Problem 1

A disk of mass m=4.0kg, and radius R=0.5m, rotates about an axis through its center, perpendicular to the plane of the disk. Calculate the moment of inertia of the disk. Use the parallel axis theorem to calculate the moment of inertia if the axis is moved a distance R/2 away from the center.

Solution 1

The moment of inertia is

$$I = \int r^2 dm$$

dm is an infinitesimal element of mass
r is the distance from the axis of rotation to the mass element dm.
For a disk use

$$dm = \frac{m}{A} 2\pi r dr$$

which represents a ring of circumference 2πr, infinitesimal width dr and mass density m/A, where A is the area of the disk.

$$I = \int r^2 \frac{m}{A} 2\pi r dr = \frac{2\pi m}{A} \int_0^R r^3 dr = \frac{2\pi m}{\pi R^2} \left[\frac{r^4}{4} \right]_0^R = \frac{1}{2} mR^2$$

$$= 0.5 kgm^2$$

The parallel axis theorem

$$I_P = I_{cm} + md^2$$

where d is the distance from the center of mass

I_{cm} is the moment of inertia through the center of mass. For a disk the center of mass is the center of the disk.

$$I_P = I_{cm} + md^2 = 0.5 kgm^2 + (4.0kg)(\frac{R}{2})^2 = 16.25 kgm^2$$

Problem 2

A thin rod has a mass m=3.0kg and length L=0.5m. The rod rotates about an axis which is through its center of mass and perpendicular to the rod with initial angular velocity $\omega = \frac{\pi}{2} rad/s$.

a)What is the moment of inertia for the rod?
b) What is the kinetic energy of the rod?
c) If the rod's angular velocity is 2πrads^{-1} at t= 4s, what is the rod's constant angular accerleration?
d) What is the initial linear velocity of one of the ends of the rod?

Solution 2

First the moment of inertial of the rod needs to be found

$$I = \int r^2 dm$$

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{m}{L} dx = \frac{m}{L} \left[\frac{x^3}{3} \right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{m}{3L} \left[\frac{L^3}{8} + \frac{L^3}{8} \right] = \frac{1}{12} mL^2$$

Then the initial kinetic energy can be found

$$K = \frac{1}{2}I\omega^2 = \frac{1}{24}mL^2\omega^2 = \frac{1}{24}(3)(.5)^2(\frac{\pi}{2})^2 = 0.077J$$

The rotational kinematic equation,

$$\omega = \omega_0 + \alpha t$$

$$2\pi = \frac{\pi}{2} + \alpha(4)$$

$$\alpha = 1.2rads^{-2}$$

Linear velocity is related to angular velocity by

$$v = r\omega$$

$$v = (L/2)(\pi/2) = 0.39m/s$$

Problem 3

A yo-yo is released from rest and has a string length L and radius R. Find the rotational acceleration of the yo-yo.

Solution 3

From conservation of energy the final angular velocity of the yo-yo can be found

$$K_1 + U_1 = K_2 + U_2$$

The yo-yo is released from rest, the initial kinetic is zero.

The initial potential is taken as zero.

The final kinetic comes from the linear and rotational motion of the yo-yo.

The finial potential is from gravity and has magnitude –mgL.

$$0 + 0 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 - mgL$$

The linear velocity is related to the angular velocity by v=Rω, and the yo-yo is considered a disk with $I = \frac{1}{2}mR^2$.

$$0 + 0 = \frac{1}{4}mR^2\omega^2 + \frac{1}{2}mR^2\omega^2 - mgL$$

$$\omega = \sqrt{\frac{4gL}{3R^2}}$$

From the rotational kinematic equations

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

The rotational acceleration is found.

The final angle is $\theta = L/R$

The initial angle is 0.

The initial angular velocity is zero.

Plugging in

$$\frac{L}{R} = \frac{1}{2}\alpha t^2$$

$$\sqrt{\frac{4gL}{3R^2}} = \alpha t$$

Combing the two equations

$$\frac{L}{R} = \frac{1}{2}\alpha \frac{4gL}{\alpha^2 3R^2}$$

$$\alpha = \frac{2g}{3R}$$

Problem 4

A compact disk is placed in a CD ROM drive of a computer. The critical energy of a CD is the amount of energy a CD can store before shattering and is about 133J. If the mass of the CD is m=0.015kg, the inner radius is R_1=0.007m and the outer radius is R_2=0.06m, what is the minimum rotational velocity needed to break the CD? If the CD-ROM spins the CD at 30000rpm, will the CD shatter?

Solution 4

The energy of a rotating object is given by

$$K = \frac{1}{2} I\omega^2$$

The moment of inertial is calculated from

$$I = \int r^2 dm$$

$$I = \int_{R1}^{R2} r^2 \frac{m}{A} 2\pi r dr = \frac{2\pi m}{A}\left[\frac{r^4}{4}\right]_{R1}^{R2} = \frac{m}{2(R_2^2 - R_1^2)}(R_2^4 - R_1^4) = 2.74\times 10^{-5} kgm^2$$

From the energy equation

$$133J = \frac{1}{2}(2.74x10^{-5})\omega^2$$

$$\omega = 3116 rad/s$$

If the CD-ROM spins at 30000rpm the angular velocity is

$$\frac{30000rev}{\min}\times\frac{\min}{60s}\times\frac{2\pi rad}{rev} = 3142 rad/s$$

the CD will explode.

Problem 5

Assume that the orbits of the earth and moon are circular. What is the magnitude of the gravitational force of the sun on the earth? Of the earth on the moon? Calculate the gravitational constant for each system (earth-sun and earth-moon), if the force of gravity is given by $F_g = \frac{-GMm}{R^2}$, where M is the mass of the central body, m is the mass of the orbiting body and R is the orbital radius.

Given:

$m_e = 5.98x10^{24} kg$

$m_m = 7.36x10^{22} kg$

$M_{sun} = 1.99x10^{30} kg$

$R_e = 1.5x10^{11} m$

$R_m = 3.84x10^8 m$

Solution 5

The angular velocity of the earth is

$$\omega = \frac{2\pi rad}{365 days} \times \frac{days}{24 hours} \times \frac{hours}{60 \min} \times \frac{\min}{60s} = 1.99x10^{\wedge}-7 rad/s$$

Use Newton’s second law in the radial direction

$$\sum F_r = ma_r$$

$$F_g = -mR\omega^2$$

$$F_g = -(5.98x10^{24} kg)(1.5x10^{11} m)(1.99x10^{-7} 1/s)^2 = 3.55x10^{22} N$$

The angular velocity of the moon is

$$\omega = \frac{2\pi rad}{27.3 days} \times \frac{days}{24 hours} \times \frac{hours}{60 \min} \times \frac{\min}{60s} = 2.66x10^{-6} rad/s$$

Use Newton’s second law in the radial direction

$$\sum F_r = ma_r$$

$$F_g = -mR\omega^2$$

$$F_g = -(7.36x10^{22} kg)(3.84x10^8 m)(2.66x10^{-6} 1/s)^2 = 2.00x10^{20} N$$

$$G = \frac{-F_g R_e^{\ 2}}{M_{sun} m_e} = \frac{(3.55x10^{22})(1.5x10^{11})^2}{(1.99x10^{30})(5.98x10^{24})} = 6.71x10^{-11} Nm^2/kg^2$$

$$G = \frac{-F_g R_m^{\ 2}}{m_m m_e} = \frac{(2.00x10^{20})(3.84x10^8)^2}{(7.36x10^{22})(5.98x10^{24})} = 6.70x10^{-11} Nm^2/kg^2$$

The accepted value is

$G = 6.67x10^{-11} Nm^2/kg^2$