

PHY509: HOMEWORK 9. (due 11/28/05)

P1. (a) As discussed in the class, prove that the Poisson bracket for conjugate variables is unity, regardless of a particular canonical transformation, i.e., $[Q, P]_{q,p} = 1$ by using the generating function $F = F(q, Q)$.

(b) (Part of FW prob. 6.18) Prove that the angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ satisfies the Poisson bracket relation $[L_i, L_j] = L_k$ with i, j, k in cyclic order.

P2. (Same as FW 6.6) Consider a harmonic oscillator in (q, p) by a Hamiltonian,

$$H(q, p) = \frac{1}{2m}(p^2 + m^2\omega^2q^2).$$

(a) By using the Poisson bracket relation $[Q, P] = 1$, determined the constant C in the transformed variables $Q = C(p + im\omega q)$ and $P = C(p - im\omega q)$.

(b) If we had used a generating function $S(q, P)$ for this transformation, what is S ?

(c) Find the transformed Hamiltonian in the new system (Q, P) .

(d) Find the Hamilton's equations of motion for (Q, P) and solve them as a function of time.

(e) *Using the result from (d)*, write (q, p) as a function of time.

This transformation is a classical counterpart of creation and annihilation operators (a^\dagger, a in $[a, a^\dagger] = 1$) in quantum mechanics.