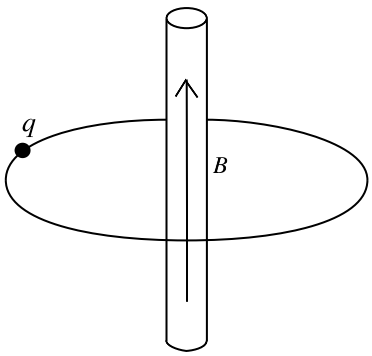
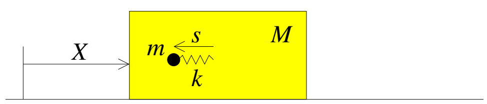
**PHY509: HOMEWORK 6. (due 10/21/05)**




**1.** (a) Show that a Lagrangian $L = \frac{1}{2}m\dot{\mathbf{x}}^2 - q\phi + q\mathbf{v} \times \mathbf{A}$ with electrostatic potential $\phi$ and vector potential $\mathbf{A}$ is consistent with the Newton's equation $m\boldsymbol{a} = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

(b) At any given time, a vector potential $A(\mathbf{r}, t)$ is translationally invariant and the scalar potential $\phi = 0$. Prove that the canonical momentum, $\mathbf{p} = m\mathbf{v} + q\mathbf{A}$, of a particle of mass $m$ and charge $q$ is indeed conserved as we turn up $\mathbf{A}$ linearly in time, by explicitly from Newton's equation with $\mathbf{F} = m\boldsymbol{a}$, **without using the Lagrangian method**.

(c) Imagine an infinitely long solenoid with magnetic field $\mathbf{B}$ pointing along its axis, as shown here. $\mathbf{B} = 0$ outside the solenoid. The charge is constrained to move frictionlessly along the circular path. Prove that the canonical angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times (m\mathbf{v} + q\mathbf{A})$, is also conserved as we turn up $\mathbf{B}$ at a constant rate.




**P2**. A mass $m$ inside a box of mass $M$ is tied to a spring of the force constant $k$. Initially the system is at rest and at time $t = 0$ the box is given an instant impulse that the box gains an initial velocity $v_0$. All motions are one-dimensional and use the generalized coordinates $s, X$. $s$ is the extension of the spring (with initial condition $s(t = 0) = 0$) and $X$ is the displacement of the box.

(a) What is the initial condition $\dot{s}(t = 0)$ right after the impulse? Justify your answer.

(b) Set up your Euler-Lagrange equations of motion for $s$ and $X$.

(c) What is the oscillation frequency of the mass inside the box?

(d) Now, assume that there is friction to the mass $m$ given by $-\alpha\dot{s}$. Write down the modified Euler-Lagrange equations with the friction force.

(e) What is the terminal velocity of the box?