

Chapter 10-11 Problems

1. A thin uniform rod is positioned in the vertical direction, with its lower end attached to a frictionless axis that is mounted on the floor. The rod has a length of 2.00 m and is allowed to fall, starting from rest. Find the tangential speed of the free end of the rod, just before the rod hits the floor after rotating through 90 degrees.

Answer: The conservation of energy gives $(1/2)I\omega^2 = (1/2)mgL$. Now the tip of the rod has, as it hits the ground, a speed of $v = L\omega$, so $\omega = v/L$. The moment of inertia of the rod is given by $I = (1/3)mL^2$. Substituting the last two equations into the first and simplifying yields $v = \sqrt{3gL} = 7.67$ m/s.

2. A uniform board is leaning against a smooth vertical wall. The board is at an angle θ above the horizontal ground. The coefficient of static friction between the ground and the lower end of the board is 0.650. Find the smallest value for the angle θ , such that the lower end of the board does not slide along the ground.

Answer: The ground exerts a vertical normal V on the lower end of the board. The maximum force of static friction has a magnitude of $\mu_s V$ and acts horizontally on the lower end of the board. The weight W acts downward at the boards center. The vertical wall applies a force P to the upper end of the board, this force begin perpendicular to the wall. Since the horizontal forces balance, $\mu_s V = P$. Since the vertical forces balance, $V = W$. Using an axis through the lower end of the board we express the fact that the torques balance to zero as

$$PL\sin\theta - W\left(\frac{L}{2}\right)\cos\theta = 0.$$

Rearranging this equation gives

$$\tan\theta = \frac{W}{2P}.$$

But, $P = \mu_s V$ and $W = V$. Substituting these results in,

$$\tan\theta = \frac{V}{2\mu_s V} = \frac{1}{2\mu_s}.$$

Therefore, $\theta = \tan^{-1}(1/2\mu_s) = 37.6^\circ$.

3. A tray is moved horizontally back and forth in simple harmonic motion at a frequency of $f = 2.00$ Hz. On this tray is an empty cup. Obtain the coefficient of static friction between the tray and the cup, given that the cup begins slipping when the amplitude of the motion is 5.00×10^{-2} m.

Answer: The cup slips when the force of static friction is overcome. So $F = ma = \mu_s mg$, where the acceleration is the maximum value for the simple harmonic motion,

$$a_{max} = A\omega^2 = A(2\pi f)^2.$$

Then we find $\mu_s = A(2\pi f)^2/g = 0.806$.

4. Pendulum A is a physical pendulum made from a thin, rigid, and uniform rod whose length is d . One end of this rod is attached to a ceiling by a frictionless hinge, so the rod is free to swing back and forth. Pendulum B is a simple pendulum whose length is also d . Obtain the ratio T_A/T_B of their periods for small-angle oscillations. *Answer:* The frequency of oscillation for the physical pendulum is $2\pi f = \sqrt{mgL/I}$. With $f = 1/T$, the period of the physical pendulum A is

$$T_A = 2\pi\sqrt{\frac{1}{mgL}}.$$

The moment of inertia of the rod is $I = (1/3)md^2$. Since the rod is uniform, its center of gravity lies at the geometric center, $L = d/2$. Thus, the period is

$$T_A = 2\pi\sqrt{\frac{2d}{g}}.$$

For the simple pendulum, we have

$$T_B = 2\pi\sqrt{\frac{d}{g}}.$$

The ratio is therefore $T_A/T_B = \sqrt{2/3} = 0.816$.