Problem 1
Two objects are moving in the x-y plane. The first has mass 3.0kg, and a velocity $\vec{v}_1 = (-2.3m/s)\hat{i} + (-3.1m/s)\hat{j}$. The second object has mass 1.5kg and a velocity given by $\vec{v}_2 = (1.6m/s)\hat{i} + (-2.0m/s)\hat{j}$.
a) What is the total momentum of the system?
b) If at some later time the velocity of the first object is observed as $\vec{v}'_1 = (2.3m/s)\hat{i}$, what is the velocity of the 1.5kg object?
Solution 1
a) The momentum of each object

$$\vec{p}_1 = m_1\vec{v}_1 = (-6.9kgm/s)\hat{i} + (-9.3kgm/s)\hat{j}$$
$$\vec{p}_2 = m_2\vec{v}_2 = (2.4kgm/s)\hat{i} + (-3.0kgm/s)\hat{j}$$

The total momentum

$$\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = (-4.5kgm/s)\hat{i} + (-12.3kgm/s)\hat{j}$$

b) Momentum is conserved

$$\vec{p}_{total} = \vec{p}_1 + \vec{p}_2$$
$$\vec{p}_2 = \vec{p}_{total} - \vec{p}_1$$
$$\vec{p}_2 = (-4.5kgm/s)\hat{i} + (-12.3kgm/s)\hat{j} - (6.9kgm/s)\hat{i}$$
$$\vec{p}_2 = (-11.4kgm/s)\hat{i} + (-12.3kgm/s)\hat{j}$$

Problem 2
The burnout velocity of a rocket is its final velocity when all the fuel has been used. Find the burnout velocity for a rocket in a uniform gravitational field. If the fuel to initial mass ratio is, $m_{fuel}/m_0 = 0.7$, the t = 120s and u_{ex} = 2600 m/s, what is the burnout velocity?
Solution 2
The differential equation for a rocket in a gravitational field is

$$-mg - u_{ex}\frac{dm}{dt} = m\frac{dv}{dt}$$
$$-\int_0^t gdt - u_{ex}\int_{m0}^{m0-mfuel}\frac{1}{m}dm = \int_0^{vburnout} dv$$
$$-gt - u_{ex}\ln\left(\frac{m_0 - m_{fuel}}{m_0}\right) = v_{burnout}$$

Just plug in the numbers.

$$-(9.8m/s^2)(120s) - (2600m/s)\ln(1-0.7) = v_{burnout}$$
$$v_{burnout} = 1954m/s$$

Problem 3
Two children sit on a frictionless surface. They play a game where they slide a rock, m = 3kg, back and forth. Each child has a mass, M=45kg, and each child gives the rock a speed of 1.0m/s when they toss it. Both children are initially at rest.
a) The first child slides the rock. What is his speed after he does so?
b) The second child catches the rock. What is his velocity?
c) The second child slides the rock back. Now what is his velocity?
d) Can this game go on forever?

Solution 3

The motion is one-dimensional, we use a coordinate system on the ice and take the positive direction from the first to the second child.

(a) When the first slides the rock, we use momentum conservation:

$p_{1i} + p_{ri} = p_{1f} + p_{rf}$;

0 + 0 = (45 kg)v_{1f} + (3.0 kg)(1.0 m/s), which gives

$v_{1f} = \boxed{-0.066 \text{ m/s}}$.

(b) When the second receives the rock, we use momentum conservation:

$p_{2i} + p_{ri} = p_{2f} + p_{rf}$;

0 + (3.0 kg)(1.0 m/s) = (45 kg + 3.0 kg)v_{2f} , which gives

$v_{2f} = \boxed{0.063 \text{ m/s}}$.

(c) When the second child slides the rock, we use momentum conservation:

$p_{2i} + p_{ri} = p_{2f} + p_{rf}$

(45 kg + 3.0 kg)[(0.063) m/s] = (45 kg)v_{2f} + (3.0 kg)[(0.063) - 1.0] m/s, which gives

$v_{2f} = \boxed{0.13 \text{ m/s}}$.

(d) For each throw, the thrower's speed increases. Because the speed of the rock is 1.0 m/s with respect to the thrower, eventually the rock will not have a velocity toward the other child with respect to the ice greater than the velocity of that child and thus will not reach that child. The game does not go on forever.

Problem 4

A person catches a ball. The pitch has velocity 50m/s when it arrives and the catcher's hand recoils 0.23m. The mass of the ball is 0.15kg. The deceleration of the ball occurs in a time, Δt, and has a constant magnitude.

a) Find the magnitude of the force exerted by the catcher.

b) How much work does the catcher do?

c) Find Δt.

Solution 4

a) We find the assumed constant acceleration from

$v^2 = v_0^2 + 2a\,\Delta x$;

$0 = (50 \text{ m/s})^2 + 2a(0.23 \text{ m})$, which gives $a = 5400 \text{m/s}^2$.

By applying Newton's second law, we get

$F = ma = (0.15 \text{ kg})(5400 \text{ m/s}^2) = \boxed{820\text{N}}$.

b) The work done is $W = F\,\Delta x = (820 \text{ N})(0.23 \text{ m}) = \boxed{190 \text{ J}}$.

c) The impulse exerted changes the momentum of the ball:

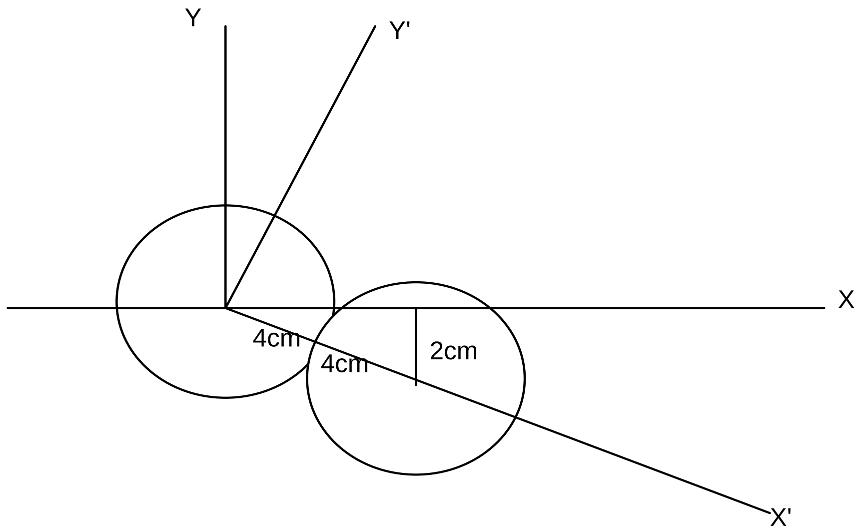
$F\,\Delta t = \Delta p$, from which we get

$\Delta t = \Delta p/F = (0.15 \text{ kg})(0 + 50 \text{ m/s})/(820 \text{ N}) = \boxed{0.0091 \text{ s}}$.

Problem 5

Two identical billiard balls each with radius 4cm move toward each other with velocities $(2.0m/s)\hat{i}$ and $(-1.0m/s)\hat{i}$. The center of one moves along the x axis in the positive x direction and the other moves in the negative x direction 2.0cm below the x axis (y= -0.02m). They collide elastically, what are the final velocities?

Solution 5



not to scale

Because the impulse is directed along the line joining the centers, we choose the $x'y'$ coordinate system indicated on the diagram.

The angle between the two systems is $\theta = \sin^{-1}[(2.0 \text{ cm})/(8.0 \text{ cm})] = 14.5°$, and the unit vectors of the two systems transfer according to

$$\hat{i}' = \hat{i} \cos\theta - \hat{j} \sin\theta, \hat{j}' = \hat{i} \sin\theta + \hat{j} \cos\theta.$$

The initial velocities are

$$\vec{v}_{A1} = (2.0m/s)\hat{i} = (2.0m/s)\cos(\theta)\hat{i}' + (2.0m/s)\sin(\theta)\hat{j}' = (1.9m/s)\hat{i}' + (0.50m/s)\hat{j}'$$

$$\vec{v}_{B1} = (-1.0m/s)\hat{i} = (-1.0m/s)\cos(\theta)\hat{i}' + (-1.0m/s)\sin(\theta)\hat{j}' = (-0.97m/s)\hat{i}' + (-0.25m/s)\hat{j}'$$

The y'-components are not changed. In the x'-direction momentum is conserved, so we have

$$m(1.9 \text{ m/s}) + m(-0.97 \text{ m/s}) = mv_{A2x'} + mv_{B2x'}.$$

Because the collision is elastic, the relative speed in the x'-direction does not change:

$$v_{A1x'} - v_{B1x'} = -(v_{A2x'} - v_{B2x'}), \quad \text{or} \quad 1.9 \text{ m/s} - (-0.97 \text{ m/s}) = -v_{A2x'} + v_{B2x'}.$$

Combining these two equations, we get

$$v_{A2x'} = -0.97 \text{ m/s}; \quad v_{B2x'} = 1.9 \text{ m/s}.$$

As expected for a perfectly elastic collision between equal masses, the velocities are exchanged.

When the final velocities are transformed back to the original system using the transformation formulas for the unit vectors, we get

$$\vec{v}_{A2} = (-0.97m/s)\hat{i}' + (0.50m/s)\hat{j}' = (-0.45m/s)i + (0.73m/s)j$$

$$\vec{v}_{B2} = (1.9m/s)\hat{i}' + (-0.25m/s)\hat{j}' = (1.78m/s)i + (-0.72m/s)j$$