Problem 1

A box of mass 30kg is pushed with a force of 95N at a downward angle of 15º. The box is initially at rest at $x_1 = 0m$ and has a speed of $v_2 = 0.50m/s$ at position $x_2 = 2.50m$

a) Calculate the coefficient of kinetic friction

b) What is the net work done?

c) How much work is done to overcome friction?

Solution 1

Because the forces are constant, the acceleration is uniform.

It can be found from the velocity kinematic equation

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$v_2^2 = v_1^2 + 2a(x_2 - x_1)$$

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{0.25}{5} = 0.05m/s^2$$

Now Newton's 2nd Law

X – component $\quad F\cos\theta - \mu_k F_N = ma.$

Y – component $\quad F_N - mg - F\sin\theta = 0$

Use the y equation to solve for Fn and plug into the x equation.

Then solve for the coefficient of friction

$$\mu_k = (ma - F\cos\theta)/(-mg - F\sin\theta) = \frac{((30)(0.05) - (95)\cos(15))}{(-(30)(9.8) - (95)\sin(15))} = 0.28$$

b) The net work is the change in kinetic energy

$$W_{net} = \Delta KE = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}(30kg)((0.50m/s)^2 - 0) = 3.75J$$

c)The net work is

$W_{net} = W_F + W_{fric}$

There is no change in position in the y direction so the y forces do no work.

The work of the applied force is $F\cos\theta\Delta x = (95N)\cos(15)(2.50m) = -229N$

Just subtract from the net work to find the work of the friction force

$W_{fric} = W_{net} - W_F = 3.75J - 229J = -225J$

The work done opposing the friction force is 225J

Problem 2

An unusual spring exerts a force $F = -k_1 x - k_2 x^3$. The values for the spring constants are k1 = 6 N/m and k2 = 14 N/m^3. Calculate the work done to stretch the spring from 0.09m to 0.15 m.

Solution 2

The work done by a position dependent force is given by the equation

$$W = \int_{xi}^{xf} F(x)dx$$

Using the force for the spring

$$W = \int_{xi}^{xf} -k_1 x - k_2 x^3 dx = -k_1 \frac{x_f^{\ 2}}{2} - k_2 \frac{x_f^{\ 4}}{4} + k_1 \frac{x_i^2}{2} + k_2 \frac{x_i^4}{4}$$

Plugging in

$$W = -(6)\frac{(0.15)^2}{2} - (14)\frac{(0.15)^4}{4} + (6)\frac{(0.09)^2}{2} + 14\frac{(0.09)^4}{4} = -0.045J$$

This is the work done by the force, we need the work done in opposition to this force, which is just the negative of the quantity calculated.

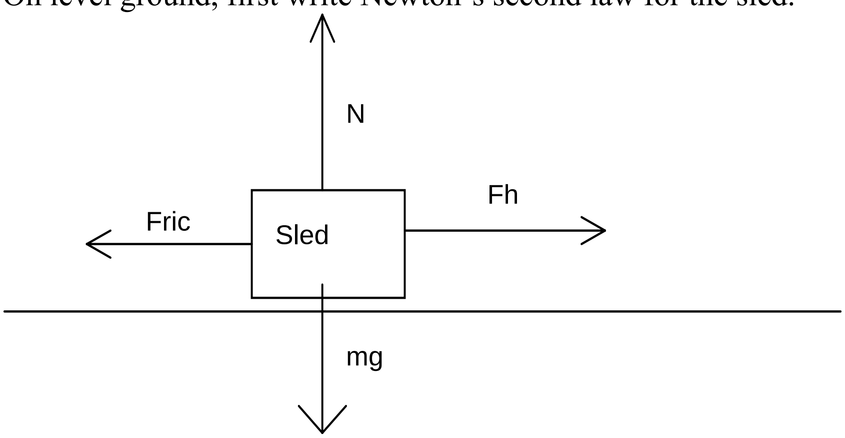
W = 0.045J

Problem 3

The power of a horse is 1 hp. With what speed can this horse pull a sled that weighs 4000N on level ground, the coefficient of kinetic friction between the sled and the ground is 0.04? What is the speed if the horse heads up an incline at 7º?

Solution 3

On level ground, first write Newton's second law for the sled.



X – direction

$$\sum F_x = ma$$

$$-F_{ric} + F_h = 0$$

$$-\mu_k N + F_h = 0$$

Y – direction

$$\sum F_y = ma$$

$$N - mg = 0$$

$$N = mg$$

Replace N in the x equation with mg

$$\mu_k mg = (0.04)(4000) = 160N = F_h$$

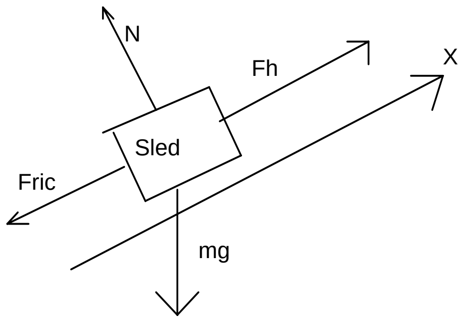
Power equation

P = Fv

$$1hp\left(\frac{746W}{hp}\right) = (160N)v$$

$$v = 4.7m/s$$

On the incline the force of gravity must be put into component form.



X – direction

$$\sum F_x = ma$$

$$-F_{ric} + F_h - mg\sin\theta = 0$$

$$-\mu_k N + F_h - mg\sin\theta = 0$$

Y – direction

$$\sum F_y = ma$$

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

Replace N in the x equation with mgcos(theta)

$$-\mu_k mg\cos\theta + F_h - mg\sin\theta = 0$$

$$F_h = \mu_k mg\cos\theta + mg\sin\theta = (0.04)(4000N)\cos(7) + (4000N)\sin(7) = 646N$$

Power equation

P = Fv

$$1hp\left(\frac{746W}{hp}\right) = (646N)v$$

$$v = 1.2m/s$$

Problem 4

A force is given by the expression $F = C|x|$. A) Calculate the work done by the force when it moves an object from x = -3m to x = 3m. B) From 0m to 6m.
Compare your answers with the work from F = Cx.

Solution 4

The work done by a position dependent force is given by the equation

$$W = \int_{xi}^{xf} F(x)dx$$

$$W = \int_{-3}^{3} C|x|dx = \int_{-3}^{0} -Cxdx + \int_{0}^{3} Cxdx$$

$$= -C\left[\frac{x^2}{2}\right]_{-3}^{0} + C\left[\frac{x^2}{2}\right]_{0}^{3} = \frac{C9}{2} + \frac{C9}{2} = C9$$

If the force were given by F = Cx

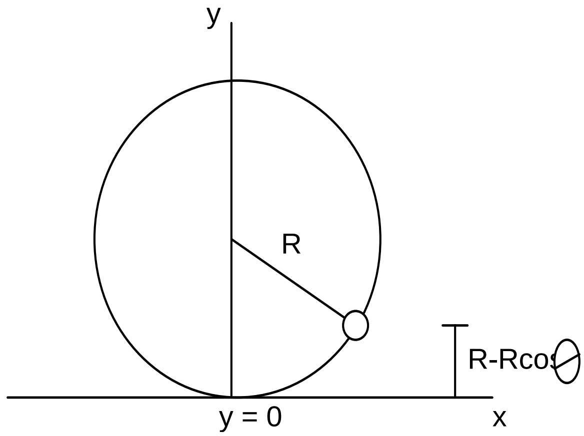
$$W = \int_{xi}^{xf} F(x)dx = \int_{-3}^{3} Cxdx = C\left[\frac{x^2}{2}\right]_{-3}^{3} = C\left(\frac{(3)^2}{2} - \frac{(-3)^2}{2}\right) = 0$$

From 0 to 6m

$$W = \int_{xi}^{xf} F(x)dx = \int_0^6 C|x|dx = \int_0^6 Cxdx = C\frac{(6)^2}{2} = 18C$$

Problem 5

A mass swings on the end of a rope in a vertical circle of radius R. The mass starts at the lowest point and is swung through an angle θ above the start. What forces are acting on the mass? Which forces do work on the mass? Calculate the work done on the mass.



Solution 5

The forces acting on the mass are the tension in the string and gravity.

The tension is always pointed perpendicular to the displacement of the mass. From the work equation $\vec{W} = \vec{F} \cdot \Delta\vec{r} = T\Delta r\cos(90) = 0$ it can be seen that the tension does no work on the mass. Gravity always points downward and does do work on the mass.

The work done by gravity is

$W = \vec{F} \cdot \Delta\vec{r} = -mg\hat{j} \cdot (\Delta x\hat{i} + \Delta y\hat{j}) = -mg\Delta y$

Theta goes from 0 to θ and if we call the lowest point y = 0, y goes from 0 to R-Rcos(θ)

$W = -mg(R - R\cos\theta - 0) = -mg(R - R\cos\theta)$