PHY 411-506 Computational Physics II

Chapter 12: Interdisciplinary Topics

Lecture 4

Monday April 14, 2008

Lecture Outline

Earthquakes and Self-Organized Criticality

Gutenberg-Richter Law

- Moment M of quake is the relative shift of fault
- Magnitude on the Richter scale

$$\mathcal{M} = \ln M$$

- Gutenberg-Richter law: probability of quake of magnitude $\mathcal{M}$

$$P(\mathcal{M}) = \frac{A}{M^b} = Ae^{-b\mathcal{M}}$$

where A is a constant and $b \simeq 0.8 - 1.5$ is a power-law exponent

- Energy released in event $E \propto M$
- Average energy released per event

$$E_{\text{average}} = \int_0^\infty EAe^{-b\mathcal{M}} \mathrm{d}\mathcal{M} \sim \int_0^\infty e^{(1-b)\mathcal{M}} \mathrm{d}\mathcal{M} = \frac{1}{1-b} e^{(1-b)\mathcal{M}} \Big|_0^\infty$$

diverges if $1 - b > 0$

- Maybe an indication that Earth is in a self-organized critical state!

Burridge-Knopoff Model

- Described in Section 12.2 of the textbook
- Top plate moves slowly with speed v_0 relative to bottom plate
- Crust between plates modeled by
  - ◇ Discrete masses m_i at positions x_i and velocities v_i
    - ○ Simplest 1-d model has N blocks in a line with equal masses m
  - ◇ Attached to top plate with leaf-springs with constant k_p
  - ◇ Attached to neighboring masses with coil springs constant k_c
  - ◇ In frictional contact with bottom plate
    - ○ Static friction has maximum magnitude F_0
    - ○ Kinetic friction given by

$$F_f = -\frac{F_0 \mathrm{sign}(v_i)}{1 + |v_i/v_f|}$$

      where v_f is a model parameter

- System undergoes stick-slip motion

- Newton's equations of motion for the blocks

$$m_i \frac{\mathrm{d}x_i}{\mathrm{d}t} = v_i$$
$$m_i \frac{\mathrm{d}v_i}{\mathrm{d}t} = k_c(x_{i+1} + x_{i-1} + 2x_i) + k_p(v_0 t - x_i) + F_f$$

- Choose a time step Δt and use Euler's algorithm
- The frictional force is a little tricky!
  - ◇ If m_i is not moving at time step n, find the net spring force
    - ◦ If $|F_{i,\mathrm{springs}}| \leq F_0$, static friction adjusts to make the net force on m_i exactly zero
    - ◦ If $|F_{i,\mathrm{springs}}| > F_0$, static friction will have magnitude F_0 and oppose the net springs force
  - ◇ If m_i is moving at time step n, find the new velocity assuming kinetic friction
    - ◦ If the new velocity has same sign as the old velocity, the block continues slipping, so proceed

  ◦ Else, set the new velocity to zero, the block sticks!

- For the simulations
  - ◇ Choose parameter, e.g., $N = 25, m = 1, k_p = 40, k_c = 250, F_0 = 50, v_0 = 100$
  - ◇ Choose a small enough time step, e.g., $\Delta t = 0.005$
  - ◇ Initialize the blocks at rest with random displacements from equilibrium of magnitude e.g., ± 0.001, otherwise the motion tends to be periodic!
  - ◇ Run the simulation for total times $t \sim 500$
- To measure the magnitude of an event:
  - ◇ The quake starts when any block begins to slip
  - ◇ The quake ends as soon as all blocks stick again
  - ◇ The moment of the quake is measured by

$$M = \sum_{\text{while slipping}} \left(\sum_{i}^{N} v_i \Delta t \right)$$

- To test the Gutenberg-Richter law:
  - ◇ Run a large number of simulations and make a histogram of magnitudes

$$\mathcal{M} = \ln M$$

  - ◇ Make a log-log plot of number of events versus $\mathcal{M}$

Cellular-Automaton Earthquake Model

- The Burridge-Knopoff model is computationally demanding
- Rundle, Jackson and Brown introduced a simplified cellular-automaton version, see Ferguson, Klein and Rundle, Computers in Physics, **12**, 34 (1998)
  - ◇ Use noninerital massless blocks with *slip deficit* variable $\phi_i(t)$, which measures the amount by which the block lags the upper plate
  - ◇ Each block has a static failure threshold $\sigma_{F,i}$ and a residual stress $\sigma_{R,i}$

- ◇ The net stress on each block is

$$\sigma_i(t) = -K_L\phi_i(t) + K_C \sum_{q \text{ nearest neighbors } j} (\phi_j(t) - \phi_i(t))$$

For a 2-d fault model $q = 4$

- ◇ A block with $\sigma_i(t) > \sigma_{F,i}$ slips to a residual stress state $\sigma_{R,i}$ using

$$\phi_i(t+1) = \phi_i(t) + \frac{\sigma_i(t) - \sigma_{R,i}}{K_L + qK_C}\theta(\sigma_i(t) - \sigma_{F,i})$$

where $\theta(x)$ is the unit step function $= 1$ if $x > 0$, zero otherwise. This is a deterministic slip model. Probabilistic slip rules can also be defined.

- ◇ A quake is simulated by repeating the slippage rule on all blocks and and incrementing $t \to t + 1$ until all $\sigma_i(t) < \sigma_{F,i}$
- ◇ Simulate time between quakes by moving the top plate by $V\Delta T$ and decreasing each ϕ_i by this amount until the next quake occurs

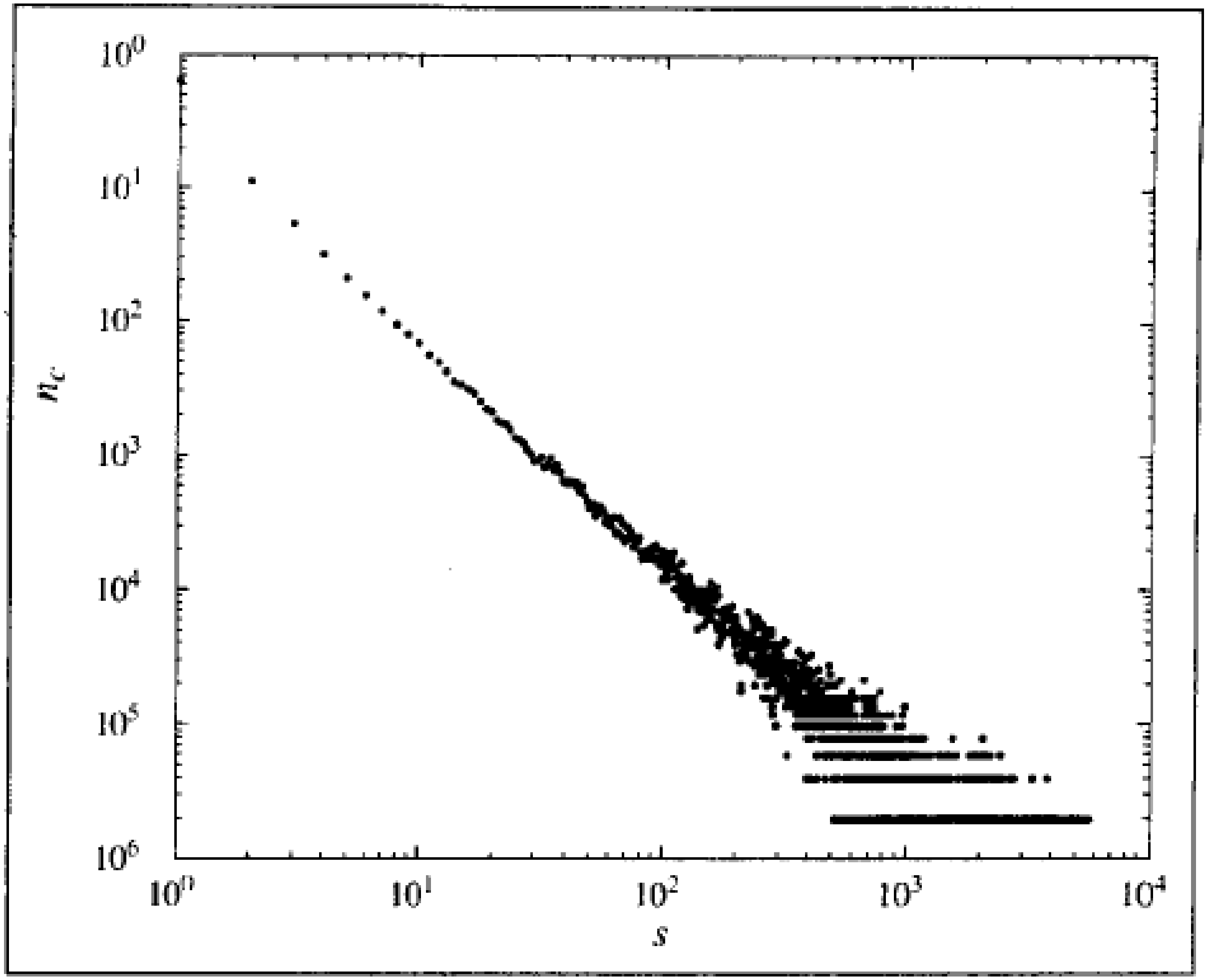

Figure 2. Log–log plot of the mean number of clusters $n_c(s)$ with s failed blocks for $L=128$, nearest-neighbor interactions, and closed boundary conditions. The parameters are $\sigma_F=50$, $\sigma_R=0$, $K_C=K_L=1$, and the zero-velocity-limit with the deterministic jump function. The negative of the slope corresponds to a cluster scaling exponent $\tau=1.6$ over the interval $1\leqslant s\leqslant 300$, where $n_c(s)\sim s^{-\tau}$, implying that $b=0.9$.