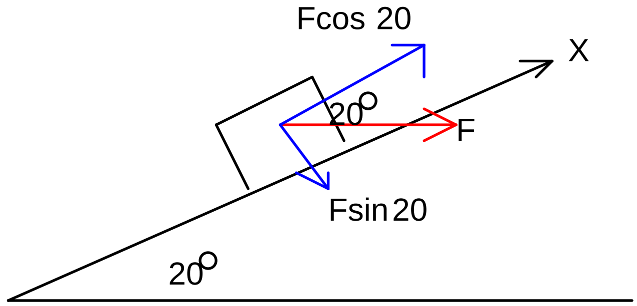
Problem 1
A horizontal force is applied to push a 30kg block up a 20° inclined plane.
Calculate the force needed to hold the block at equilibrium.
If 4 times that force is applied what will be its acceleration.

Solution 1

In equilibrium, there is no acceleration of the block.

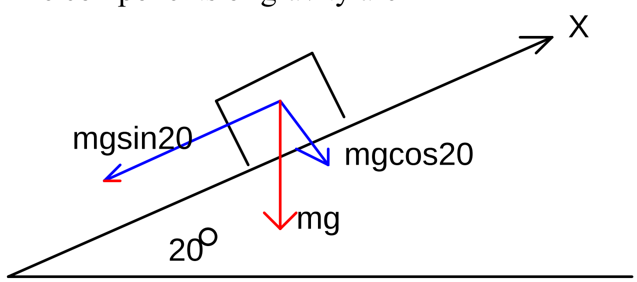
The components of the applied force are



$F_x = F\cos 20$

$F_y = F\sin 20$

The components of gravity are



$F_{gx} = -mg\sin 20$

$F_{gy} = -mg\cos 20$

And the normal force points in the +y direction

We write $\Sigma\vec{F} = m\vec{a}$

y-component: $F_N - F\sin\theta - mg\cos\theta = 0$,

x-component: $F\cos\theta - mg\sin\theta = 0$,

$F\cos 20° - (30\text{ kg})(9.8\text{ m/s}^2)\sin 20° = 0$,

which gives $F = \boxed{107\text{N}}$.

With a larger force, the block will accelerate up the plane:

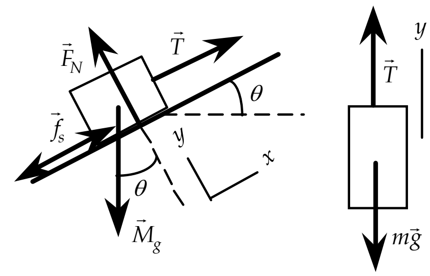
x-component: $F\cos\theta - mg\sin\theta = ma$,

$4(107\text{ N})\cos 20° - (30\text{ kg})(9.8\text{ m/s}^2)\sin 20° = (30\text{ kg})a$, which gives

$a = \boxed{10.1\text{ m/s}^2\text{ up}}$.

Problem 2

Two blocks of masses M and m are connected by a massless cord that passes through a frictionless pully. Mass M sits on an inclined plane at 25° and m hangs freely. The coefficient of static friction between the mass and the plane is 0.15. Find the largest and smallest vales for M that the system remains at rest if m = 4.0kg. Find the static friction force if M = 8.0kg.



Solution 2

For the largest value of M, the block is on the verge of slipping down the plane, so the static friction force will be up the plane and maximum, $f_s = f_{s,\max} = \mu_s F_N$.

From the force diagram, with the block M as the system, we can write $\Sigma \vec{F} = M\vec{a}$:

x-component: $T - M_{\max} g \sin\theta + f_{s,\max} = 0$;

y-component: $F_N - M_{\max} g \cos\theta = 0$; or $F_N = M_{\max} g \cos\theta$.

From the force diagram, with the block m as the system, we can write $\Sigma \vec{F} = m\vec{a}$:

y-component: $T - mg = 0$; or $T = mg$.

The x-equation becomes

$mg - M_{\max} g \sin\theta + \mu_s M_{\max} g \cos\theta = 0$, or $M_{\max}(\sin\theta - \mu_k \cos\theta) = m$;

$M_{\max}(\sin 25° - 0.15 \cos 25°) = 4.0$ kg, which gives $M_{\max} = \boxed{14\text{ kg}}$.

For the smallest value of M, the block is on the verge of slipping up the plane, so the static friction force will be down the plane and maximum, $f_s = f_{s,\max} = \mu_s F_N$. The only change will be in the x-equation. From the force diagram, with the block M as the system, we can write $\Sigma \vec{F} = M\vec{a}$:

x-component: $T - M_{\min} g \sin\theta - f_{s,\max} = 0$; $mg - M_{\min} g \sin\theta - \mu_s M_{\min} g \cos\theta = 0$, or

$M_{\min}(\sin 25° + 0.15 \cos 25°) = 4.0$ kg, which gives $M_{\min} = \boxed{7.2\text{ kg}}$.

If $M = 8$ kg, the block will remain at rest. We assume that the static friction force will be up the plane. The x-equation becomes $mg - Mg \sin\theta + f_s = 0$;

$(4.0\text{ kg})(9.8\text{ m/s}^2) - (8.0\text{ kg})(9.8\text{ m/s}^2) \sin 25° + f_s = 0$, which gives $f_s = \boxed{6.1\text{N}}$.

Problem 3

An Atwood machine consists of a massless string over a pulley which connects two masses. For this problem the masses are m1 = 1.56kg and m2 = 1.80kg. The system is released from rest with m1 on the floor and m2 2.80m above the floor.

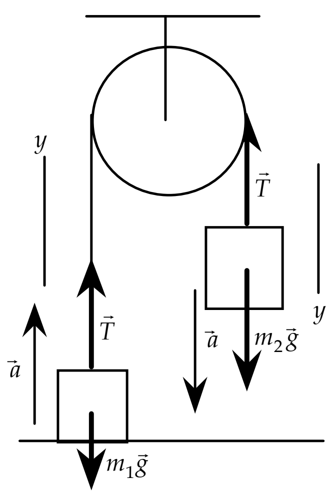
a) What are the accelerations of m1 and m2?

b) What is the velocity of m2 just before it hits the floor?

c) How long does it take m2 to hit the floor?

Solution 3

Forces are drawn for each of the blocks for the situation when m_1 leaves the floor (no normal force). Because the string doesn't stretch, the tension is the same at each end of the string, and the accelerations of the blocks have the same magnitude. Note that we take the positive direction in the direction of the acceleration for each block.



(a) We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for each block:

y-component (block 1): $T - m_1 g = m_1 a$.

y-component (block 2): $m_2 g - T = m_2 a$.

By adding the equations, we find the acceleration:

$a = (m_2 - m_1)g/(m_1 + m_2)$

$= (1.80 \text{ kg} - 1.56 \text{ kg})(9.8 \text{ m/s}^2)/(1.80 \text{ kg} + 1.56 \text{ kg})$

$= \boxed{0.70 \text{ m/s}^2}$ for both blocks.

(b) For the motion of block 2:

$v_2^2 = v_{02}^2 + 2a\,(y_2 - y_{02}) = 0 + 2(0.7 \text{ m/s}^2)\,(2.80 \text{ m} - 0)$,

which gives $v_2 = \boxed{3.92 \text{ m/s}}$.

(Note: block 1 has the same speed. Once block 2 hits the floor,

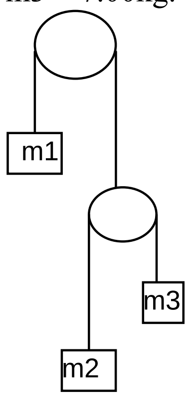
$T \to 0$ and the motions of the two blocks will differ.)

(c) To find the time to reach the floor, for block 2:

$v_2 = v_{02} + at$; $3.92 \text{ m/s} = 0 + (0.7 \text{ m/s}^2)t$, which gives $t = \boxed{5.6 \text{ s}}$.

Problem 4

Consider the double Atwood machine. The masses are m1 = 3.00kg, m2 = 11.0kg and m3 = 7.00kg. What are the tensions in the strings and the accelerations of the masses?



Solution 4

There are only two tensions. We will label the tension in the string connecting m_1 and m_3 as T_1 and the tension in the string supporting m_2 as T_2. The forces on each mass and the central pulley are shown. We choose up positive and assume that each acceleration is up.

If the mass of the pulley is negligible, we can write

$\sum F_y = ma_y$; $2T_1 - T_2 = 0$, which gives $T_2 = 2T_1$.

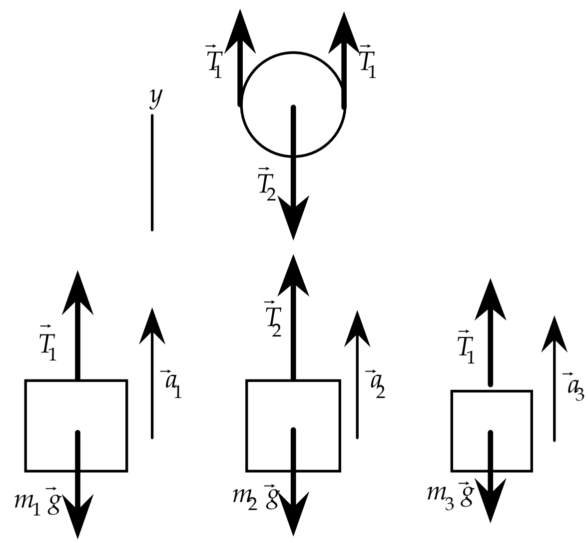
For each mass we can write $\sum F_y = ma_y$:

mass m_1: $T_1 - m_1 g = m_1 a_1$;

mass m_2: $2T_1 - m_2 g = m_2 a_2$;

mass m_3: $T_1 - m_3 g = m_3 a_3$.

It appears that we have only three equations for four unknowns: T_1 , a_1 , a_2 and a_3; however, the accelerations are related because the length of the string is constant. If we call the various segments L_1, L_2, and L_3, we have



$L_1 + 2L_2 + L_3$ = a constant. By differentiating this twice, we get $a_1 + 2a_2 + a_3 = 0$.

The three force equations can be written as

$T_1/m_1 - g = a_1$;

$4T_1/m_2 - 2g = 2a_2$;

$T_1/m_3 - g = a_3$.

From the relation between the accelerations, the sum of these three equations must be 0, from which we get

$T_1 = 4g(m_1m_2m_3)/(m_2m_3 + 4m_1m_3 + m_1m_2)$

$=4(9.8\text{m/s}^2)(3.00\text{kg})(11.00\text{kg})(7.00\text{ kg})/[(11.00\text{ kg})(7.00\text{ kg}) + 4(3.00\text{ kg})(7.00\text{ kg}) + (3.00\text{ kg})(11.00\text{ kg})]$

$= \boxed{46.7\text{ N}}$ and thus $T_2 = \boxed{93.4\text{ N}}$.

When we substitute the value of T_1 into the force equations, we get

$a_1 = T_1/m_1 - g = \boxed{5.8\text{ m/s}^2\text{ (up)}}$;

$a_2 = 2T_1/m_2 - g = \boxed{-1.3\text{ m/s}^2\text{ (down)}}$;

$a_3 = -(a_1 + 2a_2) = \boxed{-3.2\text{ m/s}^2\text{ (down)}}$.

Problem 5

A snail sits on a turn table at a radius of 3 inches. When the table rotates at 40rev/min the snail begins to slide. What is the coefficient of static friction between the snail and the turntable?

Solution 5

Until the snail begins to slide, there will be a static friction force from the turntable that provides the centripetal acceleration. The snail will begin to slide when this force reaches its maximum limit:

$f_s = f_{s,\max} = \mu_s F_N$.

We write $\Sigma \vec{F} = m\vec{a}$ for the snail:

x-component: $f_s = mR\omega^2$;

y-component: $F_N - mg = 0$.

Thus a larger R requires a larger f_s.

R will be maximum when f_s is maximum:

$f_s = \mu_s F_N = \mu_s mg$.

Thus $\mu_s mg = mR_{\max}\omega^2$, or $\mu_s = R\omega^2/g$;

$\mu_s = (3 \text{ in})(2.54 \text{ cm/in})(10^{-2} \text{ m/cm})[(40 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]^2/(9.8 \text{ m/s}^2) = \boxed{0.13}$.