Meson Photoproduction from the Nucleon

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A mass operator has been constructed which describes the coupling between meson-baryon, photon-nucleon, and single-baryon channels. The scattering and reaction amplitudes are obtained from three-dimensional Lippmann-Schwinger equations. The $S$-matrix elements for the various processes transform properly under inhomogeneous Lorentz transformations and moreover are gauge invariant. Within our framework we have derived the most general forms for the mass-operator interactions that describe the processes $\gamma + B \leftrightarrow B'$ and $\gamma + B \leftrightarrow \mu + B'$, where $\gamma$ is a photon, $B$ and $B'$ are baryons, and $\mu$ is a meson. These forms provide generalizations of the well known CGLN amplitudes. Our mass operator interactions have been derived from effective Lagrangians. Using the Elmessiri-Fuda model for the pion-nucleon system, we have carried out calculations on the photoproduction of pions from the nucleon for total c.m. energies from threshold up to $W = 1550$ MeV.

1. INTRODUCTION

If the inhomogeneous Lorentz transformation $x' = ax + b$ is applied successively it leads to the Poincaré group, multiplication law $(a', b') \circ (a, b) = (a'a, a'b + b')$. In relativistic quantum mechanics the state vectors must transform according to $|\psi'\rangle = U(a, b)|\psi\rangle$ where the unitary operators $U(a, b)$ provide a representation of the Poincaré group; in particular $U(a', b)U(a, b) = U(a'a, a'b + b')$. For proper inhomogeneous Lorentz transformations these operators can be parametrized in the form

$$U(a, b) = \exp(ib^\mu P_\mu) \exp\left(-\frac{i}{2}\omega^{\alpha\beta} J_{\alpha\beta}\right), \quad \omega^{\alpha\beta} = -\omega^{\beta\alpha}, \quad J_{\alpha\beta} = -J_{\beta\alpha}. \tag{1}$$

In a Bakamjian-Thomas construction [1] of the generators, $P_\mu$ and $J_{\alpha\beta}$, it is convenient to define $P = (P^0, P^1, P^2, P^3) = (H, P)$, $K = (J_{10}, J_{20}, J_{30})$, and $J = (J_{23}, J_{31}, J_{12})$. Here $H$ is the Hamiltonian, $P$ is the three-momentum operator, $K$ is the generator of rotationless boosts, and $J$ is the angular momentum operator. The generators can be expressed in terms of a mass operator, $M$, a spin operator, $S$, and the Newton-Wigner [2,3] position operator, $X$, according to the relations

$$H = \left(P^2 + M^2\right)^{1/2}, \quad J = X \times P + S, \quad K = -\frac{1}{2}(HX + XH) - \frac{P \times S}{M + H}. \tag{2}$$

The only non-zero commutators of the set $\{M, P, S, X\}$ are $[X^j, P^k] = i\delta^k_j$ and $[S^i, S^k] = i\varepsilon_{jik} S^j$, which are familiar from nonrelativistic quantum mechanics. If the members of the
set \{M, P, S, X\} satisfy the correct commutation relations, then the generators defined by (2) satisfy the Poincaré algebra. In a Bakamjian-Thomas construction, we choose P, S, and X to be the same as the operators for the relevant system of non-interacting particles; then the only commutation rules of the set \{M, P, S, X\} that remain to be satisfied are [P, M] = 0, [X, M] = 0, [S, M] = 0. We construct M according to \(M = M_0 + U\), where \(M_0\) is the mass operator for the non-interacting system, and U is an interaction.

2. THE MODEL

For our model space we choose states of the type \(|B\), \(|\mu B\), \(|\gamma B\rangle\), where B’s are baryon’s, \(\mu\)’s are mesons, and \(\gamma\) is the photon. We encounter 5 types of interaction matrix elements. \(|B\rangle U |B\rangle\) is a mass renormalization constant, \(|B'\rangle U |\mu B\rangle\) and \(|B'\rangle U |\gamma B\rangle\) are vertex interactions, and \(|\mu' B'\rangle U |\mu B\rangle\) and \(|\mu' B'\rangle U |\gamma B\rangle\) are potentials. The commutation rules restrict the forms of these matrix elements. For example, the \(\pi N - \pi N\) potential must be of the form \(\delta^3(\mathbf{p}' - \mathbf{p}) \langle t' i m' | U_{\pi N, \pi N} (\mathbf{q}', \mathbf{q}) | tim \rangle\) where \(\mathbf{q} = (\mathbf{p} - \mathbf{p}_N)_{cm} = -(\mathbf{p}_N)_{cm}\), \(\mathbf{p} = \mathbf{p}_N + \mathbf{p}\), the \(i\)’s and \(t\)’s are 3-components of isospin, and the \(m\)’s are 3-components of spin. The commutator \([P, U] = 0\) leads to the Dirac delta function, while the commutator \([X, U] = 0\) implies that \(U_{\pi N, \pi N} (\mathbf{q}', \mathbf{q})\) cannot depend on \(\mathbf{p}\). In order for \([S, U] = 0\) to be satisfied it is necessary that \(U_{\pi N, \pi N} (\mathbf{q}', \mathbf{q})\) be a rotationally invariant function of \(\mathbf{q}', \mathbf{q}\) and \(\sigma\). Structures such as these guarantee that the Poincaré algebra is satisfied and lead to \(S\)–matrix elements that transform properly in going from one inertial frame to another [4]. Transition probabilities are invariant.

Ref. [5] shows how the Bakamjian-Thomas construction has been used in developing the Elmessiri-Fuda model for the pion-nucleon system. This model includes only the strong interactions. In constructing the electromagnetic interactions we have shown that the most general \(B \leftrightarrow \gamma b\) vertex function and \(\mu B \leftrightarrow \gamma b\) potential consistent with rotational invariance and gauge invariance are given by

\[
U_{B, \gamma b} (\mathbf{q}, \lambda) = \sum_{m_B n_l} |s_B m_B\rangle \langle B, \gamma b | q, n, l\rangle \mathbf{Z}_{n_m s_B}^{m_B} (\hat{\mathbf{q}}) \cdot \mathbf{e} (\mathbf{q}, \lambda),
\]

\[
U_{\mu B, \gamma b} (\mathbf{q}'; \mathbf{q}, \lambda) = \sum_{j m l} \sum_{g l m} Y_{(g l m) B B}^{j m} (\hat{\mathbf{q}}') \langle B, \gamma b | q', g, L; q, n, l\rangle \mathbf{Z}_{s_B B}^{m_B} (\hat{\mathbf{q}}) \cdot \mathbf{e} (\mathbf{q}, \lambda),
\]

where \(|s_B m_B\rangle\) is a baryon spin vector, \(\mathbf{e} (\mathbf{q}, \lambda)\) is a photon’s polarization vector, and \(\lambda\) is its helicity. Here the \(Y\)’s are standard angular momentum eigenstates for the \(\mu B\) system, and the \(\mathbf{Z}\)’s are defined by

\[
\mathbf{Z}_{1l m} (\hat{\mathbf{q}}) = (i \nabla_{\mathbf{q}} \times \mathbf{q}) \sum_{m l l m} Y_{l m} (\hat{\mathbf{q}}) |s m\rangle \left(\frac{m s m_l l m}{\sqrt{l(l + 1)}}\right),
\]

\[
\mathbf{Z}_{2l m} (\hat{\mathbf{q}}) = -i \overline{\mathbf{q}} \times \mathbf{Z}_{1l m} (\hat{\mathbf{q}}).
\]

To first order in \(\mathbf{e}\) the \(\gamma + B \rightarrow \mu' + B'\) photoproduction amplitudes are given by

\[
T_{\mu' B'; \gamma B} (\mathbf{q}'; \mathbf{q}, \lambda; z) = V_{\mu' B'; \gamma B} (\mathbf{q}'; \mathbf{q}, \lambda; z)
\]

\[
+ \sum_{\mu' B'} \int \frac{T_{\mu' B'; \gamma B} (\mathbf{q}', \mathbf{q}''; z) d^3 q'' V_{\mu' B'; \gamma B} (\mathbf{q}''; \mathbf{q}, \lambda; z)}{\Delta_{\mu' B'} (\mathbf{q}'') 2W_{\mu' B'} (\mathbf{q}'') [z - W_{\mu' B'} (\mathbf{q}'')]},
\]
\[ V_{\mu' B' \gamma B}(q'; q, \lambda; z) = U_{\mu' B' \gamma B}(q'; q, \lambda) + \sum_{B''} \frac{U_{\mu' B' \gamma B''}(q') U_{B'' \gamma B}(q, \lambda)}{2m_{B''} \left[ \varepsilon - m_{B''}^{(0)} \right]}, \quad (7) \]

\[ W_{\mu B}(q) = \omega_{\mu}(q) + \varepsilon_B(q), \quad \Delta_{\mu B}(q) = (2\pi)^3 2\omega_{\mu}(q) \varepsilon_B(q)/W_{\mu B}(q). \quad (8) \]

Here \( T_{\mu' B' \mu' B''} \) is the strong interaction \( T \)-matrix, \( m_B \) is the physical mass of a baryon, and \( m_B^{(0)} \) is its bare mass. We see that (3)-(7) imply that \( T_{\mu' B' \gamma B}(q', q;z) \rightarrow T_{\mu' B' \gamma B}(q', q;z) \)
when \( \varepsilon(q, \lambda) \rightarrow \varepsilon(q, \lambda) + \text{const.} \cdot q \), so we have gauge invariance. We have used the Okubo method [5–7] to obtain the vertex functions and potentials from effective Lagrangians. Below, some of our results are compared with the SM95 multipoles [8].

**REFERENCES**