PHY509: HOMEWORK 2. (due 09/16/05)

P1. We re-derive the trajectory $r(\phi)$ of the elliptic planetary orbit in the inverse-square law force field. This method is simpler than that discussed in class. You may look up the derivation in the Goldstein 3rd Ed. pp. 86-87.

(a) From the energy conservation, we obtained

$$\frac{l^2}{2mr^4}\left(\frac{dr}{d\phi}\right)^2 + V(r) + \frac{l^2}{2mr^2} = E.$$

Now introducing a new variable $u(\phi) = 1/r(\phi)$, show that the differential equation of motion for u satisfies

$$\frac{d^2u}{d\phi^2} + u = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right), \tag{1}$$

where the force is $f(1/u) = f(r) = -dV/dr$.

(b) With the inverse-square force, $f = -k/r^2 = -ku^2$, find the general solution $u(\phi)$ of the above equation and show that it agrees with the form $1/r = C(1 - e\cos\phi)$.

(c) Imagine that you are Kepler and knew of the equation of motion, Eq. (1). If your observed data shows that the planetary bodies have elliptic orbits, $1/r = C(1 - e\cos\phi)$, what is your conclusion on the nature of the force acting between the heavenly masses?

(d) From the general theory of relativity, the gravitational force is known to deviate slightly from the inverse square law as

$$V(r) = -\frac{k}{r}\left(1 + \frac{\alpha}{mc^2}\frac{k}{r}\right),$$

with a very small parameter α. Using the equation of motion, Eq. (1), show that the orbit is a (very slowly) precessing ellipse. Find the advancing angle per period as a function of α.

P2. A satellite of mass m revolves around the earth in a perfectly circular orbit with radius R. The earth's gravitational field is $V = -k/r$. Suddenly the satellite gets a short thrust toward the earth and it instantly gains a (small) velocity of v_0 in the radial direction.

(a) What are the total energy and angular momentum of the satellite before and after the thrust?

(b) Now the satellite's orbit is a slightly elongated ellipse. What are the closest and farthest distance of the satellite from the center of the earth? Find the answer in the leading order of v_0.

(c) What is the eccentricity e in the leading order of v_0?

(d) If the thrust had been in the opposite direction, what would be your answers to the parts (b) and (c)?