PHY509: Mid-term Exam (10/17/05, 4:00-6:00 PM)

P1. A particle of mass m incoming at velocity v_0 with an impact parameter s from a remote distance scatters off an infinitely heavy mass. The repulsive force between the masses is

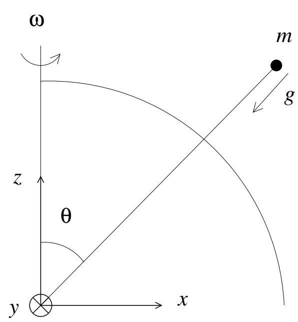
$$F(r) = +\frac{k}{r^3}.$$

(a) Using the equation of motion of central force system, $d^2u/d\phi^2 + u = -(m/l^2u^2)F$ with $u = 1/r$, find the general form of solution.

(b) With the initial conditions given above, determine the unknown parameters in $r(\phi)$.

(c) What is the scattering angle θ, expressed in terms of the impact parameter and ϕ?

(d) Find the differential scattering cross-section $\sigma(\theta) = (s/\sin\theta)(ds/d\theta)$.

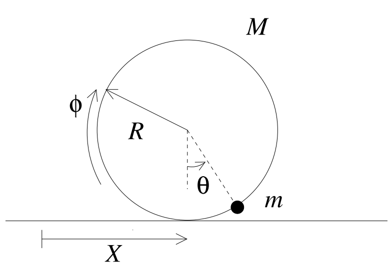


P2. An object of mass m at a height of h is falling from rest in the earth's rotating frame. During the fall the gravitational acceleration is a constant g toward the center of the earth.

(a) With the earth's rotational frequency ω, what are the Coriolis and centrifugal forces (magnitude and direction)? Use the (x, y, z) coordinates as shown.

(b) ω is small that the transverse motion is a small correction to the vertical motion. Also ignore the centrifugal force. Write down the equations of motion in the rotating frame along the x, y, z-direction.

(c) Solve the equations and find where the object lands with respect to the position straight down the release point.



P3. A bead of mass m slide frictionlessly on a ring of mass M and radius R. The ring is always up-right and moves in a straight line. As shown in the figure, the generalized coordinates are chosen as X the translational displacement of the ring, ϕ the angular displacement of the ring and θ the angular displacement of the bead from the vertical.

(a) Write down the Lagrangian of the system and obtain the equations of motion.

(b) Assume small θ so $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \theta^2/2$. What is the oscillation frequency for θ?

(c) From now on, assume that the friction from the table makes the ring move without slipping. Find the modified equations of motion by introducing a Lagrange multiplier λ.

(d) Solve the problem to obtain the new oscillation frequency for θ.

(e) What is the constraint force as a function of θ?