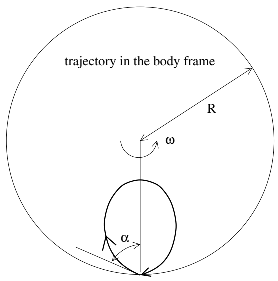
PHY509: HOMEWORK 4. (due 09/30/05)



P1. As you have seen in the video clip in class, a kid on a merry-go-round can roll a ball and get the ball back depending on the initial condition. We solve separately for such situation both in an initial and a body frame (of the merry-go-round). The merry-go-round rotates with an angular velocity ω and the kid is located at a distance R from the center. Ignore gravitational force, any friction and the rotation of the ball. The initial condition with respect to the body frame is that the boy rolls the ball with an initial *speed* v_0 at an angle α to his left as shown.

(a) In an initial frame, the boy receives the ball after he has rotated by 90^o. Find v_0 and α.

We now work in the body frame for the same situation as above, in following steps. In the cylindrical coordinate system, velocity and acceleration can be written as $\boldsymbol{v} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$ and $\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$.

(b) Get two equations from r and θ components in the equation of motion,

$$m\boldsymbol{a} = \boldsymbol{F} - 2m\boldsymbol{\omega} \times \boldsymbol{v} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}).$$

(c) Show that we have a conservation law similar to the angular momentum conservation in the inertial frame,

$$r^2(\dot{\theta} + \omega) = C.$$

Determine the constant C using the results from (a).

(d) Show that the radial part of the equation of motion becomes

$$\ddot{r} - C^2/r^2 = 0.$$

(e) The condition of the ball coming back can be written as

$$\int \dot{\theta} dt = \Delta\theta = 0,$$

for the integration over the whole path as depicted in the figure. Using (c) and (d), show that $\Delta\theta = 2\dot{r}(t) - \omega t$ where is the time t is when the ball comes back.

(f) By integrating the equation of (d), finally find the time t and show that $\Delta\theta = 0$.

P2. Fetter and Walecka Problem 2.1