

PHY509: SOLUTION HW #1.

P1. (a) For both (A) and (B), the center of mass motion is given by $(2m)\dot{v}_{CM} = F$ during $0 < t < \Delta t$. Therefore, $\Delta v_{CM} = F\Delta t/(2m)$.

(b) B has the larger kinetic energy because in addition to the CM kinetic energy it has a contribution from the rotational motion.

(c) $K = K_{CM} = \frac{1}{2}(2m)\Delta v_{CM}^2 = (F\Delta t)^2/(4m)$.

(d) During $0 < t < \Delta t$, the motion is due to a constant acceleration and therefore $\Delta x_{CM} = \frac{1}{2}a\Delta t^2 = \frac{1}{2}(F/2m)\Delta t^2 = F\Delta t^2/(4m)$.

(e) $W = F\Delta x = F \cdot F\Delta t^2/(4m)$ agrees with (c).

(f) $K_{CM} = (F\Delta t)^2/(4m)$, same as (c).

(g) Use the torque equation $\Gamma = I\alpha$, where Γ is the torque, I moment of inertia, and α angular acceleration. W.r.t. CM, the moment of inertia is $I = 2mR^2$ and the torque is $\Gamma = FR$. Therefore the angular acceleration $\alpha = F/(2mR)$ during $0 < t < \Delta t$. The final angular velocity ω is $\omega = \alpha\Delta t = F\Delta t/(2mR)$. The rotational kinetic energy $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(2mR^2)(F\Delta t/2mR)^2 = (F\Delta t)^2/(4m)$.

(h) The total kinetic energy is $K = K_{CM} + K_{rot} = (F\Delta t)^2/(2m)$, twice as large as that of (A).

(i) Since the angle moves with a constant acceleration, $\Delta\theta = \frac{1}{2}\alpha\Delta t^2 = \frac{1}{2}(F/2mR)\Delta t^2 = F\Delta t^2/(4mR)$. Therefore the (nearly straight) arc made by $\Delta\theta$ has the length $\Delta x_{rot} = R\Delta\theta = F\Delta t^2/(4m)$. Adding this to the CM displacement, we get $\Delta x_{up} = \Delta x_{CM} + \Delta x_{rot} = F\Delta t^2/(2m)$ and $\Delta x_{down} = \Delta x_{CM} - \Delta x_{rot} = 0$.

(j) $W = F_{up}\Delta x_{up} + F_{down}\Delta x_{down} = F\Delta x_{up} = (F\Delta t)^2/(2m)$, in agreement with (h). Although the impulse changes the same amount of linear momentum for (A) and (B), the work done by the force (i.e., the change in total kinetic energy) is not necessarily the same.