Problem 1
A vector is given as **A** = 5.00**i** + 2.00**j** + 1.00**k**.
a) Find the magnitudes of the x, y and z components
b) Find the magnitude of **A**
c) Find the angles the vector makes with the x and z axes

Solution 1

a) The magnitudes of the components are simply the factors in front of each of the unit vectors. The x component is denoted by the **i** vector, and has magnitude 5.00. The y component is the one preceding the **j** unit vector, magnitude 2.00. The z component is before the **k** vector, magnitude 1.00.
b) To find the magnitude of a vector take the dot product of the vector with itself, then take the square root.

$$\vec{A}\cdot\vec{A} = 5.00^2 + 2.00^2 + 1.00^2 = 30.0$$
$$\left|\vec{A}\right| = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{30.0} = 5.48$$

c)The angle with respect to the z and x axes can be found by first drawing a

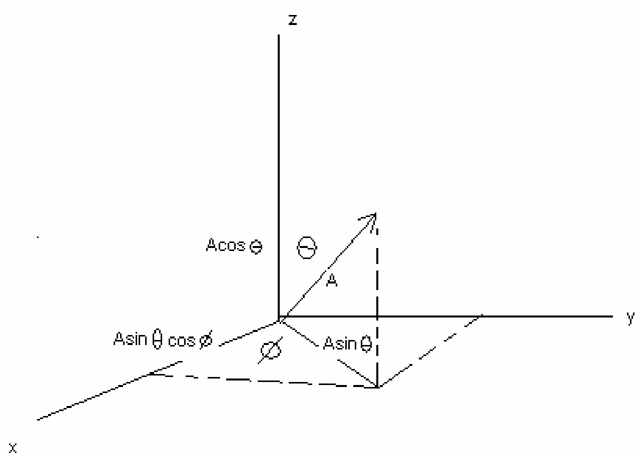


diagram.

The z component is the projection of the vector onto the z –axis, given by $A_z = \left|\vec{A}\right|\cos\theta$ where theta is the angle between **V** and the z-axis. Solve for theta.

$$\cos^{-1}\frac{A_z}{\left|\vec{A}\right|} = \theta$$
$$\cos^{-1}\frac{1.00}{5.48} = 79.5^\circ$$

To find the angle with respect to the x axis, first project the vector onto the x-y plane, then find the component along the x-axis.

The projection onto the x – y plane is given by $|\vec{A}|\sin\theta$ and is tilled from the x axis by angle φ. The x component is given by the length of the projection times cosφ.

$$A_x = |\vec{A}|\sin\theta\cos\phi$$

Solve for φ

$$\cos^{-1}\frac{A_x}{|\vec{A}|}\sin\theta = \phi$$

$$\cos^{-1}\frac{5.00}{5.48}\sin 79.5^\circ = \cos^{-1} 0.897 = 26.2^\circ$$

Problem 2

Given the vectors **A** = 2.00**i** + 1.00**j** and **B** = 2.00**j** + 3.00**k** find a new vector which is at a right angle to **A** and **B**, and the same length as **B**.

Solution 2

The cross product of two vectors always gives a third vector which is at 90 degrees to both of the original vectors.

$$\vec{A}\times\vec{B} = A_yB_z - A_zB_y\hat{i} + A_zB_x - A_xB_z\hat{j} + A_xB_y - A_yB_x\hat{k}$$

$$= (1.00)(3.00) - (0)(2.00)\hat{i} + (0)(0) - (2.00)(3.00)\hat{j} + (2.00)(2.00) - (1.00)(0)\hat{k}$$

$$= 3.00\hat{i} - 5.00\hat{j} + 4.00\hat{k} = \vec{C}$$

A unit vector in the direction of **C** can be found.

$$\vec{C} = |\vec{C}|\hat{c}$$

$$\frac{\vec{C}}{|\vec{C}|} = \hat{c}$$

$$\frac{3.00\hat{i} - 5.00\hat{j} + 4.00\hat{k}}{\sqrt{3.00^2 + 5.00^2 + 4.00^2}} = \frac{3.00\hat{i} - 5.00\hat{j} + 4.00\hat{k}}{6.16} = \frac{3.00}{6.16}\hat{i} - \frac{5.00}{6.16}\hat{j} + \frac{4.00}{6.16}\hat{k} = \hat{c}$$

$$\hat{c} = 0.487\hat{i} - 0.812\hat{j} + 0.649\hat{k}$$

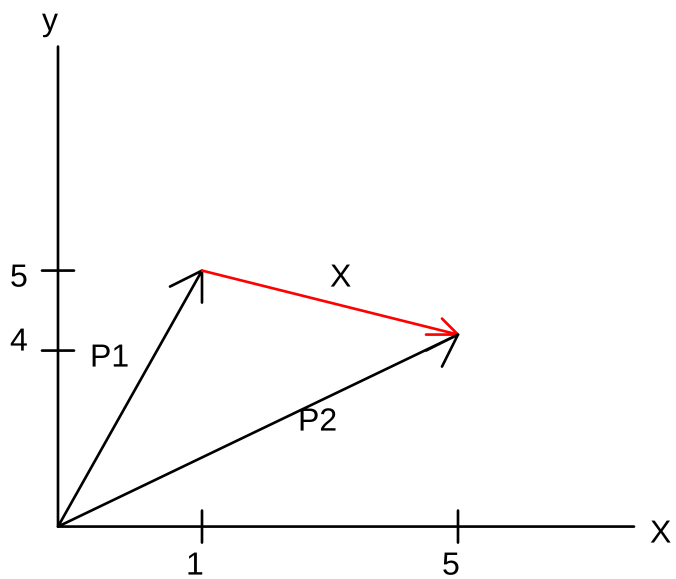
A vector the same length as B and in the C direction is given by

$$|\vec{B}|\hat{c} = (\sqrt{2.00^2 + 3.00^2})(0.487\hat{i} - 0.812\hat{j} + 0.649\hat{k}) = 1.76\hat{i} - 2.93\hat{j} + 2.34\hat{k}$$

Problem 3

A particle sits at a position given by the vector $\vec{P}_1 = 1.00\hat{i} + 5.00\hat{j}$ a second particle sits at a position given by vector $\vec{P}_2 = 5.00\hat{i} + 4.00\hat{j}$. Find the vector which gives the separation of the two particles.

Solution 3



When vectors are added graphically the tail of the second vector is placed on the head of the first and the resulting vector is the one that stretches from the tail of the first to the head of the second. From the diagram it can be seen that P1 plus the red vector, X, is equal to P2. The vector X gives the separation between the particles. Solve for X.

$$\vec{P}_1 + \vec{X} = \vec{P}_2$$

$$\vec{X} = \vec{P}_2 - \vec{P}_1$$

Now rewrite the equation in component form

$$\vec{X} = (P_{2x} - P_{1x})\hat{i} + (P_{2y} - P_{1y})\hat{j}$$

$$\vec{X} = (5.00 - 1.00)\hat{i} + (4.00 - 5.00)\hat{j} = 4.00\hat{i} - 1.00\hat{j}$$

Problem 4

A baseball player walks around a baseball diamond. As he walks he measures the distance from home to first as 27.56m, from first to second as 28.0m.If the baselines form a rectangle, find the total distance around the bases. Find the area of the field inside the base lines.

Solution 4

To find the total distance around the bases, double then add each measurement. Remember that the number of decimal places in the answer must be equal to the number of decimal places of the number with the least amount of decimal places.

$$27.56 + 27.56 + 28.0 + 28.0 = 111.1m$$

To find the field area, multiply the two measurements. Remember that the number of significant figures in the answer should equal the number of significant figure of the number with the least significant figures.

$$27.56m \times 28.0m = 772m$$

Problem 5

The equation $F = \frac{GMm}{r^2}$ represents Newton's law of gravitation. F has units $\frac{kg * m}{s^2}$, M and m are masses measured in kg, and r is a length.
What are the SI units of G?

Solution 5

Plug in the given units and solve for the units of G.

$$F = \frac{GMm}{r^2}$$

$$\frac{kg \cdot m}{s^2} = \frac{Gkg^2}{m^2}$$

$$\frac{m^2}{kg^2}\frac{kg \cdot m}{s^2} = G$$

$$G = \frac{m^3}{kg \cdot s^2}$$