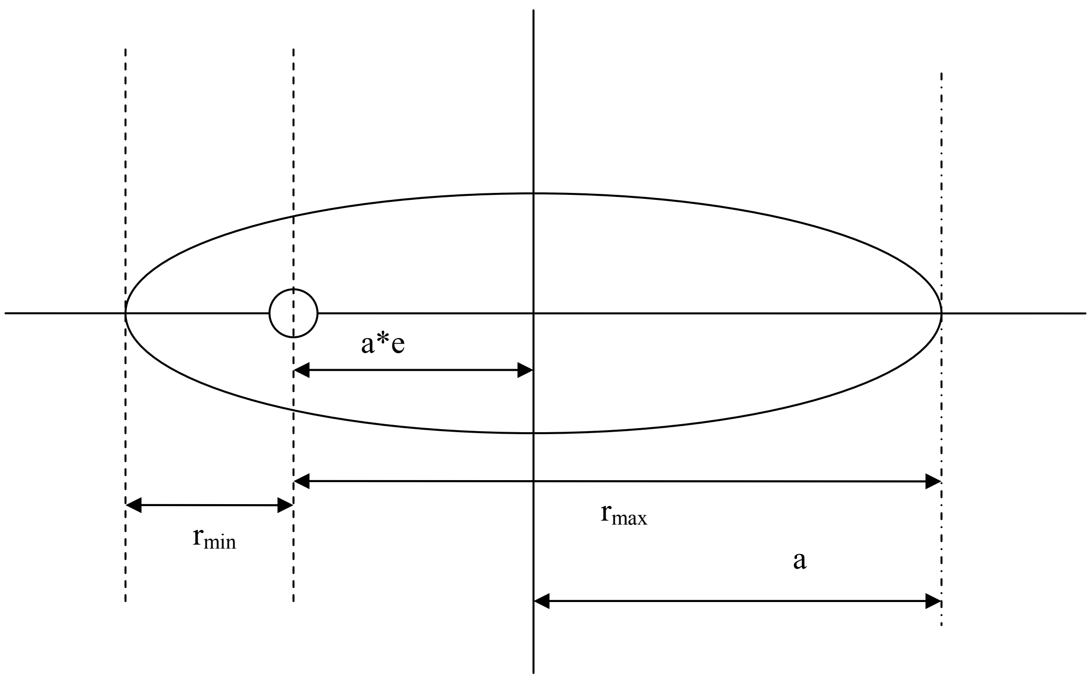
b) $F_{g,initial} = -\frac{G*M_E*m}{r^2}$

$$F_{g,final} = -\frac{G*M_E*m}{R_E{}^2}$$

$$\frac{F_{g,final}}{F_{g,initial}} = \frac{r^2}{R_E^2} \quad \text{so} \quad \frac{F_{g,final}}{F_{g,initial}} = \frac{(0.8*10^7)^2}{(6.37*10^6)^2} = 1.57$$

Problem#4

A comet is moving in an elliptical orbit around the earth. If the eccentricity of the orbit is 0.9 and the distance between aphelion and perihelion is L=10^{12} km find the distance of nearest approach of the comet.
What will the speed of the comet be at aphelion if the speed of the comet at perihelion is 100 km/s?



a) From the figure above

$r_{\min} + r_{\max} = 2a$

$r_{\max} = a + a*e = a(1+e)$

So $r_{\min} = 2a - a(1+e) = a(1-e)$

But $a = \dfrac{L}{2}$ so a= $5*10^{11}$ m

$$r_{\min} = \frac{L}{2} * (1-e) = 5*10^{10}\,m$$

b) Angular momentum is conserved

$$m * v_{aphelion} * r_{\min} = m * v_{perihelion} * r_{\max}$$

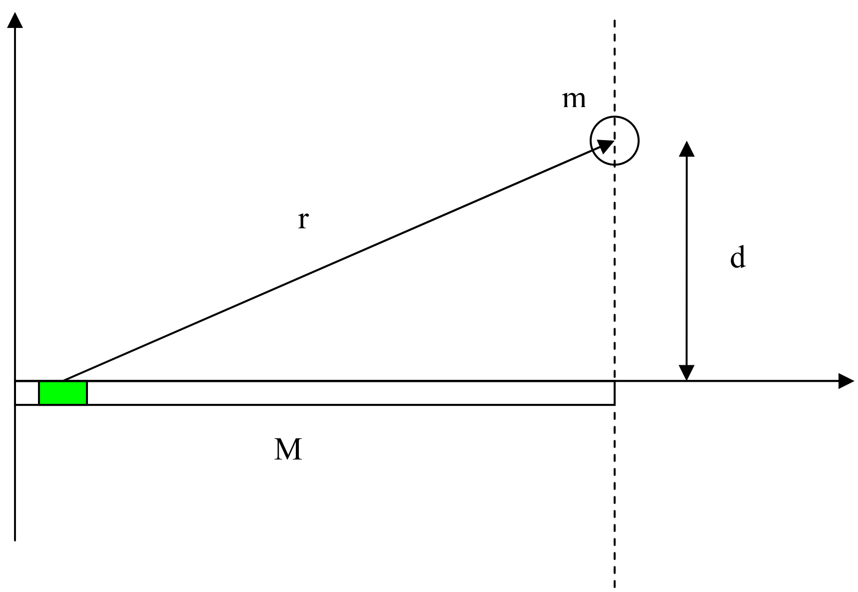
$$r_{\max} = 9.5*10^{11}\,m$$

So

$$v_{aphelion} = \frac{r_{\min}}{r_{\max}} * v_{perihelion}$$ so $v_{aphelion}$ = 52.63 km/s

Problem#5

A small sphere of mass m is placed at a distance d above one of the free ends of a uniform rod of length L and mass M. Find the gravitational potential energy of the system .Find the gravitational force between these two objects.



We divide the rod in an infinite numbers of tiny segments of mass dM and length dx
The gravitational potential energy of the system one mass element-sphere is

$$dU = -\frac{G*m*dM}{r}$$

Where $r = \sqrt{x^2 + d^2}$ and $dM = \frac{M}{L}*dx$

So

$$dU = -\frac{G*m}{\sqrt{x^2+d^2}}*\frac{M}{L}dx$$

The gravitational potential energy of the system rod-sphere is

$$U = \int dU = -\int_0^L \frac{G*m}{\sqrt{x^2+d^2}}*\frac{M}{L}dx = -\frac{G*m*M}{L}\int_0^L \frac{dx}{\sqrt{x^2+d^2}}$$

$$U = -\frac{G*M*m}{L}(\ln(x+\sqrt{x^2+d^{2)}}\Big|_0^L) = -\frac{G*M*m}{L}\ln\frac{L+\sqrt{L^2+d^2}}{d}$$

b)

$$dF = \frac{G*m*dM}{r^2}$$

$$F = \int dF = \int_0^L \frac{G*m*M}{L(x^2+d^2)}dx = \frac{G*m*M}{L}(\frac{1}{d}\arctan\frac{x}{d}\Big|_0^L)$$

$$F = \frac{G*m*M}{L}\frac{1}{d}\arctan\frac{L}{d}$$