Solar System: Planets Asteroids Comets

The Solar System

Our Sun is an average star which moves around the center of our Milky Way galaxy with a speed of 220 km/s once every 200 million years. It drags along with it numerous captive objects, 8 planets, 5 dwarf planets, asteroids, comets, and dust. Some of the planets have captive moons and other satellites. All of these objects orbit the Sun along approximately elliptical trajectories according to Newton's equation of motion and inverse square force law of gravity.

The size of a circle is determined by a single parameter, its radius r. The size and shape of an ellipse is determined by two parameters, its semimajor axis length a and its eccentricity ϵ. The semiminor axis has length $b = a\sqrt{1-\epsilon^2}$. An ellipse has two foci located on its major axis. The perihelion distance $q = a(1-\epsilon)$ is measured from a focus to the closer major axis intersection, and the aphelion distance $p = a(1+\epsilon)$ to the farther intersection. If the origin of 2-dimensional polar coordinates (r, θ) is chosen at one focus and θ measured from the major axis, then a points on the ellipse satisfies the equation

$$r = \frac{a(1-\epsilon^2)}{1-\epsilon\cos\theta} \ . \tag{1}$$

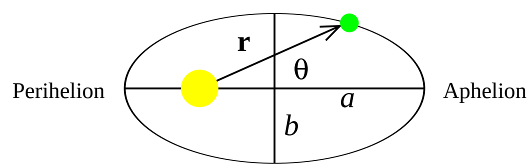

Figure 1: Elliptical orbit of a planet around the Sun at a focus.

Kepler's first law states that the planet moves along an elliptical trajectory with the Sun at a focus of the ellipse as shown in Fig. 1. Strictly speaking, the center of mass of the Sun-planet system is at the focus: in the case of the most massive planet Jupiter, this center of mass lies just outside the surface of the Sun. Assuming that the Sun is at the focus is an excellent approximation. It was shown by Newton that the elliptical orbit is a consequence of the inverse square law of gravitation

$$F = \frac{G_{\rm N}Mm}{r^2} \ , \tag{2}$$

where $G_{\rm N}$ is the same constant of universal gravitation that occurs in the equations of general relativity.

Kepler's second law says that the radius vector from the focus to the planet sweeps out equal areas in equal times. This is a consequence of conservation of angular momentum

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{mM}{m+M} r^2 \frac{d\theta}{dt}\right) = 0 \ . \tag{3}$$

Kepler's third law related the period T of the orbit to the semimajor axis

$$T^2 = \frac{4\pi}{G_{\rm N}(m+M)}\ a^3 \ . \tag{4}$$

Planets

There are 8 planets and 5 dwarf planets in the Solar System. Their physical properties and orbital parameters can be found for example at IAU and NASA websites[1].

Kepler's regular polyhedron model

Kepler's three laws were brilliantly successful in describing planetary orbits. He also discovered that the orbital radii of the six planets know at that time appeared to be related to the properties of the five regular Platonic solids[2].

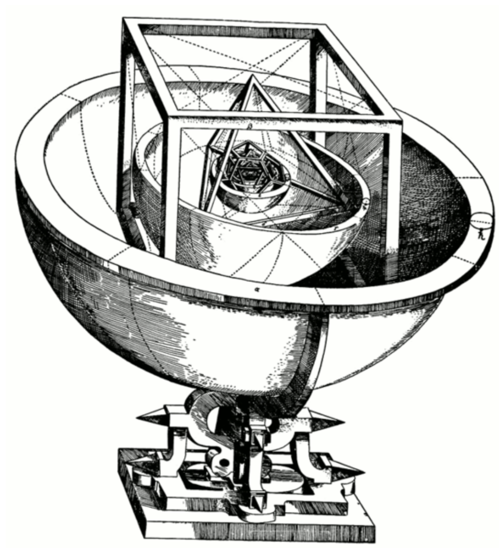

Figure 2: Kepler's model of the solar system *Mysterium Cosmographicum* (1596) located the six known planets on spheres bounded by the five Platonic solids in the order: Mercury − octahedron − Venus − icosahedron − Earth − dodecahedron − Mars − tetrahedron − Jupiter − cube − Saturn.

According to Kepler, draw spheres centered around the Sun as follows: Start with the innermost planet Mercury and construct a sphere of radius equal to the orbital radius of this planet. Construct an octahedron such that each of its faces touches the sphere. Next construct the sphere of Venus such that it touches each of the vertices of the octahedron. Continue with the planets and regular solids in the order listed in Fig. 2.

The Titius-Bode law

There are two free parameters in Kepler's laws: the semimajor axis and eccentricity. The laws do not predict the actual values of a or ϵ for the observed planets. This requires a theory about how these objects were created.

The Titius-Bode law was proposed by various astronomers and mentioned in books by Titius (1766) and

Bode (1768). It is based on the sequence of integers $n_i, i = 0, 1, 2, \ldots$

$$0, 3, 6 = 2{\times}3, 12 = 2{\times}6, 24 = 2{\times}12, 48 = 2{\times}24, \ldots, n_i = 2{\times}n_{i-1} \ . \tag{5}$$

Note that $3 \neq 2{\times}0$ is a special case. The law predicts that the mean orbital radius of the i-th planet starting with Mercury is

$$r_i = \alpha + \beta n_i \ , \tag{6}$$

where α, β are constants. The semimajor axis of the first planet Mercury is 0.387 AU. Titus and Bode chose the approximate value $r_0 = 4/10$ of the mean distance of the Earth from the Sun. Then for the Earth the equation reads

$$1 = \frac{4}{10} + 6\beta \ , \quad \Rightarrow \quad \beta = \frac{1}{10} \ . \tag{7}$$

This works reasonably well for Venus $r_1 = 0.7$, close to the observed semimajor axis 0.723 AU. For planets beyond Earth it predicts the sequence in $1.4, 2.8, 5.2, 10, 19.6, 38.8, \ldots$ in AU. This sequence fitted Mars at 1.524 AU, Jupiter at 5.204 AU, Saturn at 9.582AU. Amazingly, Uranus at 19.23AU was determined in 1781 to be a planet an not a star. The gap at 2.8 is located in the belt of asteroids between Mars and Jupiter. The dwarf planet Ceres at 2.766AU was discovered in 1801 at almost exactly the predicted location. The law breaks down however with Neptune at 30.10AU and the dwarf planet Pluto at 39.48AU.

Attempts to apply empirical laws of this type to planetary satellites, for example the moons of Uranus, have met with some success, but there has not been much success in deriving such laws from more fundamental theories of planetary evolution.

Asteroids

Asteroids are minor planets. Approximately half a million asteroids, most of them orbiting the Sun in a belt between Mars and Jupiter, have been identified and their orbital elements measured[1]. This data can be dowloaded from NASA's Jet Propulsion Laboratory website at Caltech in an ascii text file `http://ssd.jpl.nasa.gov/dat/ELEMENTS.NUMBR` that has the following format.

Num	Name	Epoch	a	e	i	w	Node	M	H	G	Ref
1	Ceres	53600	2.7659794	0.08001400	10.58604	73.39245	80.40973	86.9543947	3.34	0.12	JPL 26
2	Pallas	53400	2.7732371	0.23022192	34.85083	310.54489	173.16605	28.7016679	4.13	0.11	JPL 21
3	Juno	53600	2.6682410	0.25836604	12.97136	247.84212	170.12636	345.4894717	5.33	0.32	JPL 42

C++ Program to Generate Asteroid Histogram

The following program downloads the asteroid data file from JPL using the GNU `wget` program, which you can download and install on Windows from `http://gnuwin32.sourceforge.net/packages/wget.htm`. The data is binned and plotted using `gnuplot`.

Program 1: `http://www.physics.buffalo.edu/phy410-505/topic1/asteroid.cpp`

```
#include <cmath>
#include <cstdlib>
#include <fstream>
#include <iostream>
```

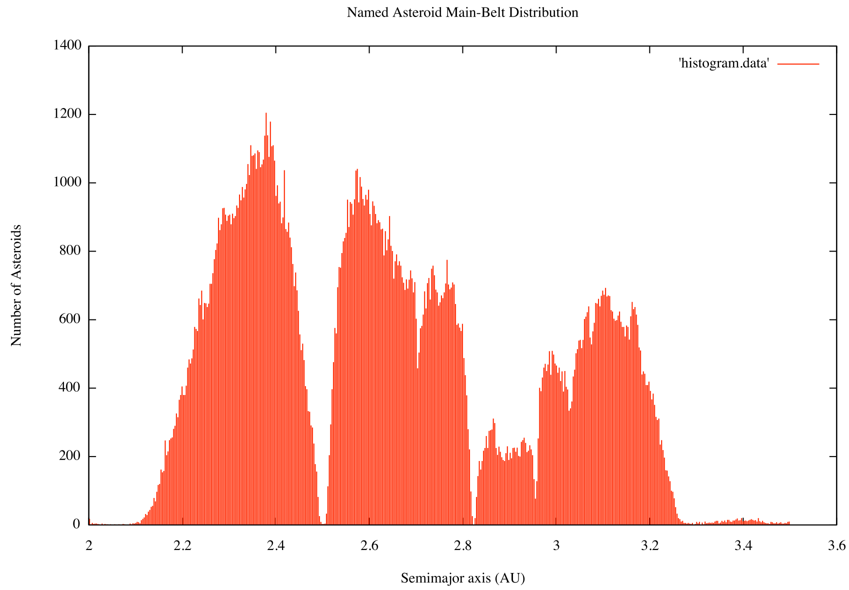

Figure 3: Kirkwood (1857) noticed gaps in the distribution of asteroids in the main belt between Mars and Jupiter. These Kirkwood gaps are believed to be due to orbital period resonances with the giant planet Jupiter that make asteroid orbits unstable.

```
#include <string>
#include <vector>
using namespace std;

// -------------------- define global constants --------------------

// locations of wget and gnuplot programs
const string
#ifdef _MSC_VER     /* typical locations on Windows using Visual C++ */
  wget("\"C:\\Program Files\\GnuWin32\\bin\\wget.exe\""),
  gnuplot("C:\\gnuplot\\bin\\wgnuplot.exe");
#else               /* typical locations on Macintosh or Linux */
  wget("/usr/bin/wget"),
  gnuplot("/usr/bin/gnuplot");
#endif

// location of asteroid data
const string
  url("http://ssd.jpl.nasa.gov/dat/ELEMENTS.NUMBR"),
  data_file_name("ELEMENTS.NUMBR");

// -------------------- function declarations --------------------

void wget_data();

void read_data(
```

```
  vector<double>& a_data
);

// -------------------- function definitions --------------------

int main() {

  // get data from JPL and read semimajor axis values
  wget_data();
  vector<double> a_data;
  read_data(a_data);

  // create a histogram
  double a_min = 2.0, a_max = 3.5;
  int bins = 500;
  double bin_width = (a_max - a_min) / bins;
  vector<double> bin_vector(bins);
  for (int i = 0; i < a_data.size(); i++) {
    double a_bin = (a_data[i] - a_min) / bin_width;
    int bin = int(a_bin);
    if (bin >= 0 && bin < bins)
      bin_vector[bin] += 1;
  }

  // plot the histogram using Gnuplot
  ofstream data_file("histogram.data");
  for (int i = 0; i < bins; i++)
    data_file << a_min + (i + 0.5) * bin_width << '\t'
              << bin_vector[i] << '\n';
  data_file.close();

  ofstream script_file("histogram.gnu");
  script_file << "set title \'Named Asteroid Main-Belt Distribution\'\n"
              << "set xlabel \'Semimajor axis (AU)\'\n"
              << "set ylabel \'Number of Asteroids\'\n";
  script_file << "plot \'histogram.data\' with impulses\n";
  script_file << "pause mouse\n";
  script_file.close();

  string command(gnuplot + " histogram.gnu");
  system(command.c_str());
}

void read_data(vector<double>& a_data) {

  // open the downloaded file
  ifstream data_file(data_file_name.c_str());
  if (data_file.fail()) {
    cerr << "Cannot open " << data_file_name << endl;
    exit(EXIT_FAILURE);
  }
```

```
  // empty the data vector and read the data
  a_data.clear();
  string line;
  while (getline(data_file, line)) {
    string s = line.substr(30, 12);
    double a = atof(s.c_str());
    if (a != 0.0)
      a_data.push_back(a);
  }
  cout << " read " << a_data.size() << " elements from "
       << data_file_name << endl;
  data_file.close();
}

void wget_data() {
  string command(wget + " " + url);
  if (system(command.c_str()) != 0) {
    cerr << "Failed to download " << url << endl;
    exit(EXIT_FAILURE);
  }
}
```

Comets

The orbital elements of approximately 3,000 comets are available from NASA's JPL website `http://ssd.jpl.nasa.gov/?sb_elem` in the following format:

```
Num  Name                                Epoch      q           e        i         w        Node          Tp       Ref
------------------------------------- ----- ----------- ---------- --------- --------- --------- -------------- ------------
  1P/Halley                            49400  0.58597811 0.96714291 162.26269 111.33249  58.42008 19860205.89532 JPL J863/77
  2P/Encke                             51120  0.33541700 0.84859557  11.86038 186.41178 334.72243 19970523.56424 JPL J974/1
  3D/Biela                             -9480  0.87907300 0.75129900  13.21640 221.65880 250.66900 18321126.61520 SAO/1832
```

Homework Problem

Download `http://nssdc.gsfc.nasa.gov/planetary/factsheet/cometfact.html` (NASA's Comet Fact Sheet on 20 selected comets) and fit this data to Kepler's third law. Use the fit to estimate $G_{\mathrm{N}} M_{\odot}$ and compare with G_{N} and $M_{\odot}$ values listed in most physics textbooks. Download the full comet orbital element table from JPL `http://ssd.jpl.nasa.gov/?sb_elem`. Compute T for each comet using Kepler's third law and try to find a simple model numerically that relates the period T to the semiminor axis b instead of the semimajor axis a. When you are looking for a model it helps to start by simply plotting the data!

References

[1] International Astronomical Union – Minor Planet Center:
http://www.cfa.harvard.edu/iau/mpc.html, NASA Jet Propulsion Laboratory – Solar System Bodies website: http://ssd.jpl.nasa.gov/?bodies.

[2] See http://mathforum.org/dr.math/faq/formulas/faq.polyhedron.html for geometric properties of regular polyhedra.