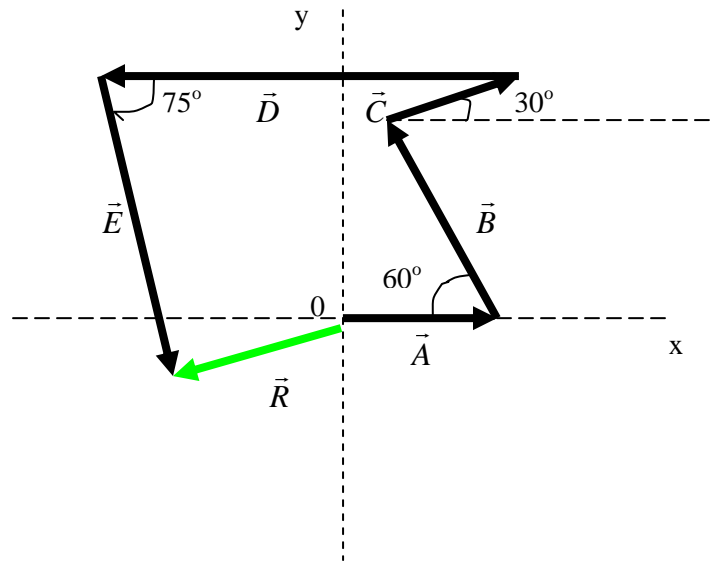


ADDING VECTORS

1 Find the magnitude of the resultant vector in the figure below



where the magnitudes of the vectors are : A = 12 units; B = 16units ; C = 9units;
D = 23units ; E = 20units

Note! The figure is not to scale.

Solution

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

$$\vec{R} = R_x * \hat{i} + R_y * \hat{j}$$

The components of the resultant vector are:

x-component

$$R_x = A_x + B_x + C_x + D_x + E_x$$

y-component

$$R_y = A_y + B_y + C_y + D_y + E_y$$

its magnitude is given by

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$A_x = A = 12 \text{ units}$$

$$B_x = B * \cos (180^\circ - 60^\circ) = -8 \text{ units}$$

$$A_y = A * \sin 0^\circ$$

$$B_y = B * \sin (180^\circ - 60^\circ) = 13.9 \text{ units}$$

$$C_x = C \cdot \cos 30^\circ = 7.8 \text{ units}$$

$$D_x = D \cdot \cos 180^\circ = -23 \text{ units}$$

$$E_x = E \cdot \cos (360^\circ - 75^\circ) = 5.2 \text{ units}$$

$$C_y = C \cdot \sin 30^\circ = 4.5 \text{ units}$$

$$D_y = D \cdot \sin 180^\circ = 0$$

$$E_y = E \cdot \sin (360^\circ - 75^\circ) = -19.3 \text{ units}$$

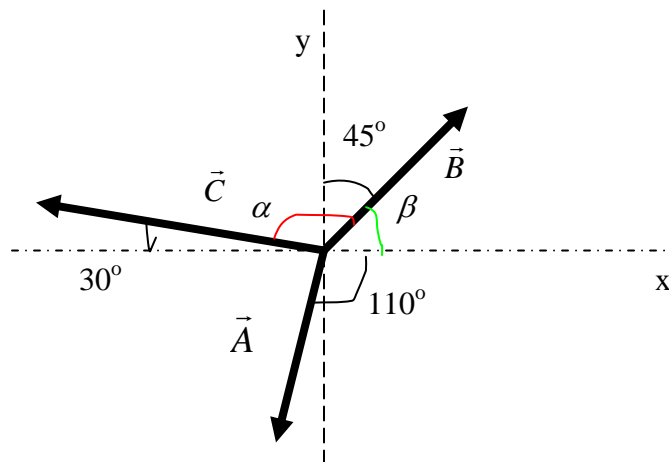
$$R_x = -6 \text{ units}$$

$$R_y = -23.9 \text{ units}$$

$$R = 143.4 \text{ units}$$

DOT PRODUCT

2 Find the scalar product $\vec{A} \cdot \vec{D}$ where $\vec{D} = \vec{B} + \vec{C}$, $A = 5$ units, $B = 5$ units, $C = 8$ units



Note! The figure is not to scale.

Solution

$$\vec{D} = \vec{B} + \vec{C}$$

$$B_x = B \cdot \cos \beta = B \cdot \cos (90^\circ - 45^\circ) \quad \text{where}$$

$$\beta = 90^\circ - 45^\circ = 45^\circ$$

$$C_x = C \cdot \cos (\alpha + \beta) \quad \text{where}$$

$$\alpha = 180 - \beta - 30 = 105^\circ$$

and

$$\alpha + \beta = 150^\circ$$

$$B_x = 3.5 \text{ units}$$

$$C_x = -6.9 \text{ units}$$

$$B_y = B \sin \beta = B \sin(90^\circ - 45^\circ) = 3.5 \text{ units}$$

$$C_y = C \sin(\alpha + \beta) = 4 \text{ units}$$

$$B_x = 3.5 \text{ units}$$

$$C_x = -6.9 \text{ units}$$

$$B_y = 3.5 \text{ units}$$

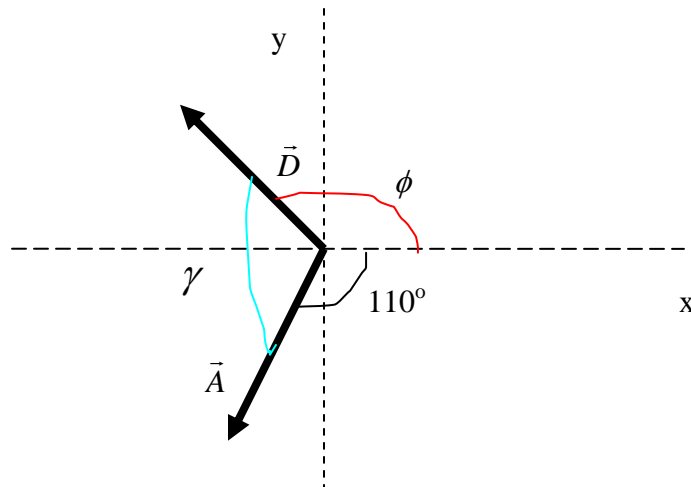
$$C_y = 4 \text{ units}$$

$$D_x = B_x + C_x = -3.4 \text{ units}$$

$$D_y = B_y + C_y = 7.5 \text{ units}$$

$$D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(B_x + C_x)^2 + (B_y + C_y)^2}, \quad D = 8.2 \text{ units}$$

$$\tan \phi = \frac{D_y}{D_x}, \quad \phi = \tan^{-1} \frac{D_y}{D_x}, \quad \phi = 114.4^\circ$$



Note! The figure is not to scale.

$$\gamma + \phi + 110^\circ = 360^\circ \quad \gamma = 360^\circ - \phi - 110^\circ = 135.6^\circ$$

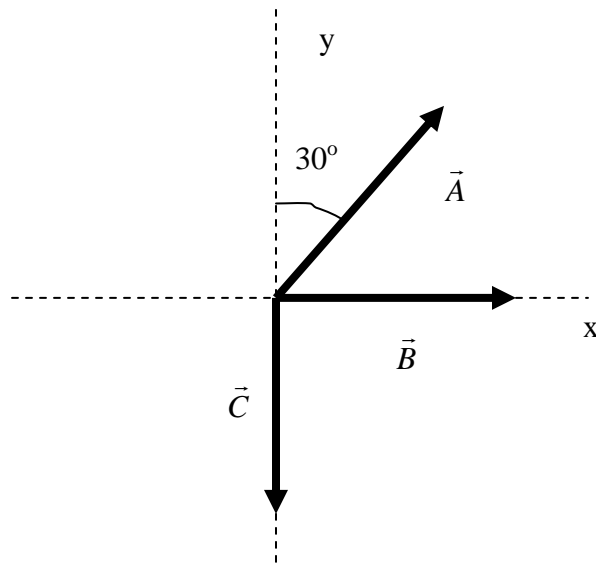
$$\vec{A} \cdot \vec{D} = A \cdot D \cdot \cos \gamma = 5 \cdot 8.2 \cdot \cos 135.6^\circ = -29.3 \text{ units}$$

CROSS PRODUCT

Find the magnitude and the direction of the following cross products:

- $\vec{A} \times (-\vec{A})$
- $\vec{A} \times (\vec{B} + \vec{C})$

where $A = 4$ units, $B = 3$ units, $C = 5$ units



Note! The figure is not to scale.

Solution

a) $\vec{A} \times (-\vec{A}) = -(\vec{A} \times \vec{A}) = 0$ *because both vectors are parallel*

b) $\vec{A} = A_x * \hat{i} + A_y * \hat{j}$

$$\vec{B} + \vec{C} = (B_x * \hat{i} + B_y * \hat{j}) + (C_x * \hat{i} + C_y * \hat{j})$$

$$A_x = A * \cos(90^\circ - 30^\circ) = 2 \text{ units}$$

$$B_x = B * \cos 0^\circ = B = 3 \text{ units}$$

$$C_x = C * \cos 270^\circ = 0$$

$$A_y = A * \sin(90^\circ - 30^\circ) = 3.5 \text{ units}$$

$$B_y = B * \sin 0^\circ = 0$$

$$C_y = C * \sin 270^\circ = -C = -5 \text{ units}$$

$$\vec{A} = A_x * \hat{i} + A_y * \hat{j}$$

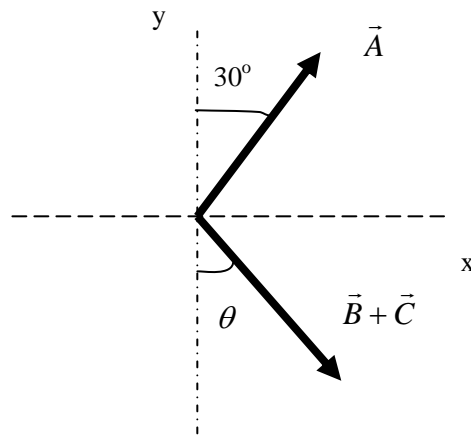
$$\vec{B} + \vec{C} = (B_x * \hat{i} + C_y * \hat{j})$$

$$\vec{A} \times (\vec{B} + \vec{C}) = (A_x * \hat{i} + A_y * \hat{j}) \times (B_x * \hat{i} + C_y * \hat{j}) = A_x * C_y * (\hat{i} \times \hat{j}) + A_y * B_x * (\hat{j} \times \hat{i})$$

$$= A_x * C_y * \hat{k} + A_y * B_x * (-\hat{k}) = (A_x * C_y - A_y * B_x) * \hat{k}$$

$$= -20.5 * \hat{k} = 20.5 * (-\hat{k})$$

The resultant vector has a magnitude of 20.5 units and points in the $-z$ direction (into the page)



Note! The figure is not to scale.

Another way of solving part b) would be

$$\tan \theta = \frac{|B_x|}{|C_y|} \quad \theta = \tan^{-1} \frac{B_x}{C_y} \quad \theta = 31^\circ$$

The angle between \vec{A} and $(\vec{B} + \vec{C})$ is $180^\circ - \theta - 30^\circ = 119^\circ$

$$|\vec{A} \times (\vec{B} + \vec{C})| = |\vec{A}| * |\vec{B} + \vec{C}| * \sin(180^\circ - \theta - 30^\circ)$$

$$|\vec{B} + \vec{C}| = \sqrt{B^2 + C^2} = 5.8 \text{ units}$$

$$|\vec{A} \times (\vec{B} + \vec{C})| = |\vec{A}| * |\vec{B} + \vec{C}| * \sin(180^\circ - \theta - 30^\circ) = 20.5$$

The direction is given by the right hand rule. The resultant vector points in the $-z$ direction (into the page)

SIGNIFICANT FIGURES

Multiplication

$$0.32 * 2.45 = 0.78 = 78 * 10^{-2}$$

$$0.65 * 9.01 = 5.9 = 59 * 10^{-1}$$

Division

$$\frac{8.62}{4.3} = 2.0$$

$$\frac{256}{5.256} = 48.7 = 487 * 10^{-1}$$

Addition

$$21.43 + 19.5 = 40.9 = 409 * 10^{-1}$$

$$425.15 + 10 = 435$$

Subtraction

$$89.65 - 23.4 - 18.165 = 48.1$$

$$23 - 87.15 = -64$$

All combined

$$423.356 - 24.5 * 120.82 + \frac{16.86}{19.82} = 424.356 - 2960 + 0.878 = 3385$$

$$23.62 * \frac{123.03}{133} + 1.43 * (17.9 - 12.39) + 24.12 * 0.45 + 18.72 = 21.8 + 143 * 5.5 + 11 + 18.72 = \\ = 21.8 + 79 * 10^1 + 18.72 = 831$$

UNIT CONSISTENCY and CONVERSIONS

The following equation is given:

$$a = b * c - d * e + f * \frac{g}{h}$$

1) If **a** is measured in cm , **c** in minutes , **e** in (seconds)² and $\frac{g}{h}$ in m which will be the SI measurement units for b , e and f ?

Solution

The SI measurement unit for length is **m** (meter)

The SI measurement unit for time is **s** (second)

$$1 \text{ cm} = 10^{-2} \text{ m}$$

$$1 \text{ minute} = 60 \text{ s}$$

The SI measurement unit for **a** is **m**

The SI measurement unit for **b*c** must be **m** so the measurement unit for b must be **m/ s**

$$\frac{m}{s} * s = m$$

The SI measurement unit for d*e must be **m** so the measurement unit for b must be **m/s²**

$$\frac{m}{s^2} * s^2 = m$$

The SI measurement unit for $f * \frac{g}{h}$ must be **m** so f must be unitless (e.g a number)

2) If $a = 58000 \text{ cm}$
 $b = 10 \text{ m/s} = 10 \text{ m*s}^{-1}$
 $c = 2 \text{ minutes}$
 $d = 4 \text{ m/s}^2 = 4 \text{ m*s}^{-2}$
 $e = 200 \text{ s}^2$
 $f = 18$
 $h = 3 \text{ s}^{-1}$

Find the value and the measurement unit for g.

Solution

First we have to convert cm into m and minutes in seconds. Let us take the measurement unit for g to be **x**.

Then

$$58000 * 10^{-2} (m) = 10 \left(\frac{m}{s} \right) * 2 * 60 (s) - 4 \left(\frac{m}{s^2} \right) * 200 (s^2) + 18 * \frac{g(x)}{3 \left(\frac{1}{s} \right)}$$

$$580(m) = 1200(m) - 800(m) + 6 * g(x * s) \text{ where } \mathbf{x*s} \text{ must be } \mathbf{m}$$

So the SI measurement unit for g must be $m \cdot s^{-1}$ because $m \cdot s^{-1} \cdot s = m$

$$580(m) = (400 + 6 \cdot g)(m)$$

$$580 - 400 = 6 \cdot g, \quad g = \frac{180}{6}, \quad g = 30 \frac{m}{s}$$