Problem 1
A particle moves in one dimension according to the equation

$$x(t) = 3.00t + 6.00t^2 + 4.00$$

where x is in meters and t is in seconds. At t = 2.00 seconds find the
a) position b) velocity c) acceleration of the particle.

Solution 1

a) The equation given tells the position, x, at all times. Just replace t with 2.00secs and calculate the value for x.

$$x(t) = 3.00t + 6.00t^2 + 4.00$$
$$x = 3.00 \times 2.00 + 6.00 \times 2.00^2 + 4.00$$
$$x = 6.00 + 6.00 \times 4.00 + 4.00$$
$$x = 6.00 + 24.0 + 4.00$$
$$x = 34.0m$$

b) The equation that tells the velocity is the derivative of the position.

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(3.00t + 6.00t^2 + 4.00)$$
$$v(t) = \frac{d}{dt}3.00t + \frac{d}{dt}6.00t^2 + \frac{d}{dt}4.00$$

The derivative of the first term is 3.00. The second term has the derivative 12t. The third term is a constant so its derivative is zero.

$$v(t) = 3.00 + 12.0t$$

Plug in the time, t = 2s.

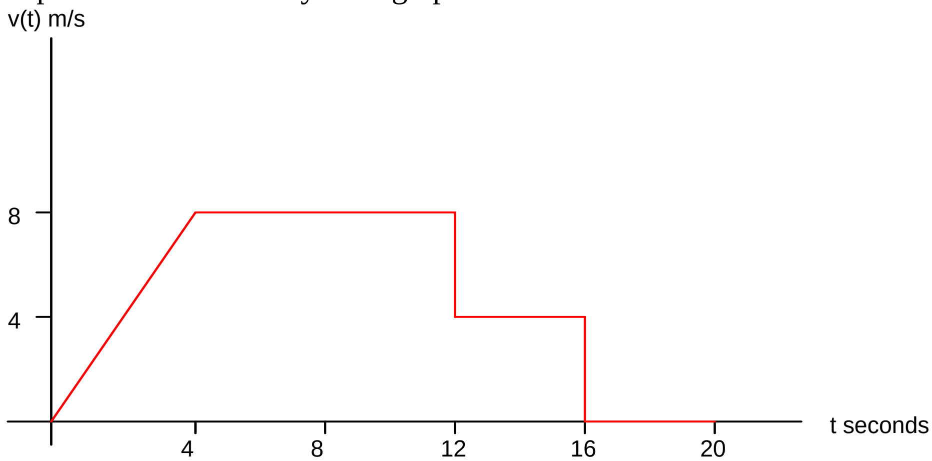
$$v = 3.00 + 12.0 \times 2.00$$
$$v = 3.00 + 24.0 = 27.0\frac{m}{s}$$

c) The acceleration is the derivative of the velocity equation.

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}3.00 + 12.0t$$
$$a = 12.0\frac{m}{s^2}$$

Taking the derivative shows that the acceleration is a constant $12m/s^2$ at all times.

Problem 2
A particle has a velocity-time graph as shown.



Draw the acceleration-time graph.

Solution 2
In order to find acceleration graph, first find the velocity as a function of time.
In the interval 0 to 4 seconds, the graph shows that the velocity is a straight line.
The equation for a straight line is

$$v(t) = mt + b$$

Where m is the slope and b is the intercept.
From the graph b = 0, and the slope of the line is 2.

$$v(t) = 2t$$

In the interval 4 to 12 seconds, velocity is a constant.

$$v(t) = 8\frac{m}{s}$$

For 12 to 16 seconds , velocity is still a constant.

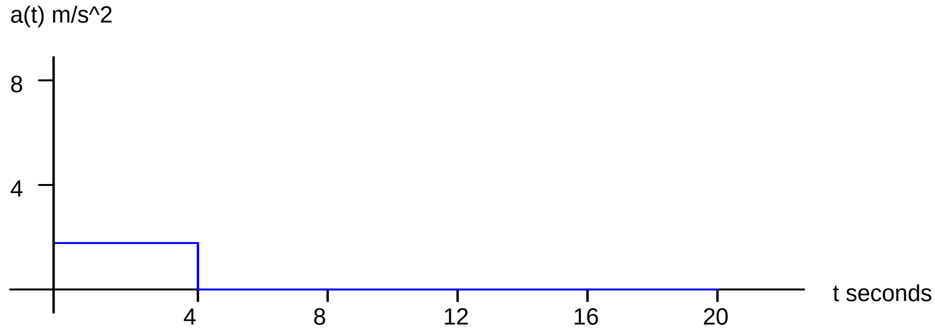
$$v(t) = 4\frac{m}{s}$$

During the last interval, 16 to 20 seconds v = 0.
Acceleration is the derivative of velocity.
For the first interval.

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}2t = 2\frac{m}{s^2}$$

For the rest of the time the velocity is a constant and its derivative is zero.

$$a = 0$$

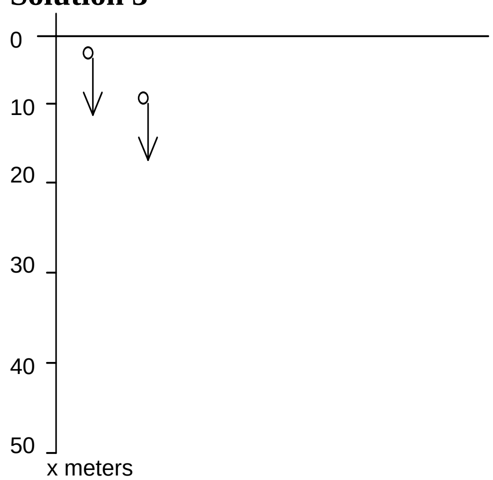


Problem 3

Two coins are thrown straight down from a height of 50.0m. The first is thrown with an initial velocity of 2.00 m/s. The second is thrown down 1.00s later. Both hit the ground at the same time.

a) How long after the release of the first coin did the coins take to hit the ground?
b) What was the initial velocity of the second coin?
c) What was the final velocity of each just as it hit?

Solution 3



a) The coins undergo one dimensional straight line motion with constant acceleration. The equation for this is given as

$$x_f - x_i = v_i t + \frac{1}{2} g t^2$$

where g is the acceleration due to gravity.

For the first coin xf = 50.0m, xi = 0, vi = 2.00m/s.

$$50.0m = 2.00t + \frac{1}{2} 9.8 t^2$$

Solve for t

$$4.9t^2 + 2.00t - 50.0 = 0$$

Quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-2.00 \pm \sqrt{4.00 + 980}}{9.8}$$
$$t = 3.00s$$
Disregard the negative root, time must be positive.
b) The equation for the second coin, in terms of the known quantities is
$$x_f - x_i = v_i t + \frac{1}{2} g t^2$$
$$50.0m = v_i (3.00s - 1.00s) + \frac{1}{2} 9.8(3.00s - 1.00s)^2$$
Where the t = 3 – 1 seconds, because the second is thrown 1.00s after the first.
Solve for vi
$$v_i = \frac{50.0 - 19.6}{2.00s} m = 15.2m/s$$
c) To find the final velocities use
$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$
For the first coin
$$v_f^2 = 4.00 \frac{m}{s^2}^2 + 2 * 9.8 \frac{m}{s^2}(50.0m) = 984 \frac{m^2}{s^2}$$
$$v_f = 31.4m/s$$
For the second coin
$$v_f^2 = 231 \frac{m}{s^2}^2 + 2 * 9.8 \frac{m}{s^2}(50.0m) = 1210 \frac{m^2}{s^2}$$
$$v_f = 34.8m/s$$
Problem 4
The acceleration of a particle is given by the equation
$$a(t) = 4.00t + 1.00$$
Find the velocity and position of the particle as a function of time, assuming the particle starts from rest at x = 0, t = 0.
Solution 4
To get the velocity the acceleration equation must be integrated.
$$a(t) = \frac{dv}{dt} = 4.00t + 1.00$$
$$\int_0^v dv = \int_0^t 4.00t + 1.00 dt$$
$$v = \left[\frac{4.00}{2} t^2 + 1.00t\right]_0^t = 2.00t^2 + 1.00t$$
To get the position the velocity formula is integrated

$$v = \frac{dx}{dt} = 2.00t^2 + 1.00t$$
$$dx = (2.00t^2 + 1.00t)dt$$

$$\int_0^x dx = \int_0^t 2.00t^2 + 1.00t dt$$

$$x = \left[\frac{2.00}{3} t^3 + \frac{1.00}{2} t^2 \right]_0^t = \frac{2}{3} t^3 + \frac{1}{2} t^2$$

Problem 5

A dog walks directly north at a constant velocity for 6.00m then turns and walks directly south for 4.00m. The trip north takes 1.00s, as does the trip south.

a) What is the dog's average speed over the entire trip?

b) What is the average velocity over the entire trip?

Solution 5

a) Average speed is the ratio of the total distance traveled over the time it takes to travel.

$$speed_{av} = \frac{6.00m + 4.00m}{1.00s + 1.00s} = 5.00m/s$$

b) Average velocity is the displacement divided by the time interval.

$$v_{av} = \frac{6.00m - 4.00m}{1.00s + 1.00s} = 1.00m/s$$