Chapter 1

Introduction

The first reasonably successful calculation of the photoproduction of pions from the nucleon was carried out by Chew and Low [1], based on a straightforward extension of their static, cutoff model for pion–nucleon scattering. The lack of Lorentz and gauge invariance in the Chew–Low model was remedied in a seminal paper by Chew, Goldberger, Low, and Nambu [2] who formulated relativistic dispersion relations for the photoproduction amplitudes. As an important byproduct of their analysis, they established the general form of the complete Lorentz and gauge invariant amplitudes for single pion photoproduction from the nucleon, i.e., the by now well known CGLN amplitudes. A detailed study based on the CGLN approach was carried out by Berends, Donnachie, and Weaver [3], who showed that dispersion relations can account for the main features of the data up through the energy of the $\Delta(1232)$ resonance. The early work on π, η, K, and vector–meson photo – production has been thoroughly reviewed by Donnachie [4].

Dispersion relations continue to be used in the analysis of the photo– and electroproduction of mesons from the nucleon. Aznauryan [5] has used such relations to extract the $\Delta(1232)$ resonance contributions from the amplitudes for pion photo – and electro production of pions from the nucleon. Hanstein, Drechsel, and Tiator [6] have carried out a multipole analysis of pion photoproduction based on fixed t dispersion relations and unitarity.

Obviously the photoproduction of mesons is a multichannel process, and for such processes it is in general important to ensure that the S–matrix is unitary. One of the first applications of the principle of unitarity to photoproduction led to the so–called Watson theorem [7]. This is the statement that in the energy range in which only a single pion is produced, the phase of a multipole transition amplitude to a final pion–nucleon state of well–defined total isospin and total angular momentum is equal to the scattering phase shift of that pion–nucleon state. A solution of the multichannel unitarity equations was obtained some time ago by Olsson [8], and applied to a pole model that accounts for the resonant multipoles in the $\Delta(1232)$ region. The importance of unitarity has been demonstrated by Wittman, Davidson, and Mukhopadhyay [9], who showed that it is necessary to unitarize the nonrelativistic Born terms of Blomqvist and Laget [10] in order to get good agreement with experiment. Davidson *et al.* [11] have also used a unitarized model to estimate the ratio of the electric quadrupole to magnetic dipole transition amplitudes in the process $\gamma + N \Leftrightarrow \Delta(1232)$.

It is possible to build into pole models for pion photoproduction the low energy theorems of current algebra by starting with a chirally symmetric Lagrangian and gauging it so as to include coupling to the electromagnetic field [12]. Such models favor pseudo – vector coupling of pions and nucleons. Olsson and Osypowski [13] have investigated models of this type and have found that in order to obtain satisfactory agreement with experiment it is necessary to include the exchange of vector mesons, as well as the Δ degrees of freedom. Davidson *et al.* [14] have also pursued this approach in some detail and have found, in particular, a large sensitivity to the method used to unitarize the tree approximation amplitudes. Chiral symmetry also plays a role in the approach to pion photo – production studied by Araki and Afnan [15], who have derived a set of equations for the various coupled amplitudes starting from a chirally symmetric and gauge invariant bag model Lagrangian.

Models in which the tree diagrams of effective Lagrangians are unitarized by various schemes continue to be popular [16–19]. Feuster and Mosel [16] have used this approach to study the electromagnetic couplings of nucleon resonances and to explore channels other than just πN and γN; in particular, $\pi\pi N$, ηN, and $K\Lambda$ [17]. Drechsel *et al.* [18] have developed a unitary isobar model for pion photo– and electroproduction on the proton. Their model contains Born terms, vector mesons, and nucleon resonances up to and including the $D_{33}(1700)$. A model based on unitarized tree diagrams has been used recently [19] to study the model dependence of extracting the amplitudes for the process $\gamma + N \Leftrightarrow \Delta(1232)$ from the reaction $\gamma + N \rightarrow \pi + N$.

Of course unitarity is not the only general restriction on the amplitudes for photo – and electroproduction; gauge invariance is also an essential constraint on these amplitudes. In treating the problem of gauge invariance in pion photoproduction several authors have focused on the Ward–Takahashi (WT) identities [20]. Ohta [21] has derived an electromagnetic current operator from the most general form of the extended pion–nucleon vertex function, using the minimal substitution prescription, and has shown that the obtained current operator and the isolated pole contribution satisfy the WT identities. He has also shown [22] that it is possible to derive electromagnetic interactions that are non–local and at the same time maintain local gauge invariance. Naus, Koch, and Friar [23] have used the WT identities to enforce gauge invariance at the operator level, rather than on the amplitude level. Besides developing relations for the off–shell behavior of strong and electromagnetic vertices, they have also reexamined the derivation of the Kroll–Ruderman theorem [24] for pion photoproduction. This theorem states that the threshold amplitude, to zeroth order in the photon energy, is model independent. Van Antwerpen and Afnan [25] have derived coupled channel integral equations that lead to photoproduction amplitudes that satisfy both two–body unitarity and generalized WT identities.

Implementing gauge invariance in models of the pion–nucleon system based on the Bethe–Salpeter equation [26], or one of its three-dimensional reductions [27, 28], requires some care. Gross and Riska [29] have shown that the WT identities play a central role in guaranteeing that the electromagnetic coupling to a two–body system described by one of these equations will lead to a conserved current and thereby to gauge invariance. Surya and Gross [30] have developed a unitary, gauge invariant, relativistic model for pion photo – production starting from a model of the pion–nucleon system [31] based on the Gross equation [28]. The Born terms and kernels of their integral equations include nucleon (*N*), delta (Δ), Roper (N^*), and D_{13} direct and crossed poles, as well as π, ρ, and ω exchange terms. They find a good fit to all $L \leq 2$ multipoles up to a photon energy of 770 MeV.

Haberzettl [32] has developed a gauge invariant model of pion photoproduction starting with an effective field theory of hadrons in which quantum chromodynamics is assumed to provide the necessary bare cutoff functions. His equations are nonlinear integral equations which can be difficult to solve in practice. He has discussed approximations that make the nonlinear formalism manageable and yet preserve gauge invariance. He and his collaborators have shown how to implement his formulation at tree level with form factors describing composite nucleons [33].

Several authors [34–41] have constructed Hamiltonian models for pion photo – production. With these models, the Hamiltonian acts in a limited Hilbert space such as $\mathcal{H} = N \oplus \Delta \oplus \pi N \oplus \gamma N$. Since these models are essentially coupled–channel potential models, they satisfy two-particle unitarity exactly and therefore lead to photo production amplitudes that satisfy Watson's theorem [7]. The electromagnetic parts of the Hamiltonian are defined in terms of matrix elements which describe the transitions $\gamma N \Leftrightarrow \pi N$ and $\gamma N \Leftrightarrow \Delta$. These matrix elements are calculated from effective Lagrangians in lowest–order perturbation theory. The strong–interaction part of the Hamiltonian provides a model for πN scattering in the absence of electromagnetic couplings and accounts for the rescattering that occurs after the photon has interacted with the nucleon. It contains $\pi N \Leftrightarrow \pi N$ potentials as well as vertex interactions, e.g., $\pi N \Leftrightarrow N$ or Δ. The potentials are either purely phenomenological separable potentials [34–36], or are taken from a meson–exchange model [37–41]. All of these Hamiltonian models are three–dimensional in character, with the total three-momentum conserved in intermediate states, but not the four–momentum. In spite of this, Nozawa, Blankleider, and Lee [36] were able to construct from Feynman amplitudes $\gamma N \Leftrightarrow \pi N$ matrix elements that are gauge invariant. This was done by assuming that in the second order matrix elements the four–momentum is conserved at the γN vertex, but not necessarily at the πN vertex.

The model for meson photoproduction that is developed here is closest in spirit to the Hamiltonian models . It differs in two important aspects. There is a more careful treatment of relativity, and gauge invariance is implemented in a very general way. Some features of the model have already been presented [42, 43]. The model is developed within the framework of relativistic quantum mechanics. By relativistic quantum mechanics is meant a theory in which it is required that the quantum–mechanical state vectors of a system transform according to a unitary representation of the Poincaré group [44]. The Poincaré group is the set of inhomogeneous Lorentz transformations, $x' = ax + b$, that map from one inertial frame to another. In the subgroup of continuous transformations, the so–called *proper subgroup*, these transformations can be expressed in terms of ten generators; four of which generate translations in space–time ($x' = x + b$), while the other six generate the homogeneous Lorentz transformations ($x' = ax$). These ten generators satisfy a set of commutation relations known as the Poincaré algebra. Several subsets of these generators have the property that they satisfy a closed subset of these commutation relations and thereby generate a subgroup of the proper Poincaré transformations. Some of these subgroups are associated with three–dimensional hypersurfaces in Minkowski space that do not contain timelike directions. Each *form* of relativistic quantum mechanics is associated with such a hypersurface and its corresponding subgroup [44, 45]. In relativistic quantum mechanics the generators are operators in the Hilbert space of the system. The generators of spacetime translations are the four components P_μ $(\mu = 0,1,2,3)$ of the four–momentum operator, while the generators of homogeneous Lorentz transformation are the six independent components $J_{\mu\nu} = -J_{\nu\mu}$ of the angular momentum tensor. In relativistic quantum mechanics these ten operators must satisfy the Poincaré algebra. In each form of relativistic quantum mechanics the generators of the subgroup of transformations that map the forms hypersurface into itself are chosen to be noninteracting. The remaining generators contain interactions. The most obvious form, i.e. the *instant form*, is based on the hypersurface t = const., while the *front form* is based on the null plane $ct + z = 0$. The point form is based on the hypersurface $c^2t^2 - \mathbf{x}^2 =$ const. The subgroups generated by the noninteracting generators are called *kinematic subgroups* [46], or *stability groups* [47].

In the instant form the three–momentum operator **P** and the angular momentum operator **J** are noninteracting, while the Hamiltonian H and **K**, the generator of rotationless boosts, contain interactions. A rotationless boost is a Lorentz transformation that relates two inertial frames moving relative to each other with the corresponding spatial axes parallel. The operators **P** and **J** generate translations and rotations in ordinary three–dimensional space, respectively. The instant form is like nonrelativistic quantum mechanics in that quantum mechanical state vectors are specified on the t = const. hypersurfaces, however, the nonrelativistic boost operators, which generate Galilean transformations, do not contain interactions. Here, we shall work in the instant form simply because it is familiar to more people than the other forms of relativistic quantum mechanics. The general procedures we shall develop can be easily adapted to the other forms, and as a matter of fact the internal part of the instant form mass operator we shall construct can be used in the other forms as well.

In constructing instant form models it is convenient to work with the set of operators $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ where M, $\mathcal{J}$, and $\mathbf{X}$ are the mass operator, the spin operator, and the Newton–Wigner position operators [44], respectively. These operators satisfy simpler commutation rules than the generators. In fact, the only nonzero commutators are the standard position–momentum commutators for the components of $\mathbf{P}$ and $\mathbf{X}$ and the usual angular momentum commutators for the components of $\mathcal{J}$. The ten generators of the Poincaré group $\{H,\mathbf{P},\mathbf{J},\mathbf{K}\}$ can be expressed in terms of the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$, and it can be shown that if the members of the set $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ satisfy the correct commutation relations then the generators expressed in terms of them satisfy the Poincaré algebra. In general, in constructing the operators $\{M,\mathbf{P},\mathcal{J},\mathbf{X}\}$ for a system it is assumed that the operators $\mathbf{P}$, $\mathcal{J}$, and $\mathbf{X}$ are the same as those of the corresponding noninteracting system, and therefore only the mass operator M contains an interaction. This procedure for constructing the Poincaré generators is known as the Bakamjian–Thomas construction [44,48].

The outline of the thesis is as follows. In Chapter 2 the fundamentals of the Poincaré group are reviewed and the Bakamjian–Thomas construction [44,48] is presented. The general structure of the momentum space matrix elements of the mass operator is developed. The transformation of quantum mechanical states under the Poincaré transformations is developed in Chapter 3. This includes a treatment of both proper transformations and spatial inversion. The material from Chapter 3 provides the basis for the development of Chapter 4, which is central to the thesis.

Chapter 4 presents the derivations of the most general forms for the electromagnetic interactions in a Poincaré and gauge invariant mass operator. These include interactions which couple a photon–baryon state to a single–baryon state, $\gamma + b \Leftrightarrow B$, as well as interactions that couple photon–baryon states directly to meson–baryon states, $\gamma + b \Leftrightarrow \mu + B$. The general results derived here reproduce older results for photo amplitudes when applied to $\gamma + N \Leftrightarrow N$, $\gamma + N \Leftrightarrow \Delta$, and $\gamma + N \Leftrightarrow \pi + N$, however they also provide the general forms for amplitudes such as

$\gamma + N \Leftrightarrow D_{13}(1520)$, $\gamma + N \Leftrightarrow \rho + N$, $\gamma + N \Leftrightarrow \eta + N$, etc. Chapter 4 also develops the general forms for the helicity amplitudes for the various photo processes involving arbitrary mesons and baryons; as well as the relations between helicity amplitudes and the usual partial wave amplitudes. The general isospin structure for the various photo amplitudes is developed at the end of Chapter 4.

Chapter 5 presents the Lippmann–Schwinger equations that arise from the mass operator. The solutions of these equations are the amplitudes for meson–nucleon scattering and meson photoproduction. This chapter also develops the formulas for the various photoproduction cross sections in terms of our partial wave photoproduction amplitudes. The relations between our partial wave photoproduction amplitudes and the multipoles presented in the literature are also derived. Some kinematic details end Chapter 5.

Chapter 6 presents the derivation of the mass operator electromagnetic interactions from effective Lagrangians by means of Okubo's method [49]. The Okubo method is based on an attempt to block diagonalize a quantum field theory Hamiltonian in Fock space. Of course in general this cannot be done exactly so it is necessary to resort to a perturbation theory, which leads to interactions that are closely related to Feynman amplitudes. Here and elsewhere in the thesis we refer to the Roper resonance, i.e., $P_{11}(1440)$, and the $D_{13}(1520)$ resonance simply as *R* and *D*, respectively.

Chapter 7 begins with a precise summary of the electromagnetic potential used here in the calculations of the multipoles that describe the process $\gamma + N \Leftrightarrow \pi + N$. The form factors that were chosen to take into account the fact that hadrons are not point particles are described here. In this chapter the multipoles that arise from the model are compared with those obtained from experiments. The experimental multipoles are given in the SAID database [50]. SAID is an acronym for Scattering Analysis Interactive Dial–in program. This program is presently maintained at the George Washington University and contains a wealth of information on pion–nucleon reactions as well as photo– and electroproduction of mesons from the nucleon. Chapter 8 summarizes and discusses the results obtained here and also gives suggestions for future research.