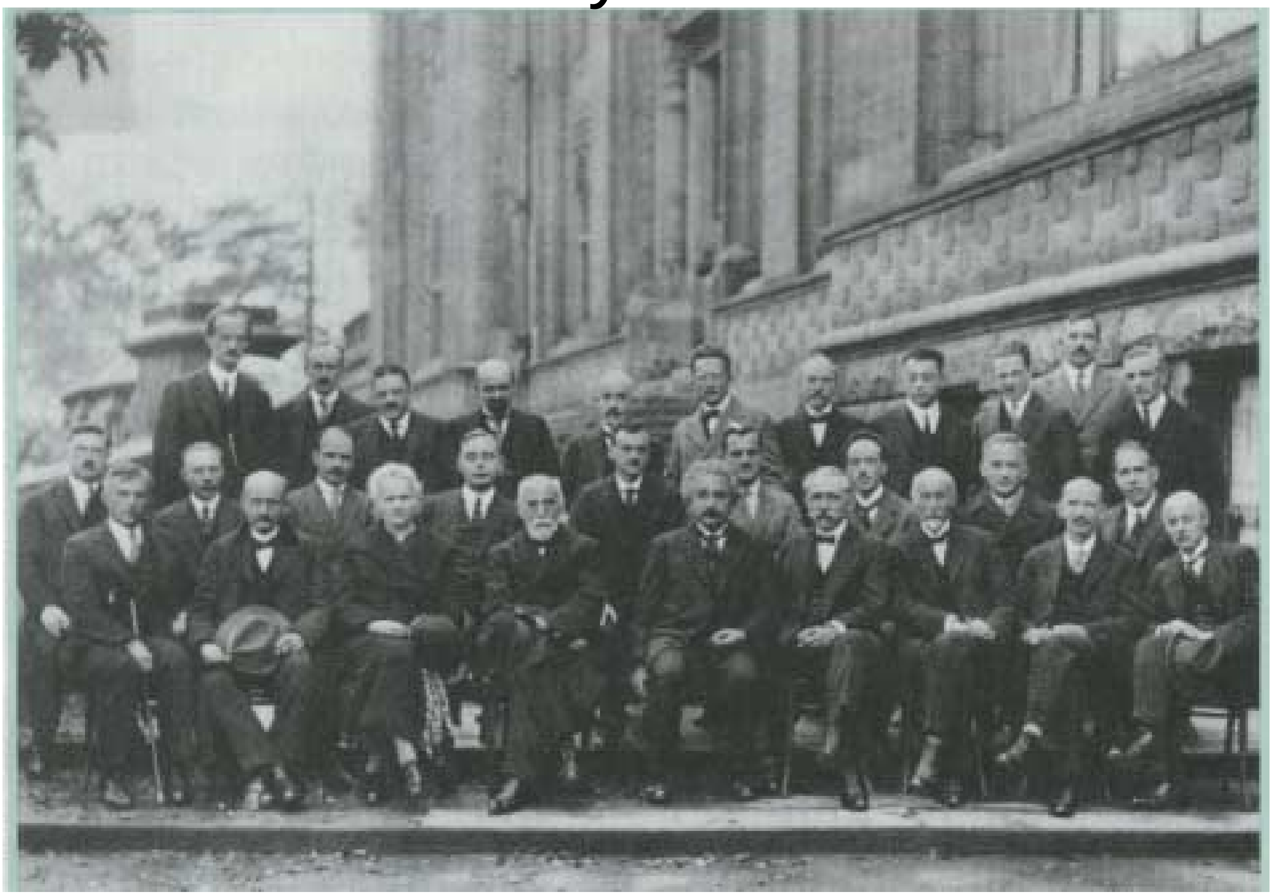
Chapter 40: Quantum Physics

Quantum mechanics deals with the Physics of *microscopic systems* which are beyond our ordinary physical senses (and conventional understanding). In quantum mechanics, particles (e.g., electrons) can behave like waves, and radiation (electromagnetic waves) can behave like a particle. In classical Physics every aspect of a system (e.g., energy, momentum, position, etc) can be known simultaneously, in quantum mechanics this is not the case! By realizing the ***particle-wave duality*** of matter, and by accepting the limited information allowed to the observer, quantum mechanics has become one of the most successful (and thoroughly tested) theories ever.

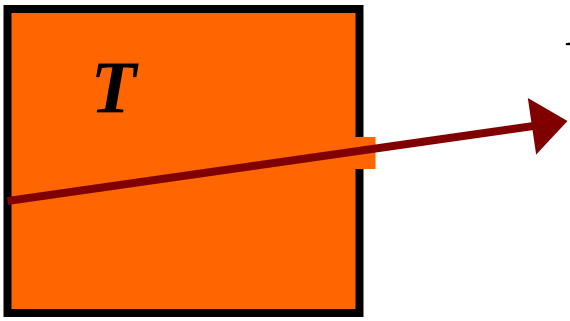
1927 Solvay Conference



- End of 19th century-Reductionism: classical mechanics can explain everything?
- Wrong: new experiments produce startling, paradoxical results that cannot be explained classically
- Quantum mechanics provides mathematical framework (Schroedinger's wave mechanics, Heisenberg's operator formalism)
- But what does it mean? Bohr-Einstein debates

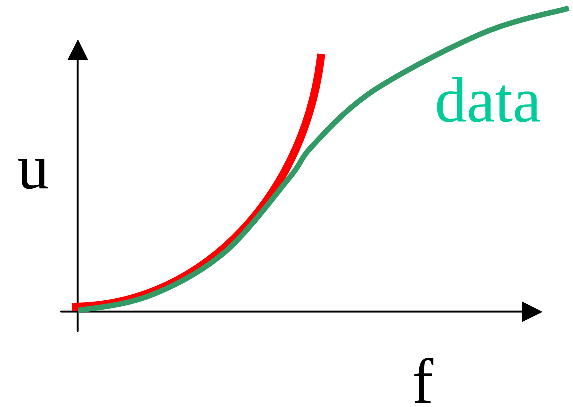
Particle nature of radiation:

Blackbody radiation



Energy density

$$u(f,T)$$



Rayleigh – Jeans:

$$u(f,T)=\frac{8\pi f^2}{c^3}kT$$

Planck: assumed that light is quantized in packets

$$u(f,T)=\frac{8\pi h}{c^3}\cdot\frac{f^3}{e^{\frac{hf}{kT}}-1}$$

fits blackbody radiation curve perfectly!

Electromagnetic radiation consists of indivisible quanta with energy

$$E=hf$$

"photon"

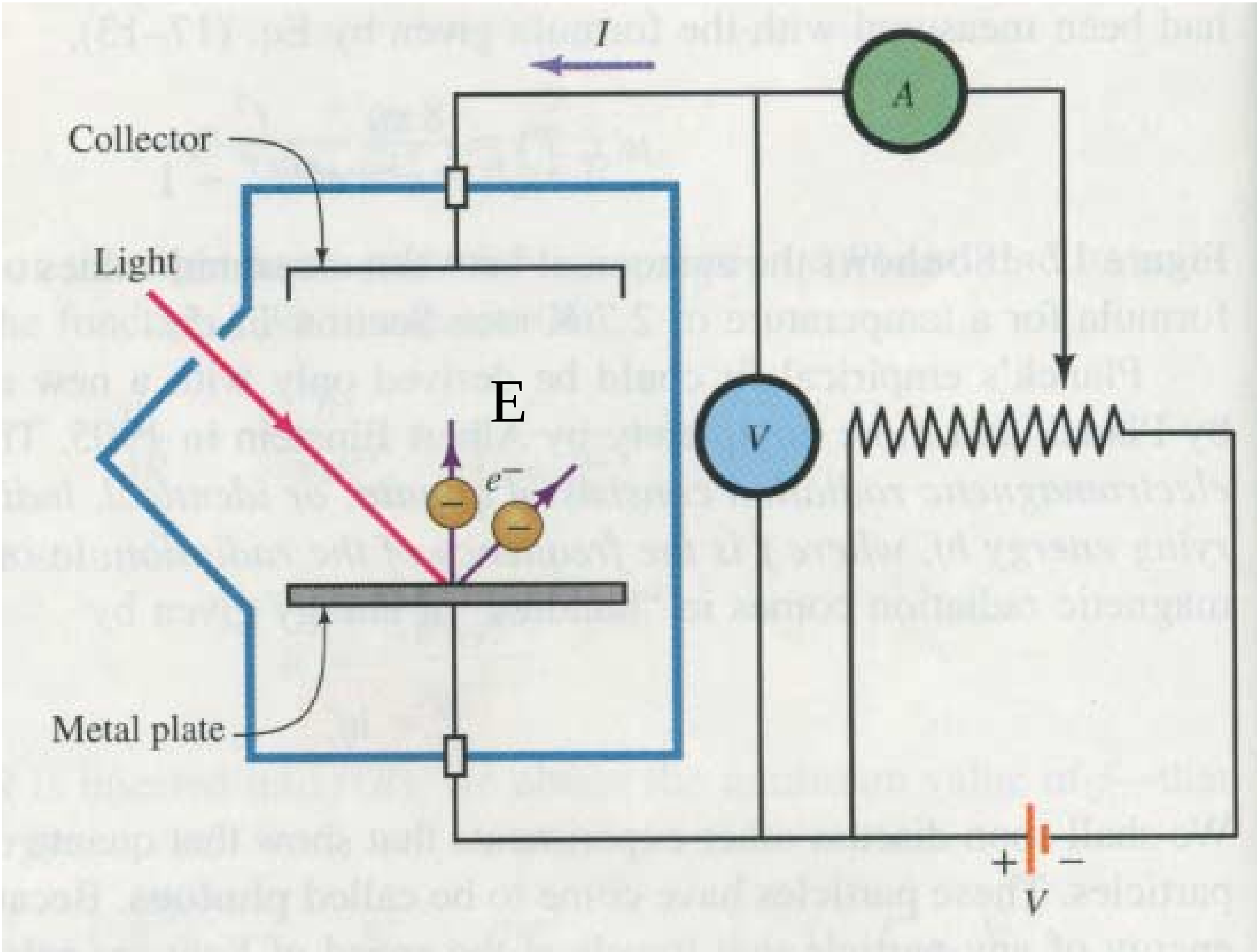
and momentum

$$p=\frac{E}{c}=\frac{hf}{c}=\frac{h}{\lambda}$$

$$\lambda=\frac{h}{p}$$

Example 40-3: Bright star I=1.6x10^{-9} W/m^2 at Earth's surface and $\lambda = 560$ nm. Rate of photons entering a night-adapted eye?

Photoelectric Effect



1. EM radiation hits metal plate → photoelectrons (PE)

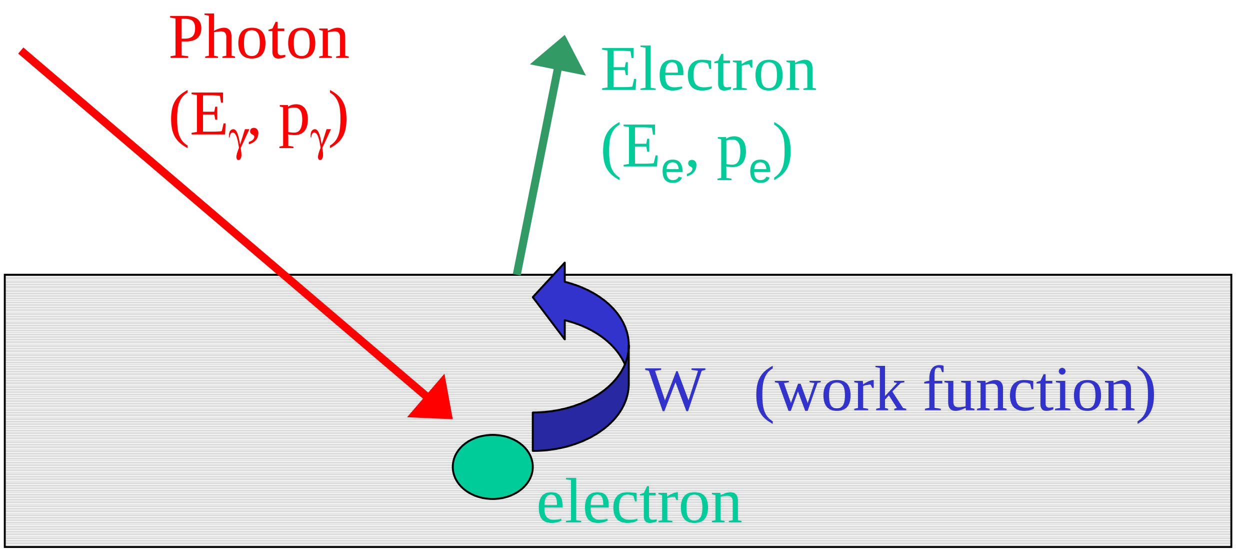
2. PE emitted only if $f > f_0$ (f_0 depends on metal)

3. If f held constant, number of PE depends linearly on intensity I

4. Max kinetic energy of PE independent of I, but varies linearly with f

5. No measurable time delay between arrival of photon and emission of PE

Energy and momentum are **always** conserved



Energy delivered by photon:

For a single photon:

$$E_\gamma = hf$$

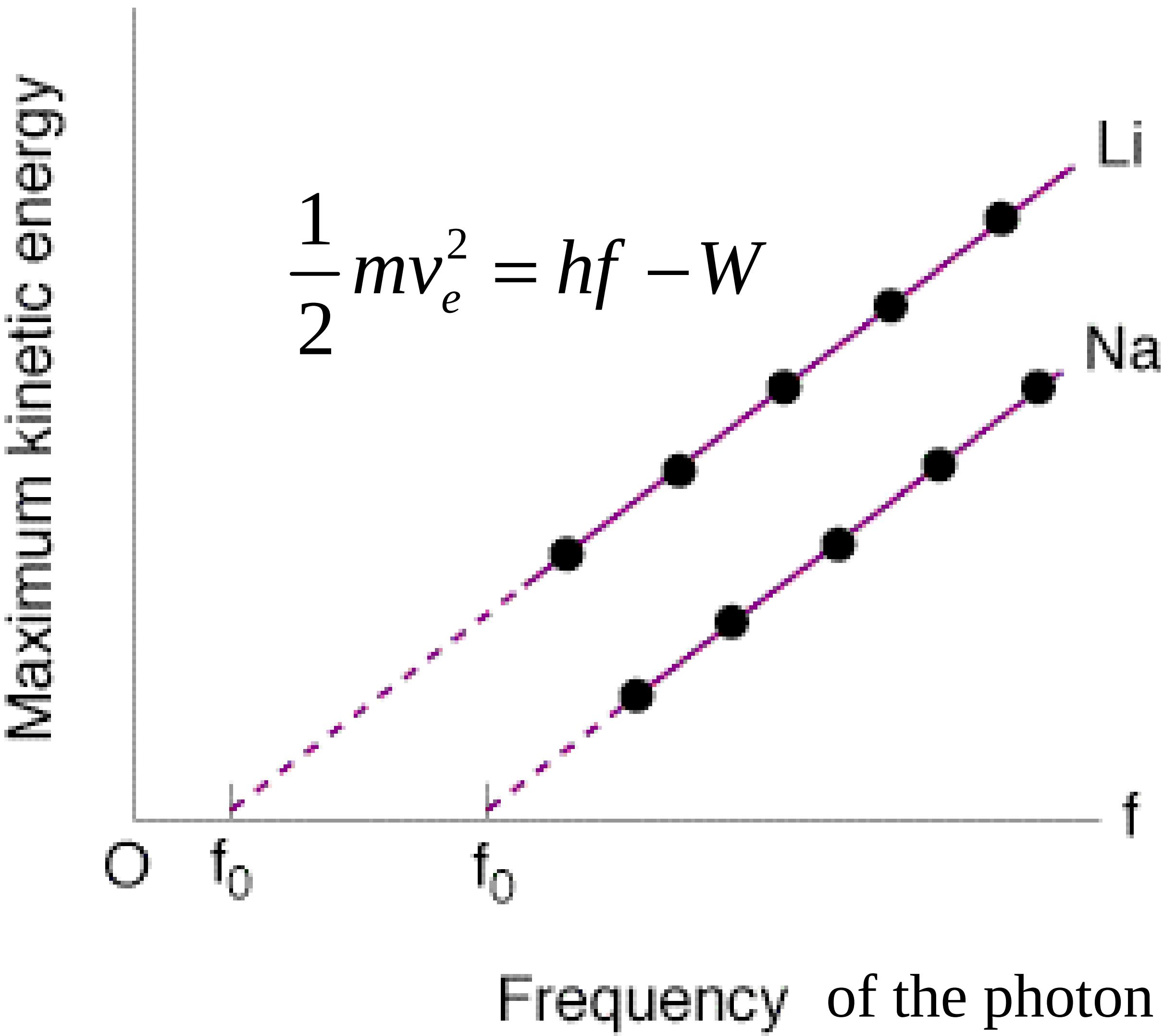
For *N* photons:

$$E_{total} = N_\gamma E_\gamma \Rightarrow I$$

Energy delivered to exiting electron:

$$E_{kin} = \frac{1}{2} m v^2 = hf - W$$

Max kinetic energy of photoemitted electron



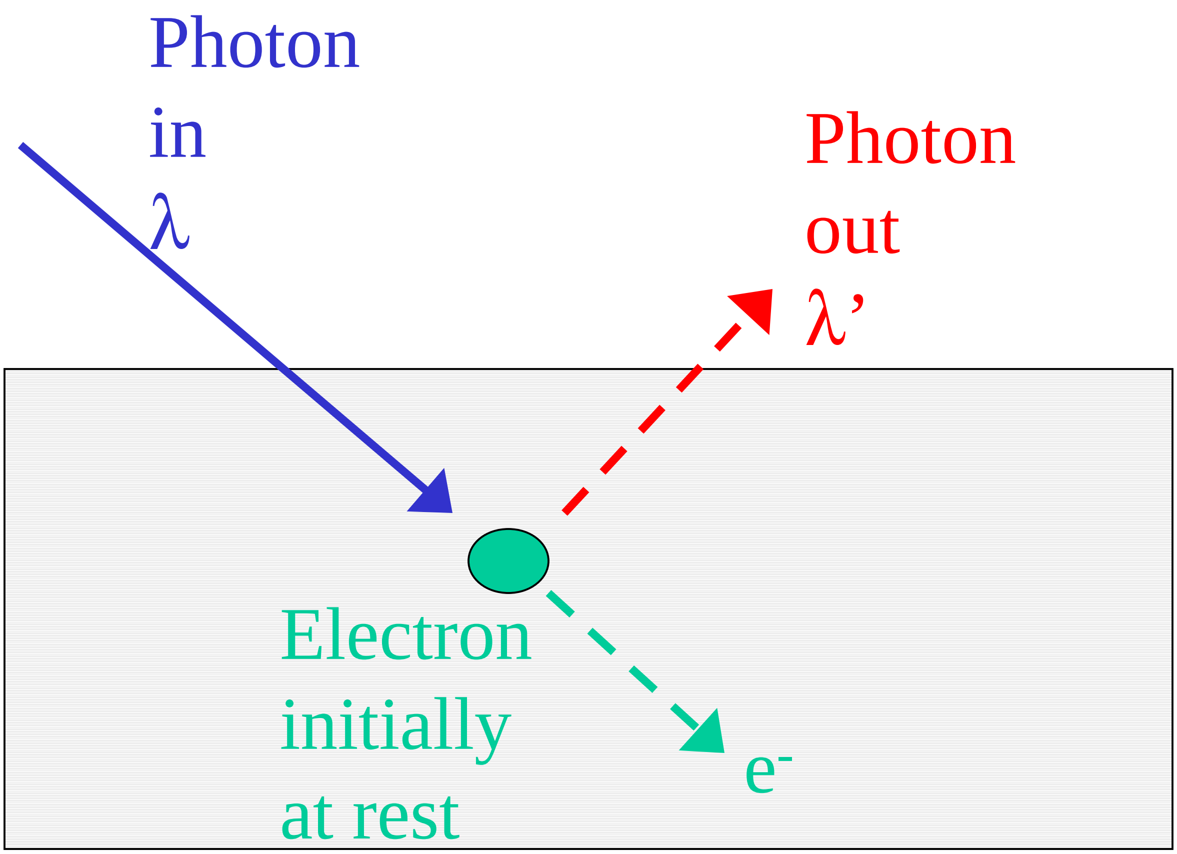
$$\frac{1}{2}mv_e^2 = hf - W$$

of the photon (Hz)

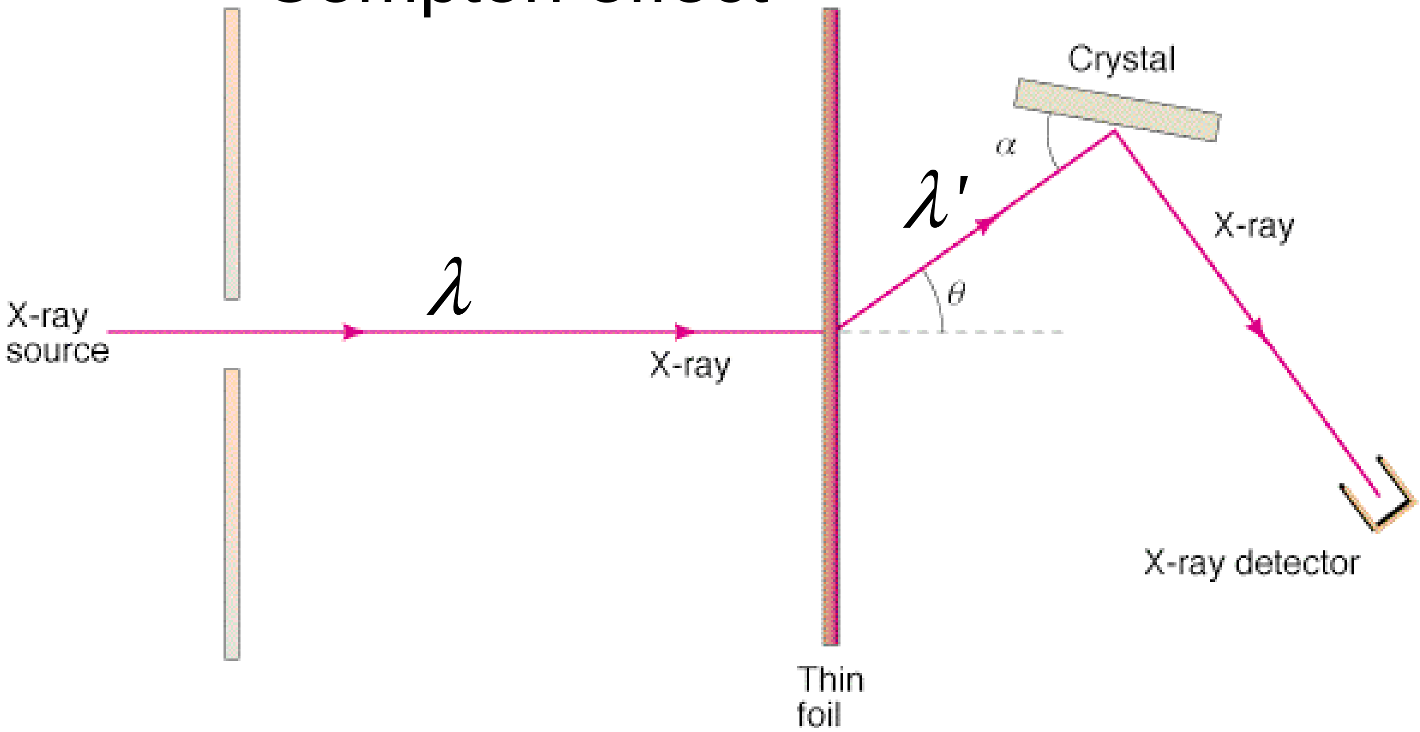
What about the number N_e of photoemitted electrons?

$N_e \ \alpha$

Compton effect



Compton effect

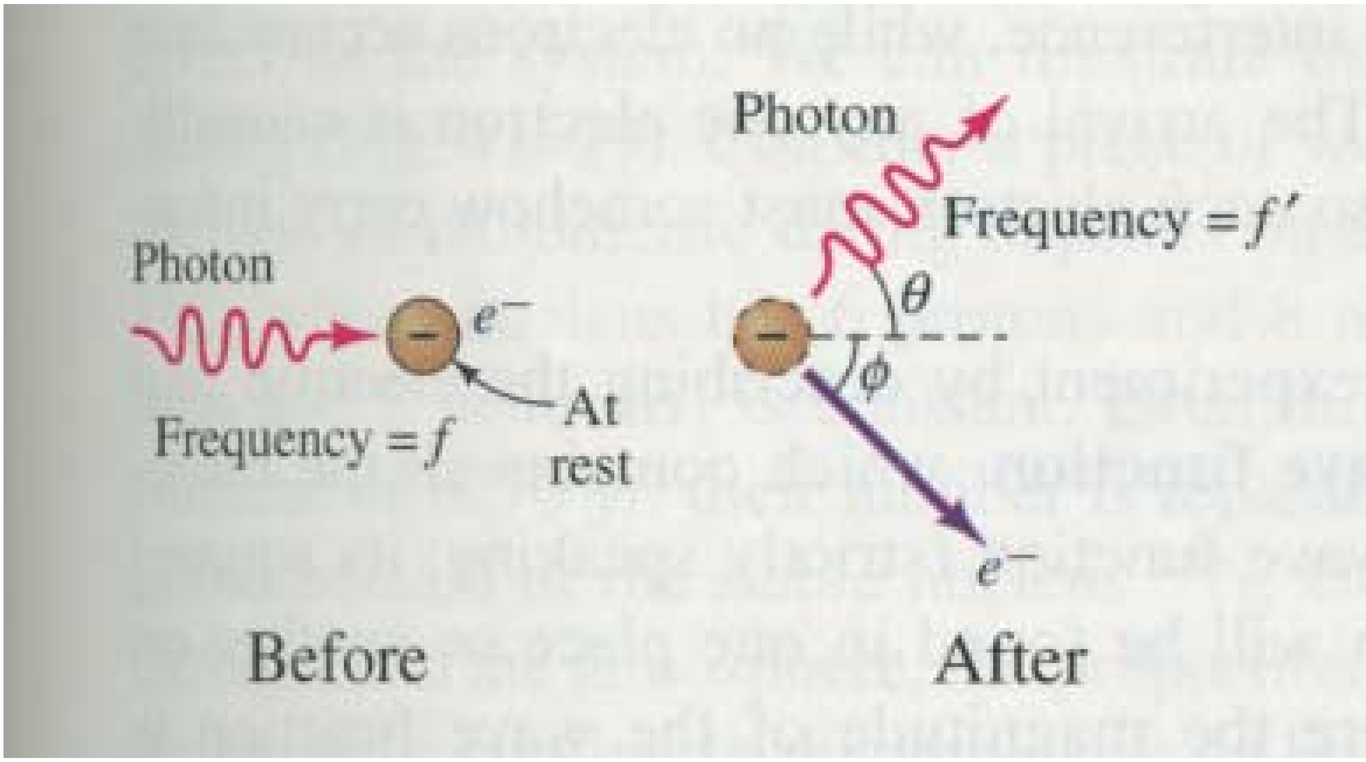


Warning in dealing with high energy particles!

No particle can travel faster than the speed of light c. For particles that are travelling at relativistic speeds (approaching c) one cannot use the classical expressions for energy and momentum (see Ch. 40). Instead on must use relativistic expression that insure that $v \leq c$.

$$E = \sqrt{p^2c^2 + m^2c^4} \cong pc \neq \frac{p^2}{2m} \text{ when } pc \gg mc^2$$

for kinetic energy pc much greater than rest mass energy mc^2 (homework 29d, 55, 22)


Photon
Frequency = f
e⁻
At rest
Before
Photon
Frequency = f′
θ
φ
e⁻
After

Momentum conservation:

x-direction: $p = p'\cos\theta + p_e \cos\phi$

y-direction: $0 = p'\sin\theta - p_e \sin\phi$

$p - p'\cos\theta = p_e \cos\phi$

$p'\sin\theta = p_e \sin\phi$ square and add

$\rightarrow p^2 - 2pp'\cos\theta + p'^2\cos^2\theta + p'^2\sin^2\theta =$

$p_e^2\cos^2\phi + p_e^2\sin^2\phi$

$p^2 - 2pp'\cos\theta + p'^2 = p_e^2$ **(A)**

Energy conservation $\left(E = \sqrt{p^2c^2 + m^2c^4}\right)$:

$E + mc^2 = E' + E_e'$

$E = pc, \quad E' = pc, \quad E_e = \sqrt{p_e{}^2c^2 + m^2c^4}$

$\rightarrow pc - p'c + mc^2 = \sqrt{p_e{}^2c^2 + m^2c^4}$ square both sides

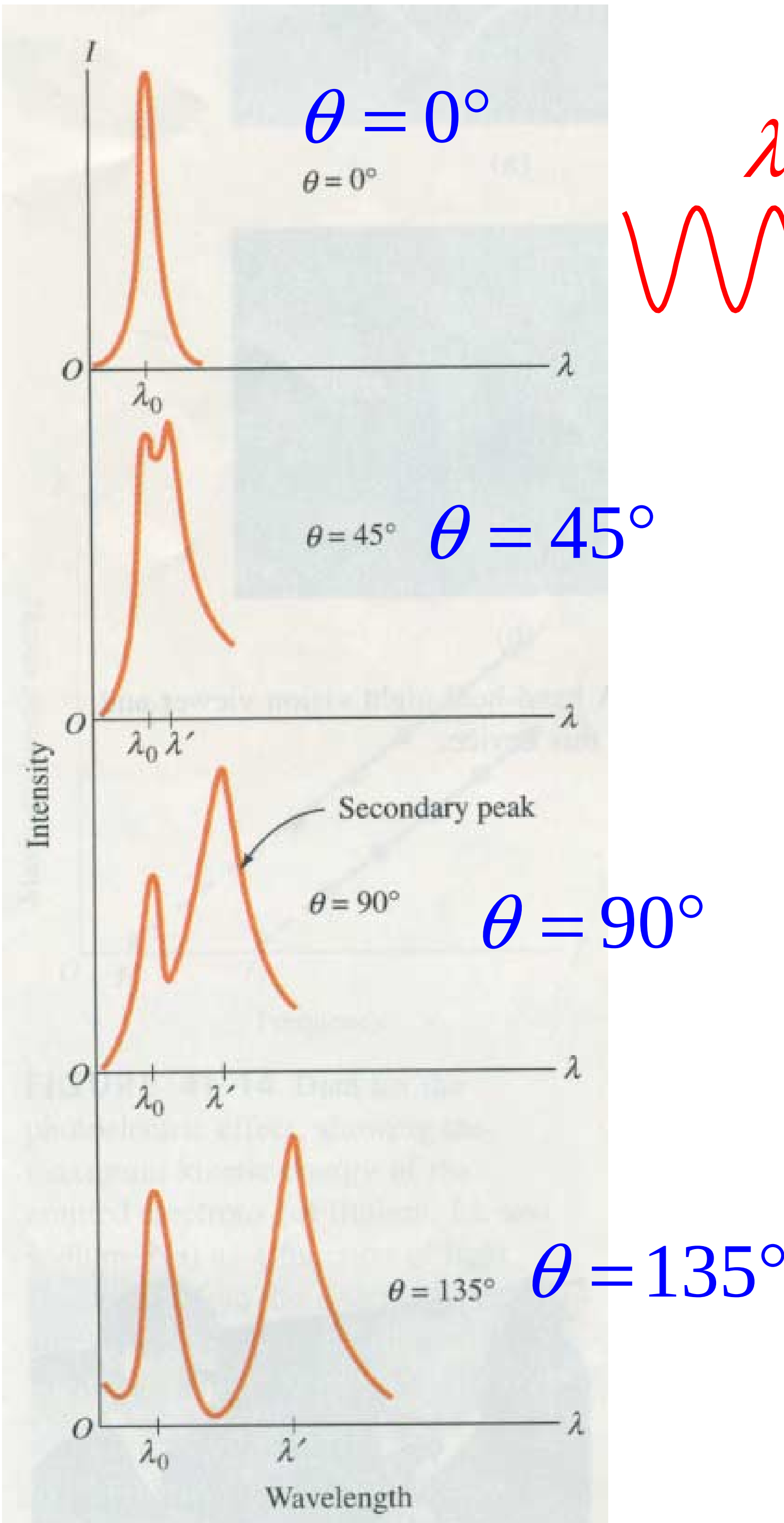
$p^2 + p'^2 + m^2c^2 - 2pp' + 2mpc - 2mp'c = p_e{}^2 + m^2c^2$ **(B)**

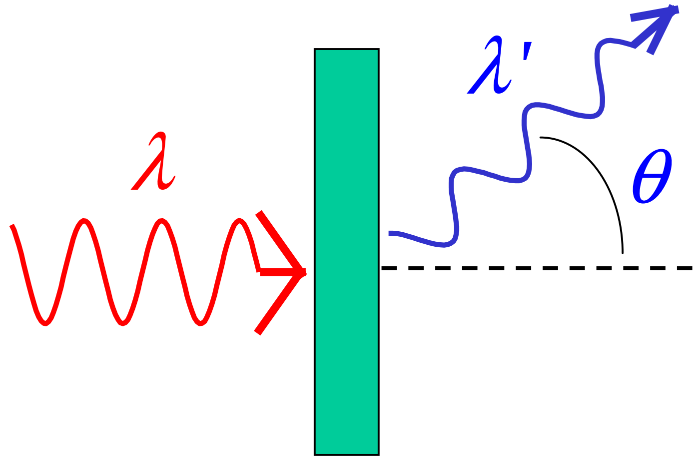
subtract Eq **(B)** from Eq. **(A)**

$2pp'(1-\cos\theta) - 2mpc = 2mp'c = 0$

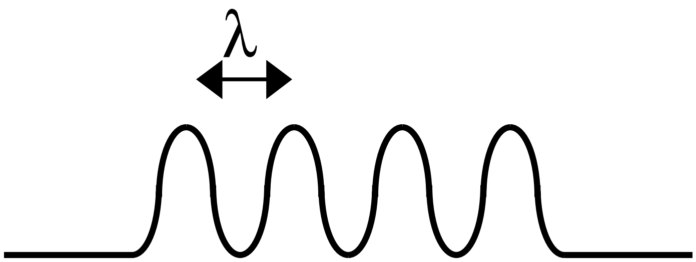
$$\frac{1}{p'} - \frac{1}{p} = \frac{1}{mc}(1\text{-}\cos\theta) \rightarrow \boxed{\lambda' - \lambda = \frac{\hbar}{mc}(1\text{-}\cos\theta)}$$

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$





Wave nature of matter



$$\lambda = \frac{h}{p}$$ deBroglie wavelength

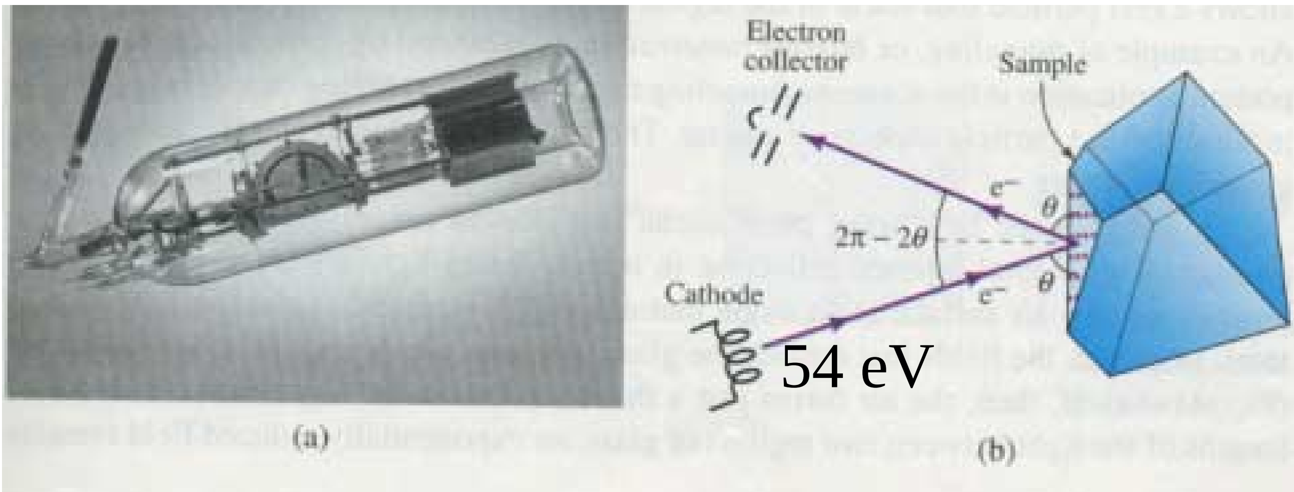
$h = 6.63 \times 10^{-34}\,\mathrm{J \cdot s}$ Planck's constant

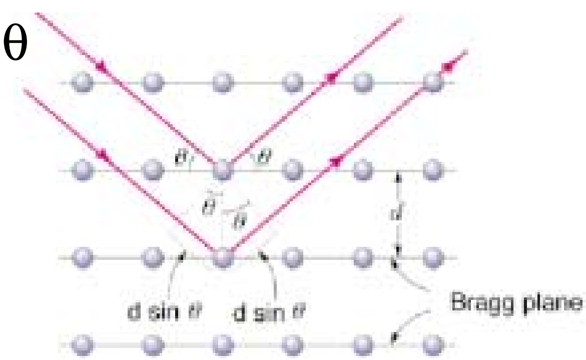
$$p = \hbar k = \hbar \frac{2\pi}{\lambda} = mv \qquad \hbar \equiv \frac{h}{2\pi}$$

Example 40-5: de Broglie wavelength of a neutron: mass=1.6x10^{-27} kg, v=1500 m/s (35 K gas)

Experimental evidence for wavelike behavior of matter

Davison-Germer electron diffraction experiment 1927





Bragg Peak: $2d\sin\theta = n\lambda \rightarrow \lambda = \frac{2d}{n}\sin\theta$

$d = 0.091$ nm

first diffraction peak (n=1) at 65°

$\rightarrow \lambda = \frac{2(0.091\text{nm})}{1}\sin 65^\circ = 0.165$ nm

What about $\lambda = \frac{h}{p}$?

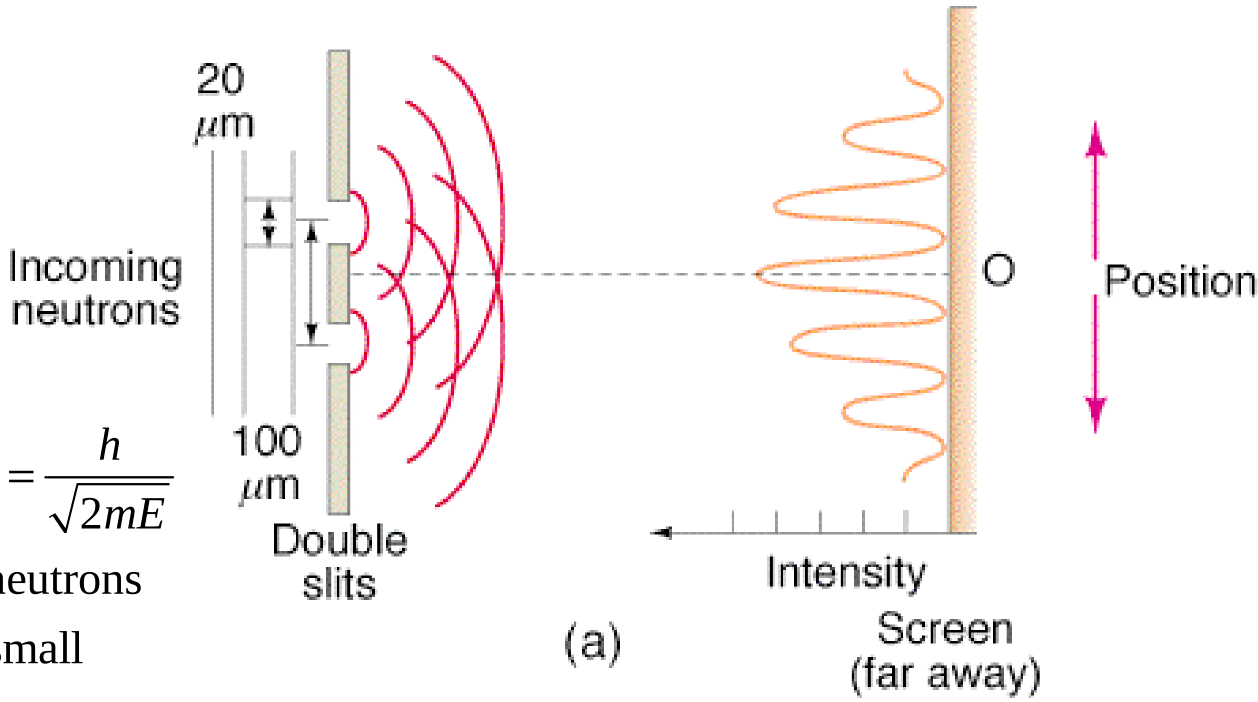
$E = 54\text{ eV} = 86.4\times10^{-19}\text{ J} = \frac{p^2}{2m}$

$p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31}\text{kg})(86.4\times10^{-19}\text{ J})} = 39.7\times10^{-25}\frac{\text{kg}\cdot\text{m}}{\text{s}}$

$\lambda = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{39.7\times10^{-25}\text{ kg}\cdot\text{m/s}} = 0.167$ nm

Example 40-6: angles for diffraction peaks for electrons with E= 120 eV incident on crystal whose scattering planes are 0.12 nm apart?

Neutron double slit experiment



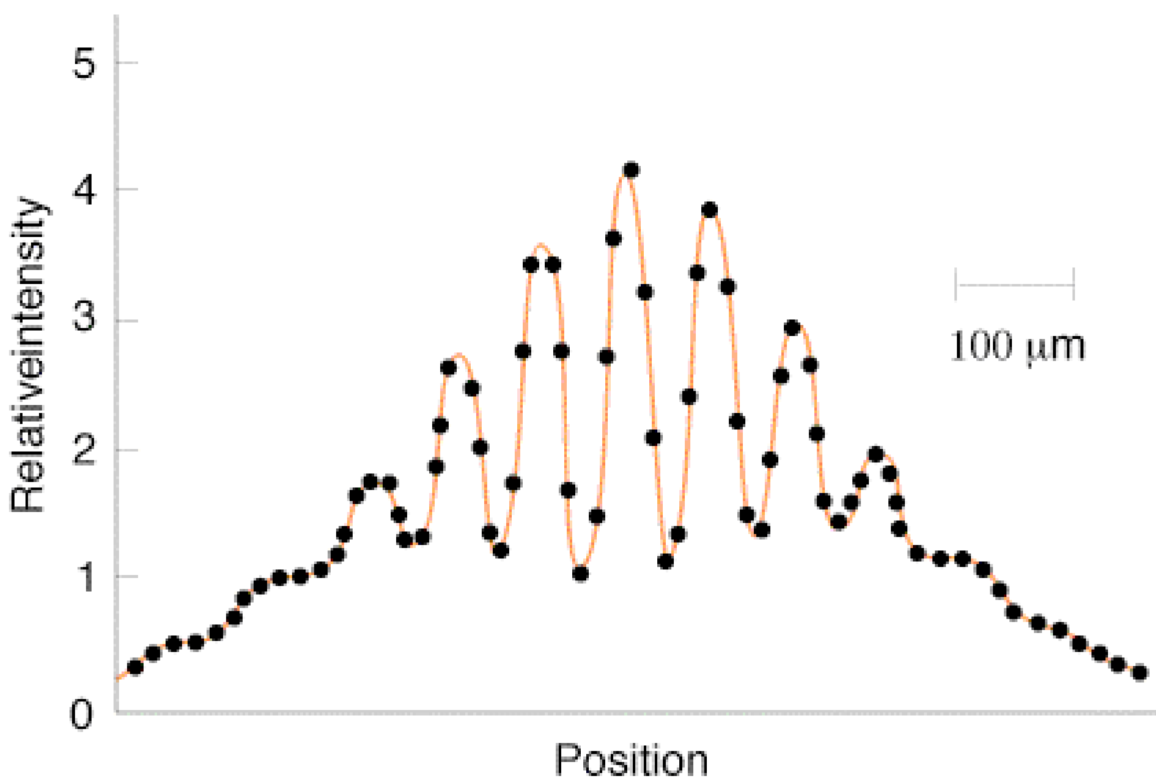
(a)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

slow neutrons

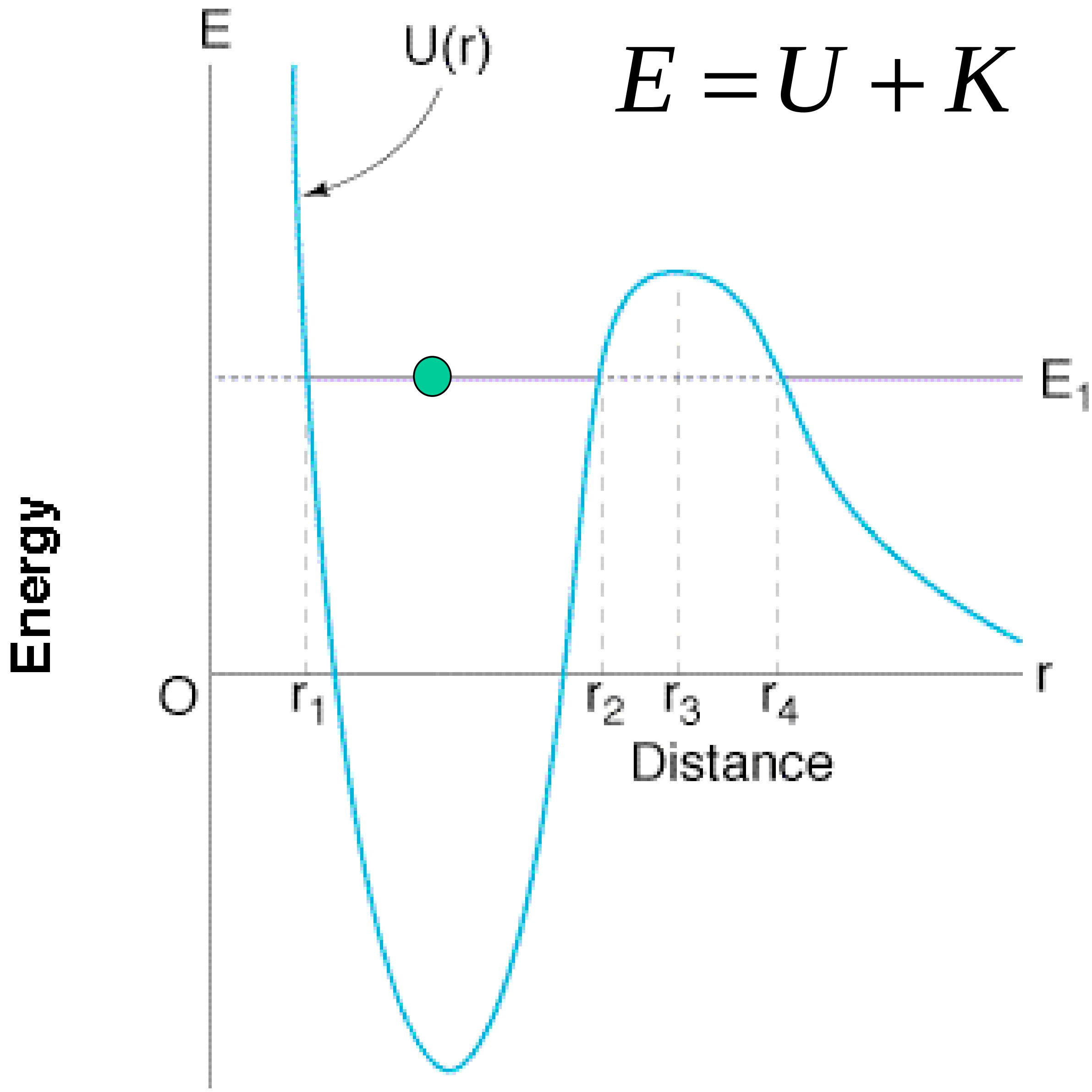
$\rightarrow p$ small

$\rightarrow \lambda$ large



(b)

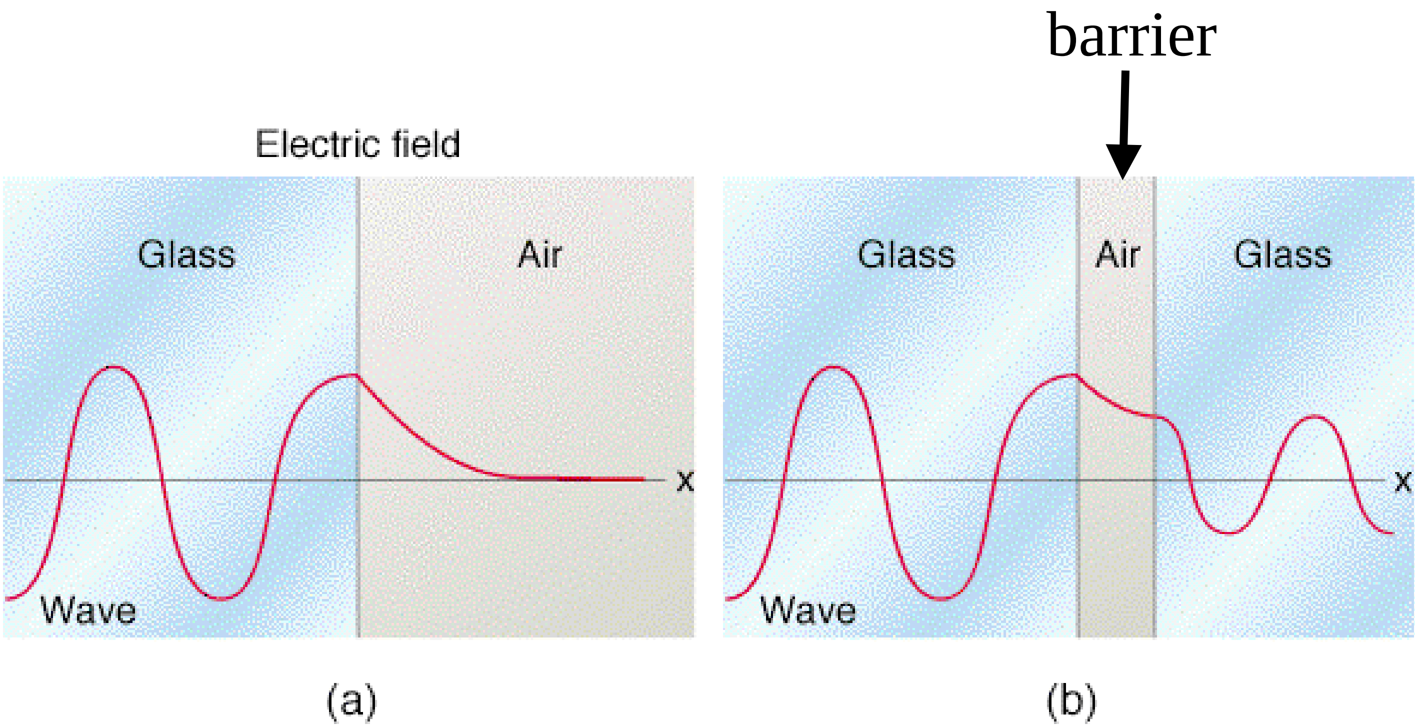
Tunneling



$$E = U + K$$

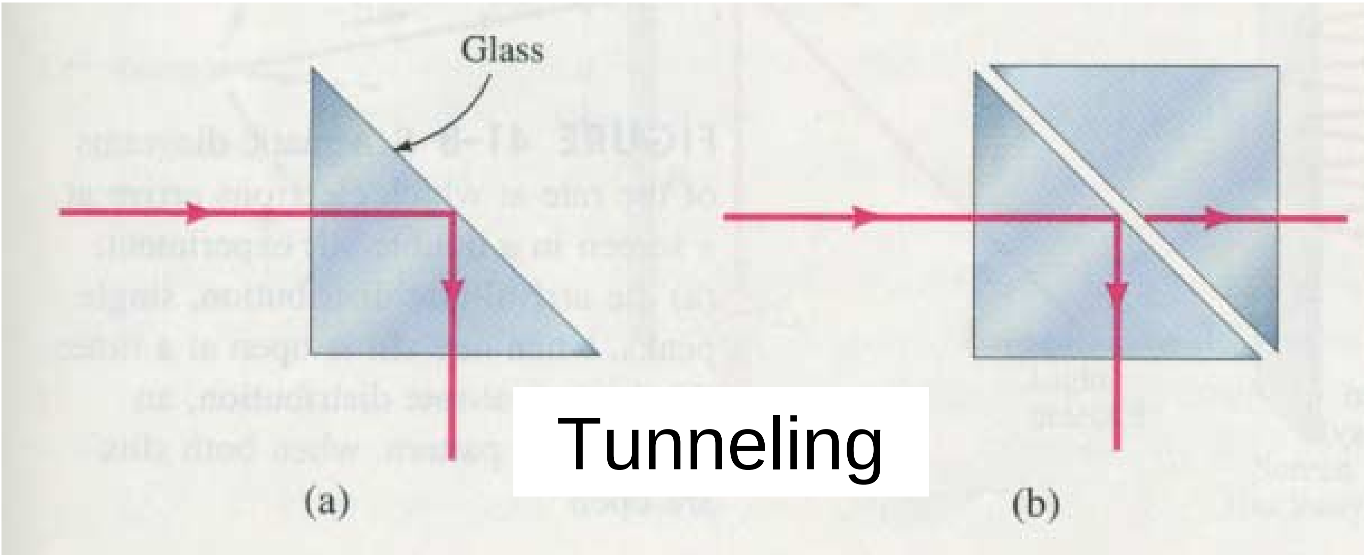
Classical particle trapped by potential

Waves can leak out of trap



Total internal reflection

Not-so-total internal reflection!



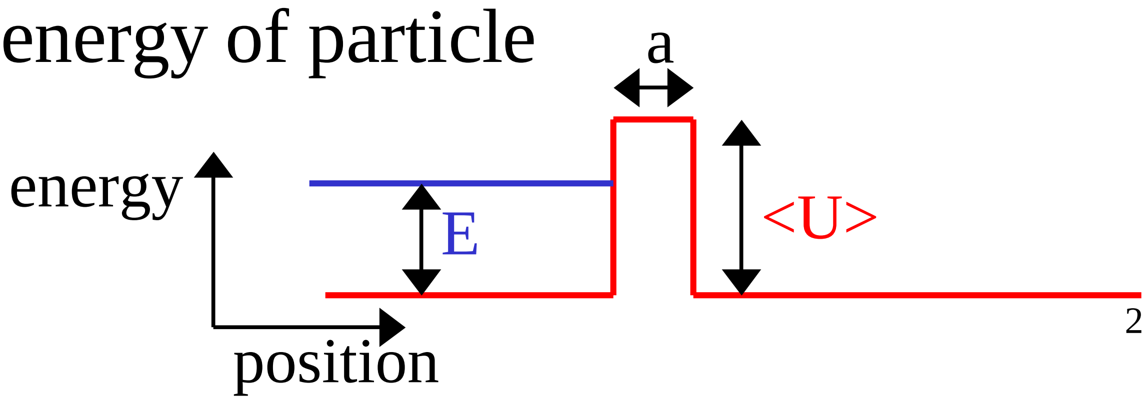
fraction (or probability) of particles getting through $\cong e^{\left[-\frac{2}{\hbar} a \sqrt{2m\left(\langle U \rangle - E\right)}\right]}$

a = barrier width

$\langle \mathrm{U} \rangle =$ average height of potential barrier

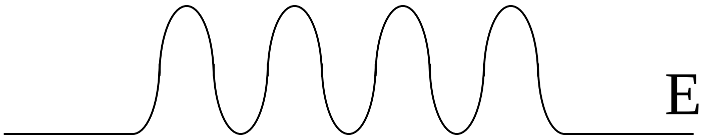
m = mass of particle

E= energy of particle



Heisenberg Uncertainty Relations

$$p = \frac{h}{\lambda}$$



$$E = \frac{p^2}{2m} \quad \text{or} \quad E_\gamma = hf$$

→ Time or position

Due to wave nature of matter $\rightarrow$

position-momentum uncertainty relation

$\Delta x \Delta p_x > \hbar$ note $x > \Delta x$ and $p_x > \Delta p_x$

time-energy uncertainty relation (see HW 40-53)

$\Delta E \Delta t > \hbar$ note $E > \Delta E$ and $t > \Delta t$

only important on atomic scales

For a dust particle, $\Delta x = 10^{-6}$ m $\rightarrow 10^{-28}$ kg $\cdot$ m/s

For an electron, $\Delta x = 10^{-11}$ m (atom) $\rightarrow 10^{-23}$ kg $\cdot$ m/s

which is 10 times larger than the electron's

momentum in its classical atomic orbit

(also v=0.1c)!

Double-slit dilemma

check "Interference experiments" on Physics Trek 2000 web site (Classical and quantum mechanical two-slit experiments demo)

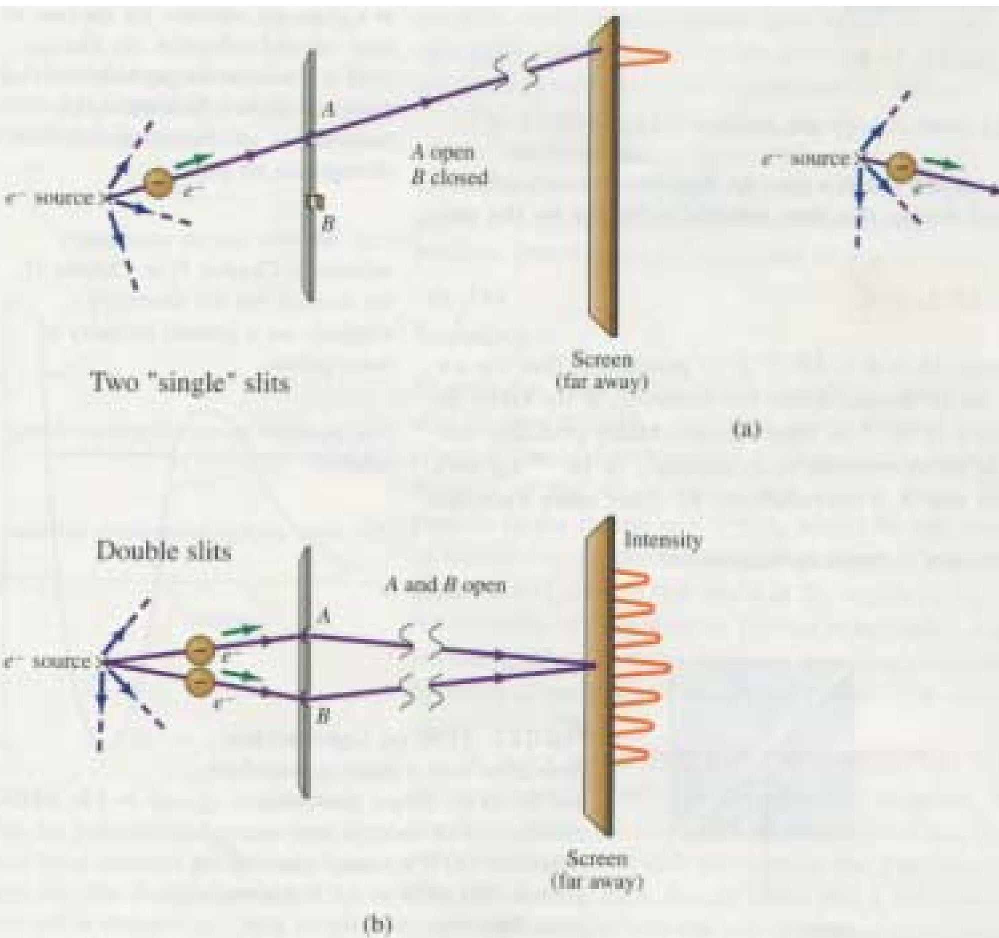
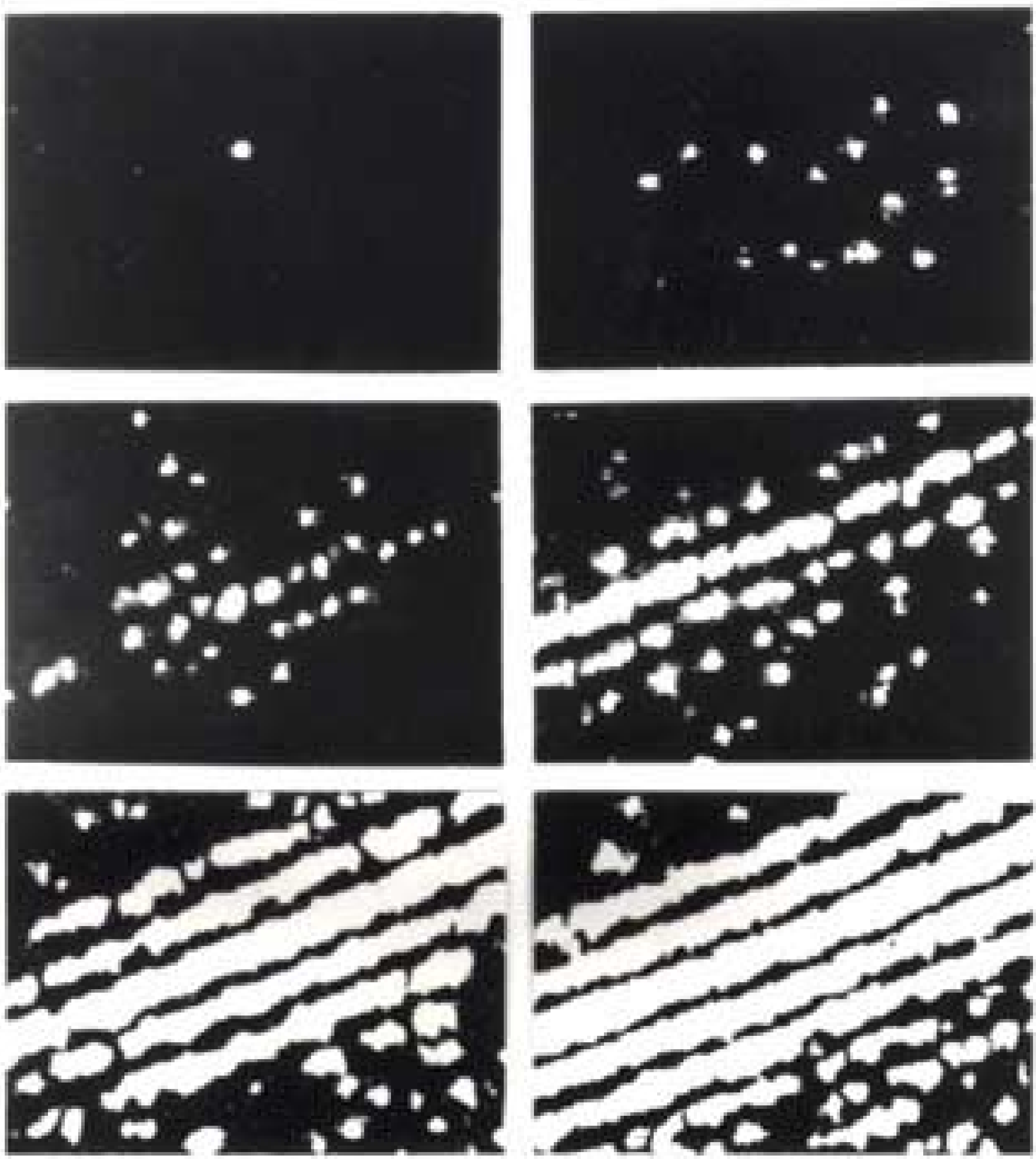


FIGURE 41–8 Schematic diagrams of the rate at which electrons arrive at a screen in a double-slit experiment: (a) the arrival-rate distribution, single peaks, when one slit is open at a time; (b) the arrival-rate distribution, an interference pattern, when both slits are open.

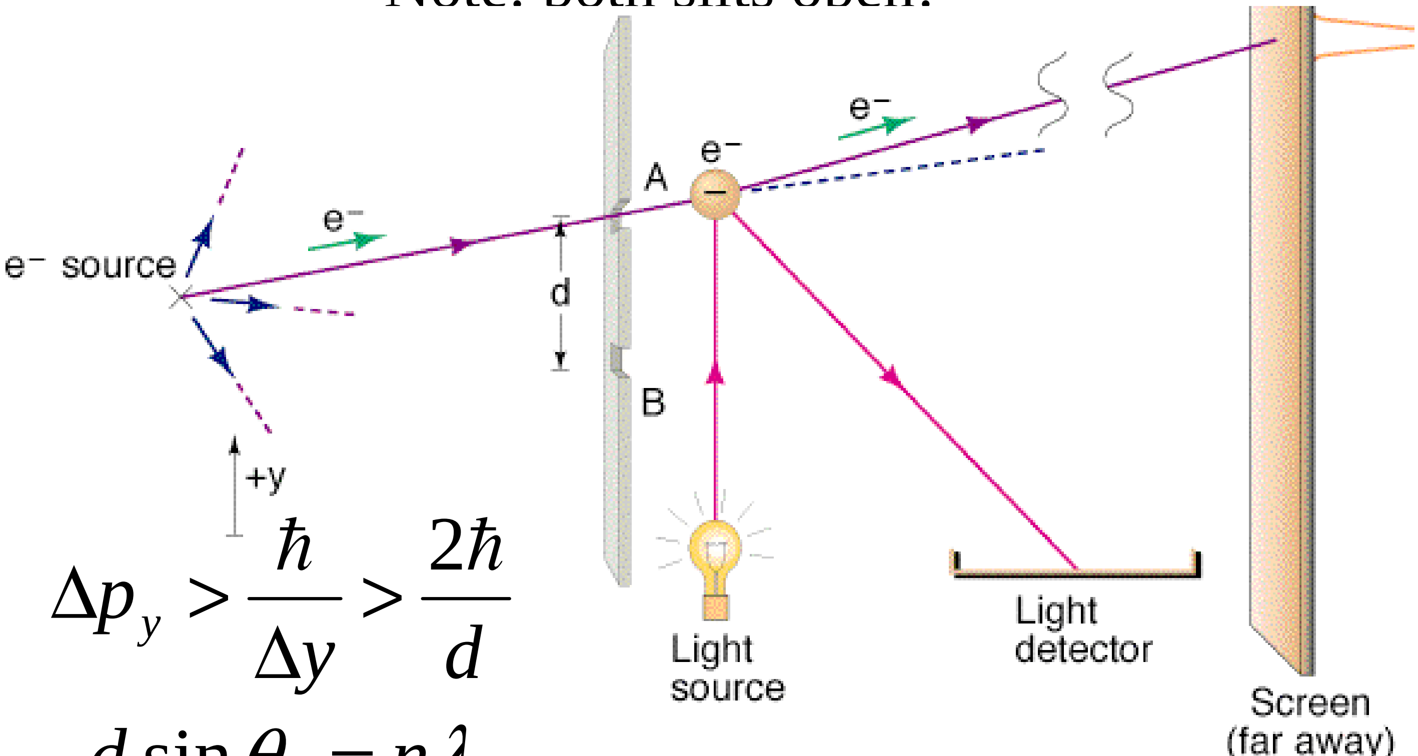
http://www.colorado.edu/physics/2000/schroedinger/two-slit2.html



Electron double-slit experiment

Resolution of double-slit dilemma

Note: both slits open!



$$\Delta p_y > \frac{\hbar}{\Delta y} > \frac{2\hbar}{d}$$

$$d \sin\theta_n = n\lambda$$

Distance between two adjacent maxima on screen

$$\Delta y_{\max} = D\sin\theta_{n+1} - D\sin\theta_n$$

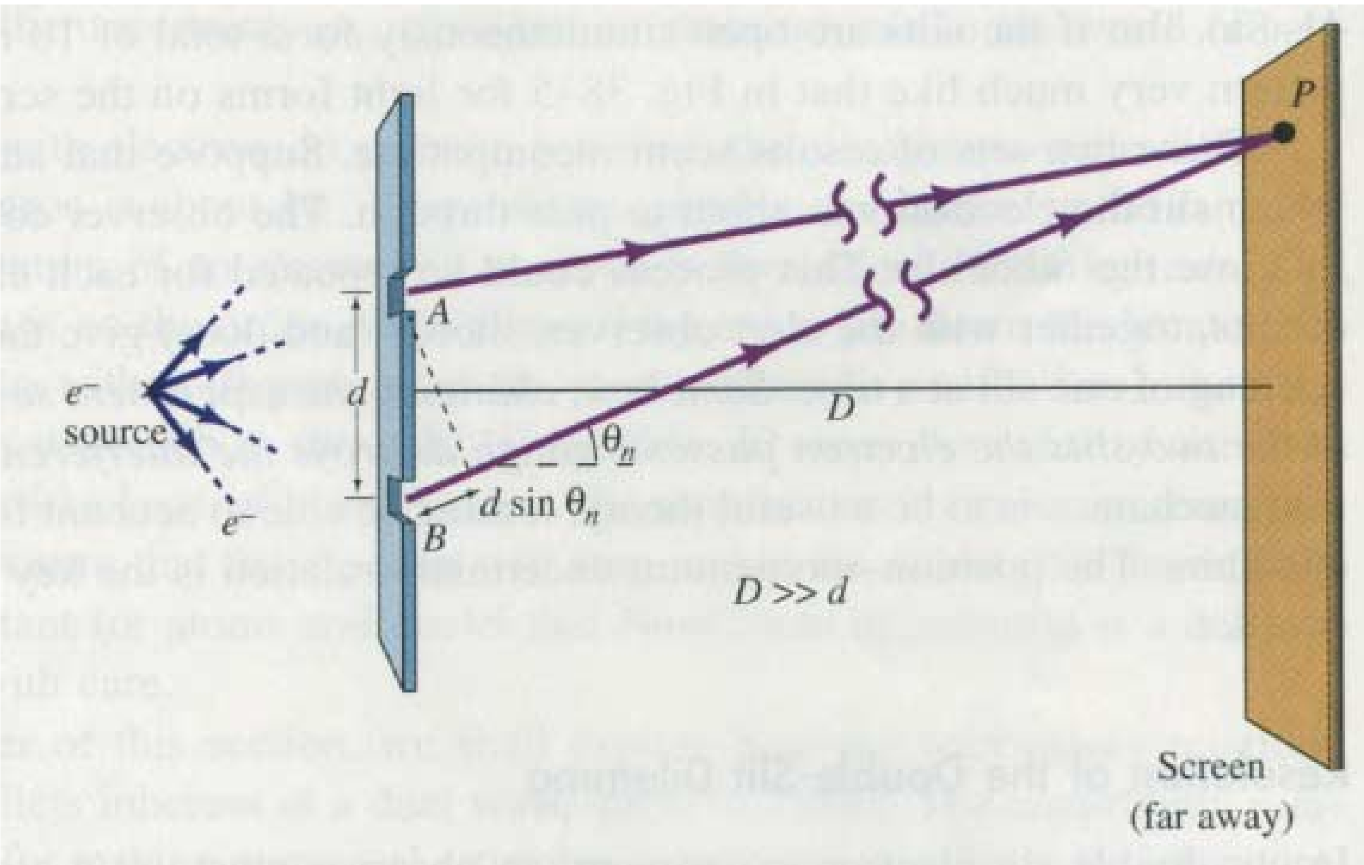
$$= \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d} = \frac{D\lambda}{d}$$

Location of peak on screen changes by $\dfrac{D\Delta p_y}{p}$

$$\frac{D\Delta p_y}{p} > \frac{D\left(\frac{2\hbar}{d}\right)}{p} = \frac{2D/d}{k} = \frac{D\lambda}{\pi d}$$

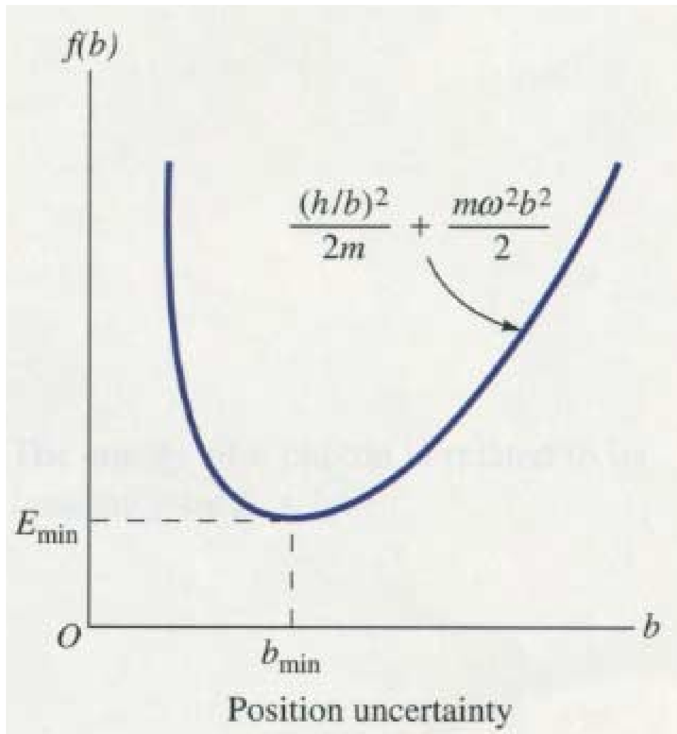
➡ **Displacement due to measurement is comparable to separation between maxima**

Double slit analysis exactly the same as for waves, but λ is very short



Ground state: Lowest energy state for a parabolic potential (simple harmonic oscillator/spring)

What is lowest energy for a classical SHO
E=? x=? p=?



$$E = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

$$\Delta x = b \rightarrow \Delta p > \frac{\hbar}{b}$$

$$\rightarrow E > \frac{(\hbar/b)^2}{2m} + \frac{m\omega^2 b^2}{2} = f(b)$$

$$\frac{df}{db} = -\frac{\hbar}{mb^3} + m\omega^2 b = 0 \rightarrow b^2_{min} = \frac{\hbar}{m\omega}$$

$$E_{min} = \frac{\hbar^2}{2m(\hbar/m\omega)} + \frac{1}{2} m\omega^2 \left(\frac{\hbar}{m\omega}\right) = \hbar\omega \neq 0$$

$\rightarrow$ zero point motion or zero point energy

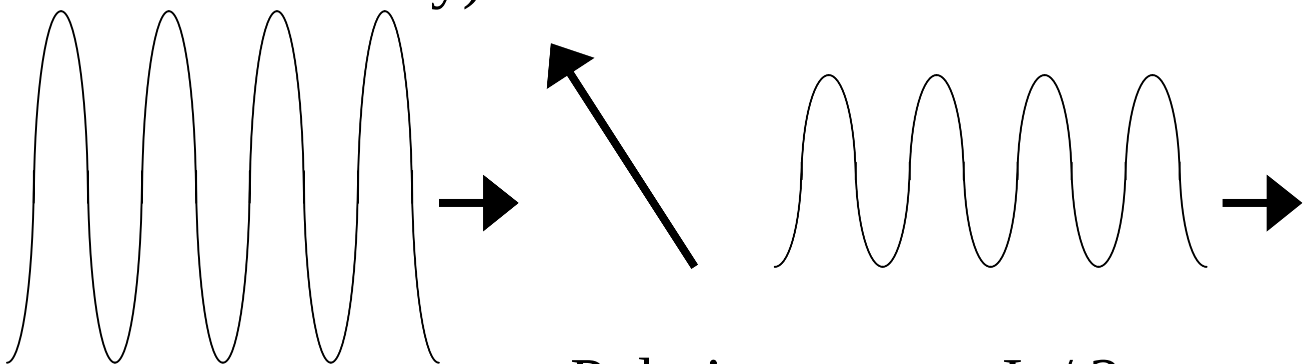
Example 40-8: Use position-momentum uncertainty relation to estimate the lowest energy of a particle with mass *m* in a 1-D box of width L.

Example: use position-momentum uncertainty relation to estimate the lowest energy of an electron (mass *m)* moving in an attractive Coulomb potential
$U(r)= -e^2/4\pi\varepsilon_o r$

Classical vs.	Quantum
Position: (x,y,z) well defined function of time	P(x,y,z) probability wave that interferes with itself
Momentum: (p_x, p_y, p_z)	$\Delta x \Delta p_x > \hbar$ $\Delta y \Delta p_y > \hbar$ $\Delta z \Delta p_z > \hbar$
Energy: E	$\Delta E \Delta t > \hbar$

Polarizer experiment

Classical EM wave
(polarized vertically)

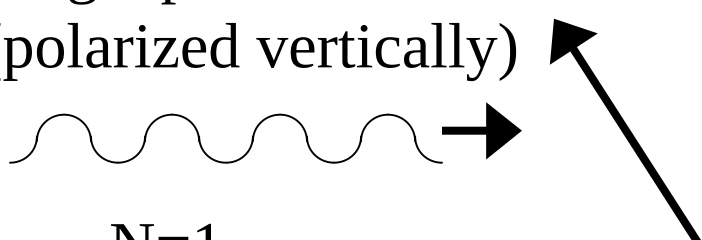


Polarizer
at 45^o to
vertical

I_0 / 2

I_0
N huge

Single photon
(polarized vertically)

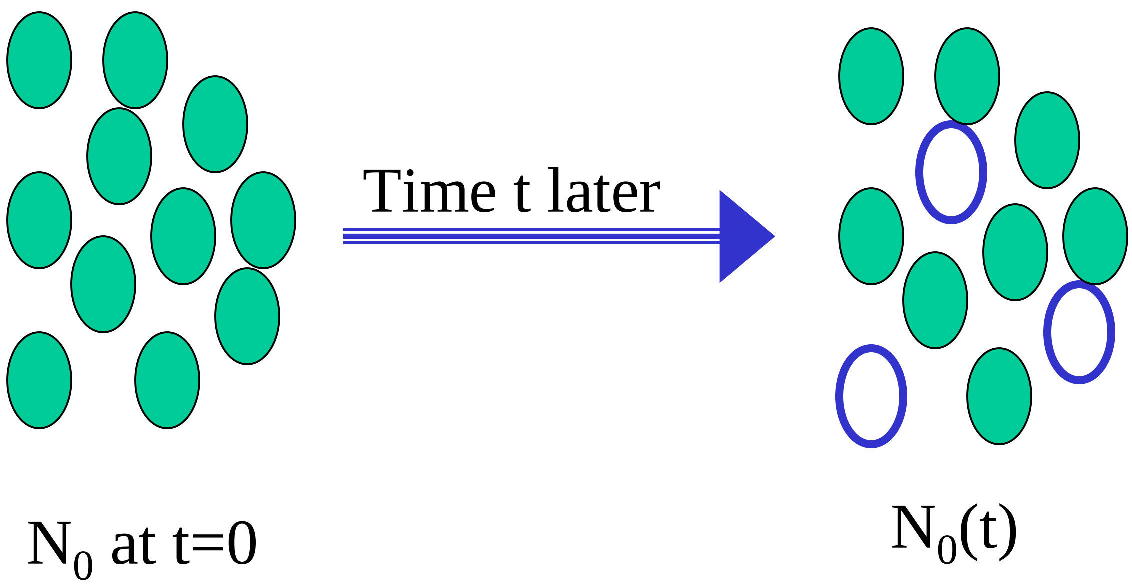


N=1

?
Probability=1/2

Polarizer
at 45^o to
vertical

Radioactive decay

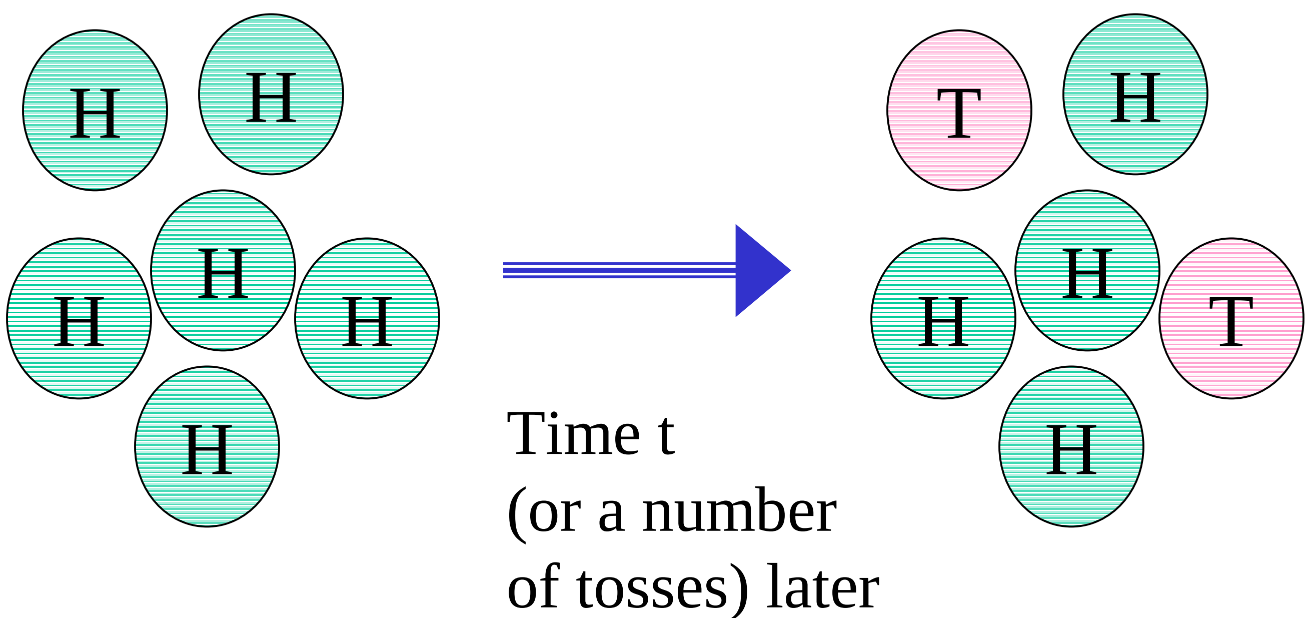


$$N(t) = N_0 e^{-t/\tau}$$

Note that $N(\tau) = N_0 e^{-1} = 0.37 N_0$

are the older A atoms more likely to decay into B atoms?

Coin tossing analogy demo: Start with all heads, for each round of tossing, flip each coin that is still heads up, ***do not flip coins that are already tails*** (P=1/2 throughout). N(t) is number of remaining heads.



N_0 at t=0
all heads

N(t)

Problem 40-63. The half life $T_{1/2}$ is the time in which half of the original set of nuclei have decayed (the other half are still in the original form). Express half life in terms of the lifetime τ .