Problem 1
Find the weight of a 150lbs person on the moon, Mars, Jupiter, Pluto and the sun.
Solution 1
The value of the constant g is given as

$$g = \frac{GM}{R^2}$$

where G is the gravitational constant, M is the mass of the planet, and R is the radius of the planet. For the moon, Mars, Jupiter, Pluto and the Sun

$$g_m = \frac{GM}{R^2} = \frac{(6.67x10^{-11} m^3 / kgs^2)(7.35x10^{22} kg)}{(1.74x10^6 m)^2} = 1.6 m/s^2$$

$$g_M = \frac{GM}{R^2} = \frac{(6.67x10^{-11} m^3 / kgs^2)(6.42x10^{23} kg)}{(3.37x10^6 m)^2} = 3.8 m/s^2$$

$$g_J = \frac{GM}{R^2} = \frac{(6.67x10^{-11} m^3 / kgs^2)(1.9x10^{27} kg)}{(6.99x10^7 m)^2} = 26 m/s^2$$

$$g_P = \frac{GM}{R^2} = \frac{(6.67x10^{-11} m^3 / kgs^2)(1.4x10^{22} kg)}{(1.5.x10^6 m)^2} = 0.42 m/s^2$$

$$g_s = \frac{GM}{R^2} = \frac{(6.67x10^{-11} m^3 / kgs^2)(1.99x10^{30} kg)}{(6.96x10^8 m)^2} = 275 m/s^2$$

To find the weight of a 150lbs person, first find their mass.

$$1lbs = 4.45N$$
$$150lbs = 667.5N$$
$$667.5N = m(9.8m/s^2)$$
$$m = 68kg$$

Now find the weight on each planet

$$w_{moon} = g_m m = 109N = 24lbs$$
$$w_{Mars} = g_M m = 258N = 58lbs$$
$$w_{Jupiter} = g_J m = 1768N = 397lbs$$
$$w_{pluto} = g_P m = 28.5N = 6.4lbs$$
$$w_{sun} = g_S m = 18700N = 4200lbs$$

Problem 2
Two 10000kg spheres are in a liquid with density ρ = 30000kg/m^2. The only forces the spheres are subject to are the gravitational attraction between the spheres and the gravity between the liquid and the spheres. Find the magnitude of the force on one of the spheres if the distance between them is R = 20m.
Solution 2
Two forces act on the spheres. The first is the gravity between the spheres, which is given by

$$F = \frac{GMm}{R^2} = \frac{(6.67x10^{-11} Nm^2 / kg^2)(10000kg)(10000kg)}{(20m)^2} = 1.67x10^{-5} N$$

The second is the gravity of the surrounding liquid. This can be found by treating the liquid as two parts, the sphere of liquid centered on one sphere and contained by the radius R and the shell of liquid with inner radius R. From the text, the liquid shell occupying the space outside the spheres contributes no net force. The liquid sphere can be treated as having all of its mass concentrated at its center. The inner liquid applies a force

$$F = \frac{G\rho Vm}{R^2} = \frac{(6.67x10^{-11}Nm^2/kg^2)(30000kg/m^3)\frac{3}{4}\pi(20m)^3(10000kg)}{(20m)^2} = 0.94N$$

to one of the spheres. The total force is $(0.94 + 1.67x10^{-5})N$

The same procedure applied to the other sphere will give the same result.

Problem 3

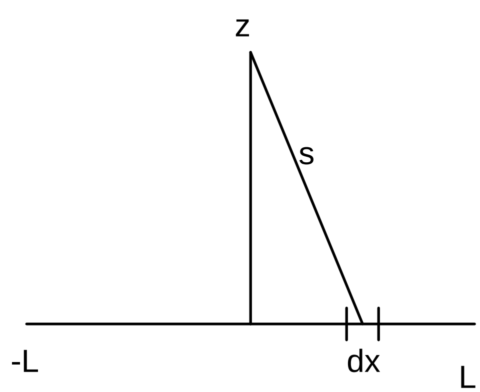
A uniform rod has a mass M and length L. What is the gravitational potential for a unit mass at a distance z above its midpoint?

Solution 3

The gravitational potential energy is given by

$$U = -Gm\int \frac{dM}{s}$$

Where G is the gravitational constant, m is the unit mass, dM is the mass of an infinitesimal piece of the rod, s is the distance from dM to the point where the potential is calculated.

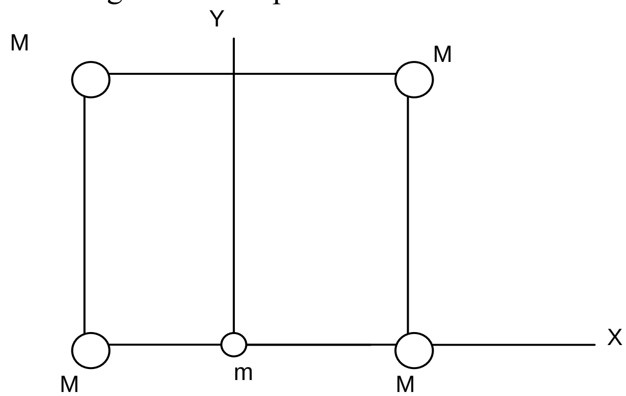


The mass element is taken as $dM = \lambda dl = \frac{M}{L}dx$. The distance s is written $s = \sqrt{x^2 + z^2}$. Plugging into the integral gives

$$U = -Gm\int_{-L/2}^{L/2} \frac{Mdx}{L\sqrt{x^2 + z^2}} = \frac{-GMm}{L}\left[\ln(x + \sqrt{x^2 + z^2})\right]_{-L/2}^{L/2} = \frac{-GMm}{L}\ln\left(\frac{L/2 + \sqrt{(L/2)^2 + z^2}}{-L/2 + \sqrt{(L/2)^2 + z^2}}\right)$$

All variables except x are treated as constants, the integral is evaluated using an integral table

Problem 4
Four planets, mass M = M_e, are located at the corners of a square with sides $10R_e$, where R_e and M_e are the radius and mass of the earth. Calculate the net gravitational force on a unit mass located on the center of one of the sides.
Solution 4
The diagram for the problem is



Each planet will apply a force $\vec{F} = -\frac{GMm}{r^2}\hat{r}$ to the unit mass. For the two planets on the x axis the forces have a magnitude

$$F = \frac{GMm}{10R_e/2}$$

The direction of the force on the unit mass is toward the planet creating the force. For the two planets on the x axis the forces are equal in magnitude but opposite in direction, the two forces will cancel.
The upper right planet

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} = \frac{GMm}{((10R_e/2)^2 + (10R_e)^2)}(\cos(63.4)\hat{x} + \sin(63.4)\hat{y})$$

The upper left planet

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} = \frac{GMm}{((10R_e/2)^2 + (10R_e)^2)}(-\cos(63.4)\hat{x} + \sin(63.4)\hat{y})$$

Adding the two gives a net force of

$$\vec{F} = \frac{GMm}{((10R_e/2)^2 + (10R_e)^2)}(-\cos(63.4)\hat{x} + \sin(63.4)\hat{y}) +$$

$$\frac{GMm}{((10R_e/2)^2 + (10R_e)^2)}(-\cos(63.4)\hat{x} + \sin(63.4)\hat{y}) =$$

$$\frac{2GMm}{((10R_e/2)^2 + (10R_e)^2)}\sin(63.4)\hat{y}$$

$$\frac{2(6.67x10^{-11})(5.97x10^{24}kg)(1kg)}{125(6.38x10^{6}m)^2}\sin(63.4)\hat{y} = 0.14N$$

Problem 5
Two stars of equal mass, M, revolve around their common center of mass in a circular orbit. Given the period of revolution, T, and the velocity, v, of the stars, write an expression for the masses of the stars.
Solution 5
The force on the star is from gravity

$$F = -\frac{GMM}{(2r)^2}$$

Where r is the radius of the orbit.
From Newton's second law, the forces on a star must sum to its mass times its acceleration

$$F = -\frac{GMM}{4r^2} = Ma$$

For a object moving in a circle the acceleration is centripetal, $a = -v^2/r$

$$-\frac{GMM}{4r^2} = -M\frac{v^2}{r}$$

$$\frac{GM}{4r} = v^2$$

The velocity relates to the orbital period through the equation

$$v = \frac{2\pi r}{T}$$

Combining the last two equations gives

$$v^2 = \frac{2\pi GM}{4vT}$$

Solve for M

$$M = \frac{2v^3T}{\pi G}$$