Chapter 2

Squeezed Phonon States

This chapter is a summary of our ideas and results on squeezed phonon states [18, 19, 20]; these are presented in detail in Chapter 3. We study squeezed quantum states of phonons, which allow the possibility of modulating the quantum fluctuations of atomic displacements below the zero-point quantum noise level of coherent phonon states. We calculate the corresponding expectation values and fluctuations of both the atomic displacement and the lattice amplitude operators, and also investigate the possibility of generating squeezed phonon states using a three-phonon parametric down-conversion process based on phonon-phonon interactions. We also study an alternative approach of squeezing quantum noise in the atomic displacement using a polariton-based approach. Furthermore, we propose a detection scheme based on reflectivity measurements.

2.1 Introduction

Photon squeezed states have attracted much attention during the past decade [11]. These states are important because they can achieve lower quantum noise than the zero-point fluctuations of the vacuum or coherent states. Thus they provide a way of manipulating quantum fluctuations and have a promising future in different applications ranging from optical communications to gravitational wave detection [11]. Indeed, squeezed states are currently being explored in a variety of non-quantum-optics systems, including classical squeezed states [16, 17]. Here we study the properties of *phonon* squeezed states and explore the possibility of generating these states through phonon-phonon interactions. After briefly presenting the quantum mechanical description of various kinds of phonon states, we study a simple model for generating phonon squeezed states, in which analytical results can be obtained [18, 19, 20]. We also propose a scheme for detecting this squeezing effect.

In most macroscopic situations, a classical description is adequate. However, the quantum fluctuations of a phonon system can be dominant at low enough temperatures. Indeed, a recent study shows that quantum fluctuations in the atomic positions can influence observable quantities (e.g., the Raman line-shape) [15] even when temperatures are not very low.

An experimentally observable quantity for a phonon system is the real part of the Fourier transform of the atomic displacement:[1]

$$Re\,(u_\alpha(\mathbf{q})) = \sum_\lambda \sqrt{\hbar/8m\omega_{\mathbf{q}\lambda}} \left\{ U^\lambda_{\mathbf{q}\alpha}(b_{\mathbf{q}\lambda} + b^\dagger_{-\mathbf{q}\lambda}) + U^{\lambda\,*}_{\mathbf{q}\alpha}(b_{-\mathbf{q}\lambda} + b^\dagger_{\mathbf{q}\lambda}) \right\} . \qquad (2.1)$$

For simplicity, hereafter we will drop the branch subscript λ, assume that $U_{\mathbf{q}\alpha}$ is real, and define a $\mathbf{q}$-mode dimensionless lattice amplitude operator:

$$u(\pm\mathbf{q}) = b_\mathbf{q} + b^\dagger_{-\mathbf{q}} + b_{-\mathbf{q}} + b^\dagger_\mathbf{q} . \qquad (2.2)$$

This operator contains essential information on the lattice dynamics, including quantum fluctuations. It is the phonon analog of the electric field in the photon case.

2.2 Phonon Quantum States

2.2.1 Phonon Vacuum and Number States

When no phonon is excited, the crystal is in the phonon vacuum state $|0\rangle$. The eigenstates of the harmonic phonon Hamiltonian are number states which satisfy

$$b_\mathbf{q}|n_\mathbf{q}\rangle = \sqrt{n_\mathbf{q}}\,|n_\mathbf{q} - 1\rangle . \qquad (2.3)$$

The phonon number and the phase of atomic vibrations are conjugate variables. Thus, due to the uncertainty principle, the phase is arbitrary when the phonon number is certain, as it is the case with any number state $|n_\mathbf{q}\rangle$. Therefore, the expectation values of the atomic displacement $\langle n_\mathbf{q}|u_{i\alpha}|n_\mathbf{q}\rangle$ and $\mathbf{q}$-mode lattice amplitude $\langle n_\mathbf{q}|u(\pm\mathbf{q})|n_\mathbf{q}\rangle$ vanish due to the randomness in the phase of the atomic displacements.

2.2.2 Phonon Coherent States

A single-mode ($\mathbf{q}$) phonon coherent state[2] is an eigenstate of a phonon annihilation operator:

$$b_\mathbf{q}|\beta_\mathbf{q}\rangle = \beta_\mathbf{q}|\beta_\mathbf{q}\rangle . \qquad (2.4)$$

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[1] A phonon with quasi-momentum $\mathbf{p} = \hbar\mathbf{q}$ and branch subscript λ has energy $\epsilon_{\mathbf{q}\lambda} = \hbar\omega_{\mathbf{q}\lambda}$; the corresponding creation and annihilation operators satisfy the boson commutation relations: $[b_{\mathbf{q}'\lambda'},\ b^\dagger_{\mathbf{q}\lambda}] = \delta_{\mathbf{q}\mathbf{q}'}\delta_{\lambda\lambda'}$, $[b_{\mathbf{q}\lambda},\ b_{\mathbf{q}'\lambda'}] = 0$. The atomic displacements $u_{i\alpha}$ of a crystal lattice are given by $u_{i\alpha} = (1/\sqrt{Nm})\sum^N_{\mathbf{q}\lambda} U^\lambda_{\mathbf{q}\alpha} Q^\lambda_\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{R}_i}$. Here $\mathbf{R}_i$ refer to the equilibrium lattice positions, α to a particular direction, and $Q^\lambda_\mathbf{q} = \sqrt{\hbar/2\omega_{\mathbf{q}\lambda}}(b_{\mathbf{q}\lambda} + b^\dagger_{-\mathbf{q}\lambda})$ is the normal-mode amplitude operator.

[2] A single-mode phonon coherent state can be generated by the Hamiltonian $H = \hbar\omega_\mathbf{q}(b^\dagger_\mathbf{q} b_\mathbf{q} + 1/2) + \lambda^*_\mathbf{q}(t)\,b_\mathbf{q} + \lambda_\mathbf{q}(t)\,b^\dagger_\mathbf{q}$ and an appropriate initial state. Here $\lambda_\mathbf{q}(t)$ represents the interaction strength between