PHY509: HOMEWORK 3. (due 09/26/05)

P1. Solve trajectory from a central force $f(r) = -k/r^3$. Use the equation of motion

$$\frac{d^2u}{d\phi^2} + u = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right).$$

You may consult Landau and Lifshitz, Prob.1 after §15.

(a) For $mk/l^2 < 1$, what is the general form of solution $u(\phi)$?

(b) Determine free parameters using the following energy conservation with energy E.

$$\frac{l^2}{2m}\left(\frac{du}{d\phi}\right)^2 + V(1/u) + \frac{l^2}{2m}u^2 = E.$$

(c) For $mk/l^2 > 1$, what is the general form of solution $u(\phi)$?

(d) Determine free parameters using energy conservation with energy E. Depending on the sign of E, there are two different types of orbits, one bounded and the other unbounded.

(e) Above solutions satisfy $u \to \infty$ (or $r \to 0$) as $\phi \to \infty$, i.e., it takes infinite number of revolutions in approacing to the center of force. But the time in the fall does not take infinitely long. Show that the time needed for an object to completely fall into the center from a radius r is

$$t = \frac{\sqrt{m}}{E}\left[\left(k - \frac{l^2}{m} + 2Er^2\right)^{1/2} - \left(k - \frac{l^2}{m}\right)^{1/2}\right],$$ by using the relation

$$\frac{dr}{dt} = \sqrt{\frac{2}{m}\left(E - V(r) - \frac{l^2}{2mr^2}\right)},$$ and $$t = \int \frac{dr}{\sqrt{\frac{2}{m}\left(E - V(r) - \frac{l^2}{2mr^2}\right)}}.$$

P2. Do Fetter and Walecka, Problem 1.14.

The cross section is $\sigma_T = \int 2\pi s ds = \pi s_{\max}^2$ where $s_{\max}$ is the maximum impact parameter when the scattering actually involves collision with the necleus.