PHY 411-506 Computational Physics II

Chapter 12: Interdisciplinary Topics

Lecture 5

Wednesday April 16, 2008

Lecture Outline

Neural Networks and the Brain

General References

- Minicourse on computational neuroscience for physicists by Sebastian Seung, M.I.T.
- Computation in the brain
- The Nervous System
- Wikipedia Neurons.
- Java applet Associative Memory: Hopfield model

Neural Network Models

- A crude binary model of a neuron is that it can be in one of two states, a resting state which can be represented by binary 0, and an active or *firing* state in which an impulse or signal is transmitted along the *axon* which is a long fiber extending from the cell body or *soma*.
- The axon of a neuron branches multiply and connects to other neurons via *synapses*, which are essentially chemical junctions.

- What determines the state of a neuron? A simple model was introduced by McCulloch and Pitts in 1943
  - ◇ The neuron sums all of the input signals from other neurons which synapse to it: if this sum is larger than a threshold value, then it fires, and otherwise it does not.
- Hopfield introduced a simple model based on these ideas in Proc. Natl. Acad. Sci. USA, **79**, 2554 (1982) which simulates the storage and retrieval of memories.
  - ◇ Hopfield network in Scholarpedia
- Consider a network of N neurons. The state of the network is defined by specifying a binary valued *potential* $V_i = 1$ or 0 at each neuron: if $V_i = 1$ then neuron i is firing, while if $V_i = 0$ it is not.
- The *synaptic strength* between neurons i and j is denoted T_{ij}. The integrated signal at neuron i is

$$S_i = \sum_{j \neq i} T_{ij} V_j \ .$$

The state of this neuron is set according to the criterion

$$V_i = \begin{cases} 1, & \text{if } S_i > 0 \\ 0, & \text{if } S_i \leq 0 \end{cases} .$$

- The network is operated by updating the neurons according to some protocol, for example by choosing neurons at random or sequentially (which is usually what is done in software networks), or by updating the whole network synchronously (which is more natural for a hardwired network controlled by a clock).
- Hopfield showed that the network tends to the global minimum of the function

  $$E = -\sum_{\text{pairs}} T_{ij} V_i V_j \ ,$$

  which represents the energy of a random spin glass with spin variables $s_i = 1 - 2V_i = \pm 1$.
- The energy landscape depends on the the synaptic strengths of the network T_{ij}. It turns out that these strengths can be used to store

patterns represented by states of the network according to *Hebb's Rule*:

$$T_{ij} = \sum_{p=1}^{P} (1 - 2V_i^{(p)})(1 - 2V_j^{(p)}) \ ,$$

where P is the number of patterns stored and $V_i^{(p)}$ is the state of neuron i in pattern p.

- Hopfield showed that
  - ◇ The network dynamics decreases the energy of the network This implies that if the network is started in an arbitrary state, then it will evolve to the nearest local energy minimum.
  - ◇ The stored states are local minima of the energy function. So if the initial state happens to be in the *basin of attraction* of one of the stored minima, the that pattern will be recalled!
- A network with N neurons has a huge number 2^N states. The network works best if the stored memories partition the space of network states into well defined basins.

- The storage capacity of the network is found to be $\sim 0.13N$. If too many memories are stored, then the minima are not well defined and memories may not be perfectly recalled.
- A simple application of neural networks is to storage and reconstruction of "associative memories", illustrated in the Hopfield Java Applet
- Neural networks have applications in many areas:
  - ◇ Artificial Neural Networks in High Energy Physics
  - ◇ Electrical Engineering
- For web resources on neural networks, see Wikipedia Neural Network.