Chapter 12 Problems

1. The amplitude of a trasnverse wave on a string is 4.5 cm. The ratio of the maximum particle speed to the speed of the wave is 3:1. What is the wavelength (in cm) of the wave?

Answer: A particle of the string is moving in simple harmonic motion. The maximum speed of the particle is given by $v_{max} = A\omega$, where A is the amplitude of the wave and ω is the angular frequency. The angular frequency is related to the frequency f by $\omega = 2\pi f$, so the maximum speed can be written $v_{max} = 2\pi f A$. The speed v of a wave on a string can also be written $v = f\lambda$. The ratio of the maximum particle speed to the speed of the wave is

$$\frac{v_{max}}{v} = \frac{2\pi f A}{f\lambda} = \frac{2\pi A}{\lambda} = 9.1cm.$$

2. A transverse wave is traveling on a string. The displacement y of a particle from its equilibrium position is given by $y = (0.021\,m)\sin(25t - 2.0x)$. Note that the phase angle $25t - 2.0x$ is in radians, t is in seconds and x is in meters. The linear density of the string is 1.6×10^{-2} kg/m. What is the tension in the string?

Answer: The tension in the string is given by $T = v^2(m/L)$ and with $v = f\lambda$ we have

$$T = (\lambda f)^2 \frac{m}{L}.$$

Since we are given the linear density of the string, (m/L), we must determine λ and f. We can compare the general expression for the displacement of a particle undergoing simple harmonic motion

$$y = A\sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right),$$

with that given in the problem, $y = (0.021\,m)\sin(25t - 2.0x)$, to make the following identifications:

$$2\pi f = 25\text{rad/s} \rightarrow f = \frac{25}{2\pi}\text{Hz} \quad (1)$$

$$\frac{2\pi}{\lambda} = 2.0\text{m}^{-1} \rightarrow \lambda = \frac{2\pi}{2.0}\text{m}. \quad (2)$$

Substituting into the expression for the tension,

$$T = \left[\left(\frac{2\pi}{2.0}\text{m} \right) \left(\frac{25\text{Hz}}{2\pi} \right) \right]^2 (1.6 \times 10^{-2}\text{kg/m}) = 2.5\text{N}.$$

3. Given that the densities of water and ice are $\rho_{H_2O} = 1.000 \times 10^3$ kg/m^3 and $\rho_i = 917$ k/m^3, respectively, show that approximately 9/10 of an iceberg lies below the surface of the water.

Answer: Archimedes Principle states that the bouyant force exerted on an object by a fluid is equal to the weight of that amount of fluid displaced by the object. For an object to float, Newton's second law tells us that the bouyant force must be equal in magnitude to the objects weight, $F_B = mg$. From Archimedes Principle, we have

$$F_B = W_{H_2O,displaced} = m_{H_2O,displaced}g = \rho_{H_2O}V_{displaced}g.$$

The volume of water displaced is equal to the fraction of the object's volume that is submerged. The weight of the iceberg is $W = m_i g = \rho_i V_i g$, where V_i is the volume of the iceberg. Equating the bouyant force and the weight yields

$$\rho_{H_2O}V_{displaced}g = \rho_i V_i g \quad (3)$$

$$\frac{V_{displaced}}{V_i} = \frac{\rho_i}{\rho_{H_2O}} = \frac{917}{1000} \approx \frac{9}{10}. \quad (4)$$

4. An airplane has an effective wing surface area of 16 m^2 that is generating the lift force. In level flight the air speed over the top of the wings is 62.0 m/s, while the air speed beneath the wings is 54.0 m/s. What is the weight of the plane?

Answer: In level flight, the lift force must balance the plane's weight W. The lift force arises because the pressure P_B beneath the wings is greater than the pressure P_T on top of the wings. The lift force, then, is the pressure difference times the effective wing surface area A, so that $W = (P_B - P_t)A$. The area is given, and we can determine the pressure difference by using Bernoulli's equation,

$$P_B + \frac{1}{2}\rho v_B^2 + \rho g y_B = P_T + \frac{1}{2}\rho v_T^2 + \rho g y_T.$$

Since the flight is level, $y_B = y_T$. We then have

$$P_B - P_T = \frac{1}{2}\rho v_T^2 - \frac{1}{2}\rho v_B^2.$$

Recognizing that $W = (P_B - P_T)A$, we can substitute for the pressure difference from Bernoulli's equation to show that

$$W = (P_B - P_T)A = \frac{1}{2}\rho\left(v_T^2 - v_B^2\right)A.$$

Plugging in the values of the velocities and the area we obtain $W = 9600$ N.