Physics 502, Problem Set #8
Due Wednesday, 9 April

(1) For this problem, use the Minkowski metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.
(a) Show that the invariant length of the four velocity

$$u^\mu \equiv \frac{dx^\mu}{ds} \tag{1}$$

is unity:

$$u_\mu u^\mu = 1, \tag{2}$$

where $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ is the proper time.
(b) Show that the four acceleration

$$a^\mu \equiv \frac{du^\mu}{ds} \tag{3}$$

is always orthogonal to the four velocity

$$u_\mu a^\mu = 0 \tag{4}$$

(c) Show that any inertial reference frame with four-velocity u^μ can be transformeed to the rest frame $u^\mu = (1, 0, 0, 0)$ by a single Lorentz transformation.

(2) In the laboratory frame, particle 1 moves along the x-axis with uniform velocity u_x, and
(a) Particle 2 moves along the x-axis with uniform velocity v_x . Find the velocity of particle 2 measured in the rest frame of particle 1.
(b) Particle 2 moves along the y-axis with uniform velocity v_y.Find the velocity of particle 2 measured in the rest frame of particle 1.
Express the answers to both parts (a) and (b) in vector form (i.e., three-velocity.)

(3) Stella boards a spaceship that travels away from the earth at a constant speed of $0.4c$. One year later (according to Earth time), her twin sister Terra boards a second spaceship that pursues the first at a constant speed of $0.8c$. When the second spaceship catches up with the first, what is the difference in age between the twins?