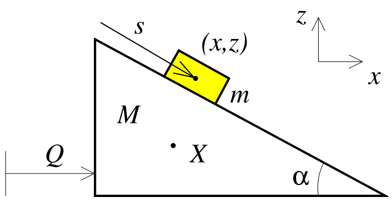
PHY509: HOMEWORK 5. (due 09/30/05)



P1. A block of mass m is sliding down a wedge of mass M without friction by the gravitational force. The wedge also slides on a table frictionlessly. The angle of the wedge is given as α.

(a) Introduce the generalized coordinates (s, Q) as shown in the figure. Express the block's center of mass coordinates (x, z) and the horizontal coordinate of the wedge X in terms of the generalized coordinates up to constants.

(b) For a virtual displacement associated with s ($\delta s \neq 0$ and $\delta Q = 0$), find the corresponding displacements for the block $(\delta x, \delta z)$ and the wedge δX.

(c) Do the same for the virtual displacement for Q ($\delta = 0$ and $\delta Q \neq 0$).

(d) What are the generalized forces Q_j for the coordinates (s, Q)?

(e) From the forces above in (d), construct the Lagrangian-Euler equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j.$$

(f) Solve for $\ddot{s}$.

P2. Consider a simple pendulum consisting of a mass m attached to a string of length l. After the pendulum is set into motion, the length of the string is shortened at a constant rate

$$\frac{dl}{dt} = -\alpha = constant.$$

The suspension point remains fixed.

(a) With the initial length l_0. write down the Lagrangian.

(b) Write down the equation of motion for the swinging angle θ.

(c) Assume that α is very small ($\sqrt{g/l} \gg \alpha/l$ for most l), i.e., it swings many times as we pull the string. Then we can approximately define the amplitude as a function of time. Explain how the amplitude changes with time.