Meson Photoproduction from the Nucleon

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A mass operator has been constructed which describes the coupling between meson-baryon, photon-nucleon, and single-baryon channels. The scattering and reaction amplitudes are obtained from three-dimensional Lippmann-Schwinger equations. The S-matrix elements for the various processes transform properly under inhomogeneous Lorentz transformations and moreover are gauge invariant. Within our framework we have derived the most general forms for the mass-operator interactions that describe the processes $\gamma + B \Leftrightarrow B'$ and $\gamma + B \Leftrightarrow \mu + B'$, where γ is a photon, B and B' are baryons, and μ is a meson. These forms provide generalizations of the well known CGLN amplitudes. Our mass operator interactions have been derived from effective Lagrangians. Using the Elmessiri-Fuda model for the pion-nucleon system, we have carried out calculations on the photoproduction of pions from the nucleon for total c.m. energies from threshold up to $W = 1550$ MeV.

1. INTRODUCTION

If the inhomogeneous Lorentz transformation $x' = ax + b$ is applied successively it leads to the Poincaré group, multiplication law $(a', b') \circ (a, b) = (a'a, a'b + b')$. In relativistic quantum mechanics the state vectors must transform according to $|\psi'\rangle = U(a, b)|\psi\rangle$ where the unitary operators $U(a, b)$ provide a representation of the Poincaré group; in particular $U(a', b) U(a, b) = U(a'a, a'b + b')$. For proper inhomogeneous Lorentz transformations these operators can be parametrized in the form

$$U(a, b) = \exp(ib^\mu P_\mu) \exp\left(-\frac{i}{2}\omega^{\alpha\beta} J_{\alpha\beta}\right) , \ \omega^{\alpha\beta} = -\omega^{\beta\alpha}, \ J_{\alpha\beta} = -J_{\beta\alpha} \ . \tag{1}$$

In a Bakamjian-Thomas construction [1] of the generators, P_μ and $J_{\alpha\beta}$, it is convenient to define $P = (P^0, P^1, P^2, P^3) = (H, \mathbf{P})$, $\mathbf{K} = (J_{10}, J_{20}, J_{30})$, and $\mathbf{J} = (J_{23}, J_{31}, J_{12})$. Here H is the Hamiltonian, $\mathbf{P}$ is the three-momentum operator, $\mathbf{K}$ is the generator of rotationless boosts, and $\mathbf{J}$ is the angular momentum operator. The generators can be expressed in terms of a mass operator, M, a spin operator, $\mathbf{S}$, and the Newton-Wigner [2,3] position operator, $\mathbf{X}$, according to the relations

$$H = \left(\mathbf{P}^2 + M^2\right)^{1/2}, \ \mathbf{J} = \mathbf{X} \times \mathbf{P} + \mathbf{S}, \ \mathbf{K} = -\frac{1}{2}(H\mathbf{X} + \mathbf{X}H) - \frac{\mathbf{P} \times \mathbf{S}}{M + H}. \tag{2}$$

The only non-zero commutators of the set $\{M, \mathbf{P}, \mathbf{S}, \mathbf{X}\}$ are $\left[X^j, P^k\right] = i\delta_{jk}$ and $\left[S^j, S^k\right] = i\varepsilon_{jkl} S^l$, which are familiar from nonrelativistic quantum mechanics. If the members of the set $\{M, \mathbf{P}, \mathbf{S}, \mathbf{X}\}$ satisfy the correct commutation relations, then the generators defined by (2) satisfy the Poincaré algebra. In a Bakamjian-Thomas construction, we choose $\mathbf{P}$, $\mathbf{S}$, and $\mathbf{X}$ to be the same as the operators for the relevant system of non-interacting particles; then the only commutation rules of the set $\{M, \mathbf{P}, \mathbf{S}, \mathbf{X}\}$ that remain to be satisfied are $[\mathbf{P}, M] = 0,\ [\mathbf{X}, M] = 0,\ [\mathbf{S}, M] = 0$. We construct M according to $M = M_0 + U$, where M_0 is the mass operator for the non-interacting system, and U is an interaction.

2. THE MODEL

For our model space we choose states of the type $|B\rangle, |\mu B\rangle, |\gamma B\rangle$, where B's are baryon's, μ's are mesons, and γ is the photon. We encounter 5 types of interaction matrix elements. $\langle B| U |B\rangle$ is a mass renormaliztion constant, $\langle B'| U |\mu B\rangle$ and $\langle B'| U |\gamma B\rangle$ are vertex interactions, and $\langle \mu' B'| U |\mu B\rangle$ and $\langle \mu' B'| U |\gamma B\rangle$ are potentials. The commutation rules restrict the forms of these matrix elements. For example, the $\pi N - \pi N$ potential must be of the form $\delta^3(\mathbf{p}' - \mathbf{p}) \langle t'\, i' m'| U_{\pi N, \pi N}(\mathbf{q}', \mathbf{q}) |tim\rangle$ where $\mathbf{q} = (\mathbf{p}_\pi)_{cm} = -(\mathbf{p}_N)_{cm}$, $\mathbf{p} = \mathbf{p}_\pi + \mathbf{p}_N$, the i's and $t's$ are 3-components of isospin, and the m's are 3-components of spin. The commutator $[\mathbf{P}, U] = 0$ leads to the Dirac delta function, while the commutator $[\mathbf{X}, U] = 0$ implies that $U_{\pi N, \pi N}(\mathbf{q}', \mathbf{q})$ cannot depend on $\mathbf{p}$. In order for $[\mathbf{S}, U] = 0$ to be satisfied it is necessary that $U_{\pi N, \pi N}(\mathbf{q}', \mathbf{q})$ be a rotationally invariant function of $\mathbf{q}', \mathbf{q}$ and $\boldsymbol{\sigma}$. Structures such as these guarantee that the Poincaré algebra is satisfied and lead to $S-$matrix elements that transform properly in going from one inertial frame to another [4]. Transition probabilities are invariant.

Ref. [5] shows how the Bakamjian-Thomas construction has been used in developing the Elmessiri-Fuda model for the pion-nucleon system. This model includes only the strong interactions. In constructing the electromagnetic interactions we have shown that the most general $B \Longleftrightarrow \gamma b$ vertex function and $\mu B \Longleftrightarrow \gamma b$ potential consistent with rotational invariance and gauge invariance are given by

$$U_{B,\gamma b}(\mathbf{q}, \lambda) = \sum_{m_B nl} |s_B m_B\rangle\, U_{B,\gamma b}(q, n, l)\, \mathbf{Z}^{m_B \dagger}_{nls_b s_B}(\hat{\mathbf{q}}) \cdot \boldsymbol{\varepsilon}(\mathbf{q}, \lambda), \tag{3}$$

$$U_{\mu B,\gamma b}(\mathbf{q}'; \mathbf{q}, \lambda) = \sum_{jm} \sum_{gLnl} Y^m_{(gs_\mu)Ls_B j}(\hat{\mathbf{q}}')\, U^{(j)}_{\mu B,\gamma b}(q', g, L; q, n, l)\, \mathbf{Z}^{m\dagger}_{nls_b j}(\hat{\mathbf{q}}) \cdot \boldsymbol{\varepsilon}(\mathbf{q}, \lambda), \tag{4}$$

where $|s_B m_B\rangle$ is a baryon spin vector, $\boldsymbol{\varepsilon}(\mathbf{q}, \lambda)$ is a photon's polarization vector, and λ is its helicity. Here the Y's are standard angular momentum eigenstates for the μB system, and the $\mathbf{Z}$'s are defined by

$$\mathbf{Z}^m_{1lsj}(\hat{\mathbf{q}}) = (i\nabla_{\mathbf{q}} \times \mathbf{q}) \sum_{m_l m_s} Y^{m_l}_l(\hat{\mathbf{q}})\, |sm_s\rangle \frac{\langle lsm_l m_s|\, jm\rangle}{\sqrt{l(l+1)}},\ \mathbf{Z}^m_{2lsj}(\hat{\mathbf{q}}) = -i\hat{\mathbf{q}} \times \mathbf{Z}^m_{1lsj}(\hat{\mathbf{q}}), \tag{5}$$

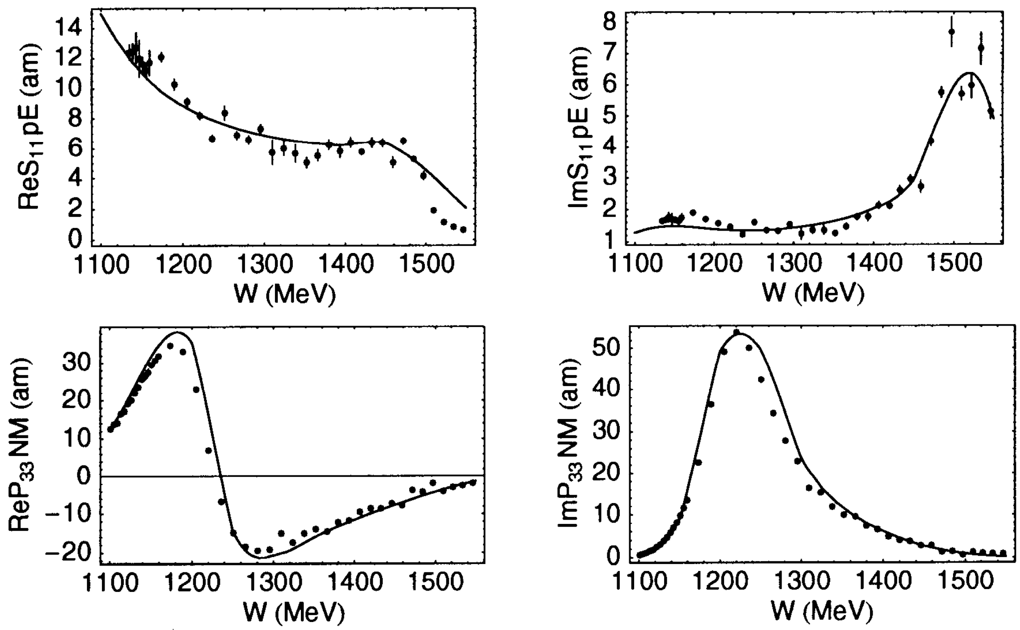
To first order in e the $\gamma + B \Longrightarrow \mu' + B'$ photoproduction amplitudes are given by

$$\begin{aligned} T_{\mu' B',\gamma B}(\mathbf{q}'; \mathbf{q}, \lambda; z) &= V_{\mu' B',\gamma B}(\mathbf{q}'; \mathbf{q}, \lambda; z) \\ &+ \sum_{\mu'' B''} \int \frac{T_{\mu' B',\mu'' B''}(\mathbf{q}', \mathbf{q}''; z)\, d^3q'' V_{\mu'' B'',\gamma B}(\mathbf{q}''; \mathbf{q}, \lambda; z)}{\Delta_{\mu'' B''}(\mathbf{q}'')\, 2W_{\mu'' B''}(\mathbf{q}'')\, [z - W_{\mu'' B''}(\mathbf{q}'')]}, \end{aligned} \tag{6}$$

$$V_{\mu' B',\gamma B}\left(\mathbf{q}';\mathbf{q},\lambda;z\right) = U_{\mu' B',\gamma B}\left(\mathbf{q}';\mathbf{q},\lambda\right) + \sum_{B''} \frac{U_{\mu' B',B''}\left(\mathbf{q}'\right) U_{B'',\gamma B}\left(\mathbf{q},\lambda\right)}{2m_{B''}\left[z - m_{B''}^{(0)}\right]}, \tag{7}$$

$$W_{\mu B}\left(\mathbf{q}\right) = \omega_{\mu}\left(\mathbf{q}\right) + \varepsilon_{B}\left(\mathbf{q}\right), \ \Delta_{\mu B}\left(\mathbf{q}\right) = \left(2\pi\right)^{3} 2\omega_{\mu}\left(\mathbf{q}\right)\varepsilon_{B}\left(\mathbf{q}\right)/W_{\mu B}\left(\mathbf{q}\right). \tag{8}$$

Here $T_{\mu' B',\mu'' B''}$ is the strong interaction $T-$matrix, m_B is the physical mass of a baryon, and $m_B^{(0)}$ is its bare mass. We see that (3)-(7) imply that $T_{\mu' B',\gamma B}\left(\mathbf{q}',\mathbf{q};z\right) \Longrightarrow T_{\mu' B',\gamma B}\left(\mathbf{q}',\mathbf{q};z\right)$ when $\boldsymbol{\varepsilon}\left(\mathbf{q},\lambda\right) \Longrightarrow \boldsymbol{\varepsilon}\left(\mathbf{q},\lambda\right) + const.\mathbf{q}$, so we have gauge invariance. We have used the Okubo method [5–7] to obtain the vertex functions and potentials from effective Lagrangians. Below, some of our results are compared with the SM95 multipoles [8].



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