Lorenz Model of Atmospheric Physics

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Introduction to Atmospheric Physics

- Atmospheric Physics: Application of Physics to study the atmosphere

  → Attempt to model Earth's atmosphere through fluid flow equations, chemical models, radiation balancing, energy transfer processes.

  → Weather system modeling requires applications of statistical mechanics, scattering theory, wave propagation models and cloud physics.

  → Atmospheric Electricity/Tide also studied on global scale

Introduction to Atmospheric Physics

- Climate modeling: One of the most difficult tasks in Atmospheric Physics
    - *Simple Models*
        - → Allow for broad, fundamental understanding of processes occurring
        - → Easy to interpret result
    - *Complex Models*
        - → Provide realistic picture of processes occurring
        - → Result much harder to interpret

E. N. Lorenz and the field of chaos

- Lorenz (1963) studied basic equations of fluid mechanics (Navier-Stokes eqs.); Describe velocity, temp., density as functions of time/position
- Rayleigh-Benard Problem: Fluid in container with surfaces at different temps.

  → Increase temp. diff. to obtain transitions between stationary state, steady flow, and eventual chaotic flow.
- Computational power not as impressive compared to today prompted Lorenz to simplify the Navier-Stokes eqs…

The Lorenz Equations

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

- Variables x, y, z: Derived from temp., density, velocity
- Variables σ, r, b: Measures of temp. diff. across fluid and other fluid parameters

Any behavior in Lorenz Model also found in weather problem

- Lorenz Model is centered around three, coupled, differential eqs.
- Behavior depends on the three parameters…

1.) r – Rayleigh number:

→ Measure of temp. diff. between top/bottom surfaces of liquid

→ Proportional to effective force on fluid

2.) σ – Prandtl number:

→ Ratio of momentum diffusivity to thermal diffusivity

3.) b – Physical proportion

Matlab to be used to analyze this chaotic system

- Analysis of 3-D systems can be done numerically/ symbolically
- Using symbolic analysis, proposed program with four comps.

1.) Set of numerical libraries for implementation of commands/functions

2.) Set of symbolic routines

→ Symbolic Matlab Toolbox

3.) Graphical commands for rep. of tasks

→ 3-D rotation, zoom in/out

4.) Graphical User Interface (GUI)

Symbolic-Numeric Analysis of Lorenz Model

- Lorenz equations may be put into Jacobian matrix form
- Once specific parameters are given, Lyapunov exponents of the system may be numerically computed

  → Possible to observe Lyapunov exponents for all 3-D as a function of time

  → Neg. L.E. (stable), pos. L.E. (chaos)
- Graphically display the attractor during system evolution

TO BE CONTINUED…