Chapter 37: Interference

Chs. 35 and 36: geometric optics => rays (lenses, mirrors, …)

Chs. 37 and 38: ***Physical Optics***, where the wave nature of light is used to explain an even wider range of phenomena.

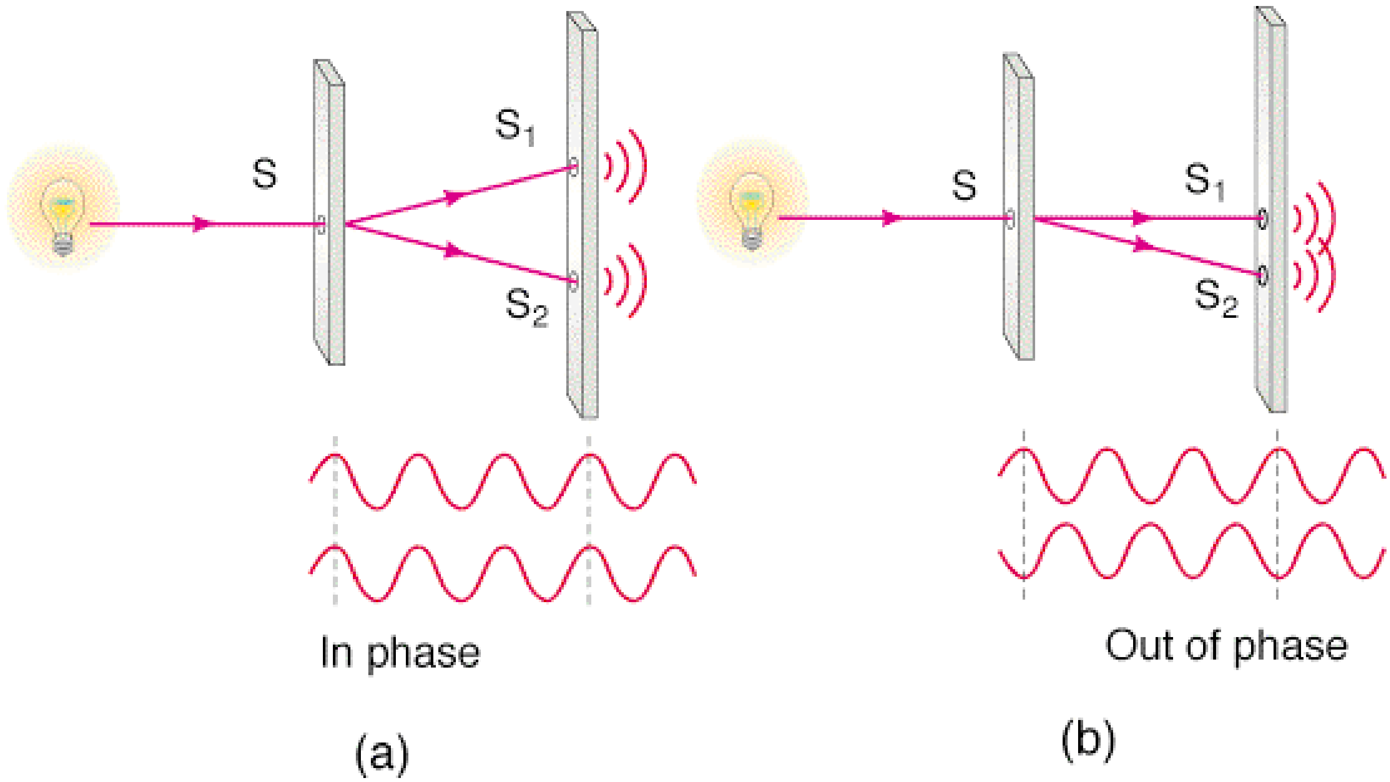
Cohererent / incoherent light

Two sources to produce an interference that is stable over time, if their light has a *phase relationship* that does not change with time

Coherent sources: $A(z,t) = A_0 \cos(kz - \omega t)$ and $B(z,t) = B_0 \cos(kz - \omega t)$. Phase θ must be well defined and constant. When waves from coherent sources meet, stable interference can occur.

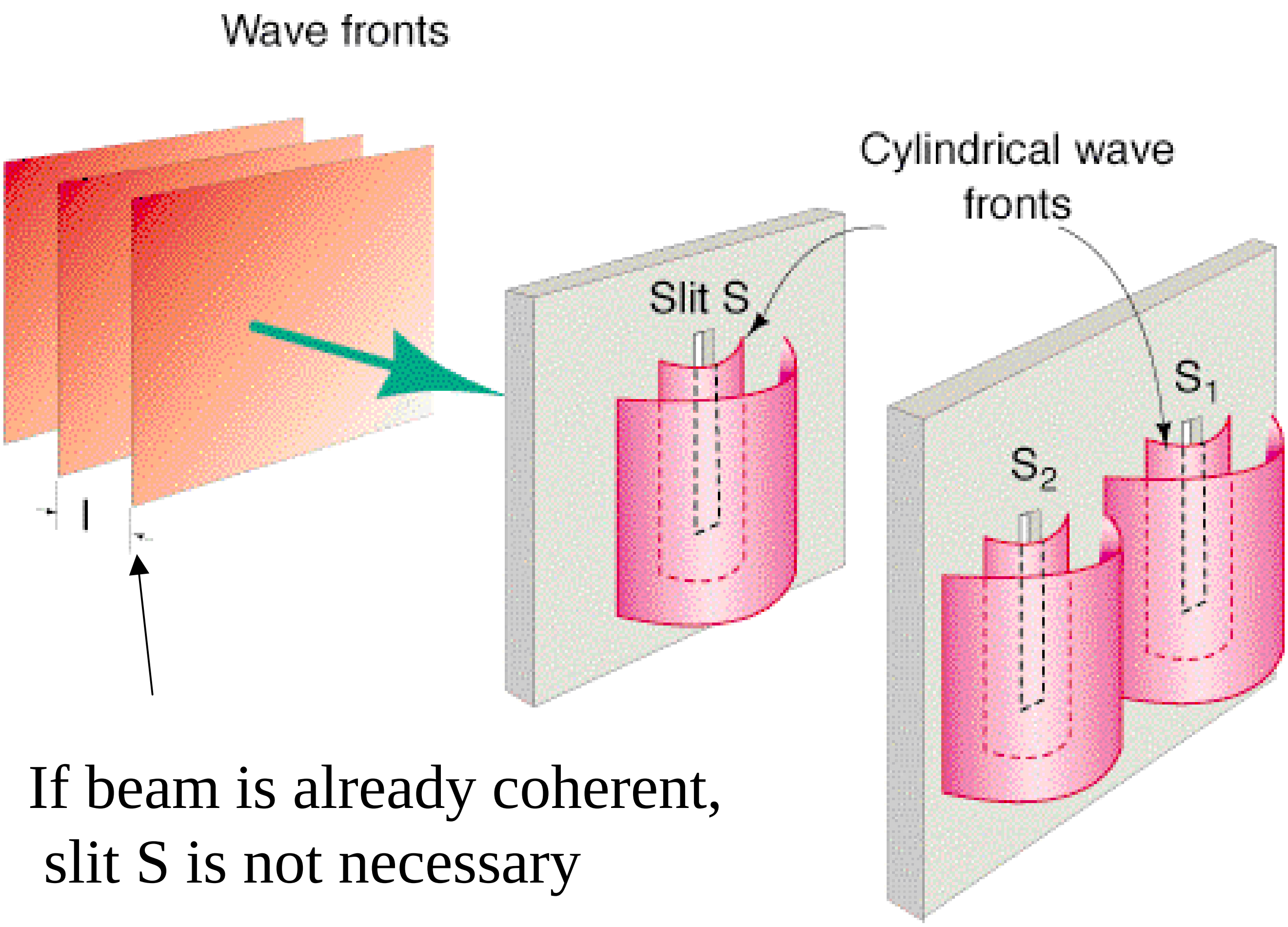
Incoherent sources: θ jitters randomly in time, no stable interference occurs

Creating two coherent sources from an incoherent light bulb



(see 3D view of double slit on next slide)

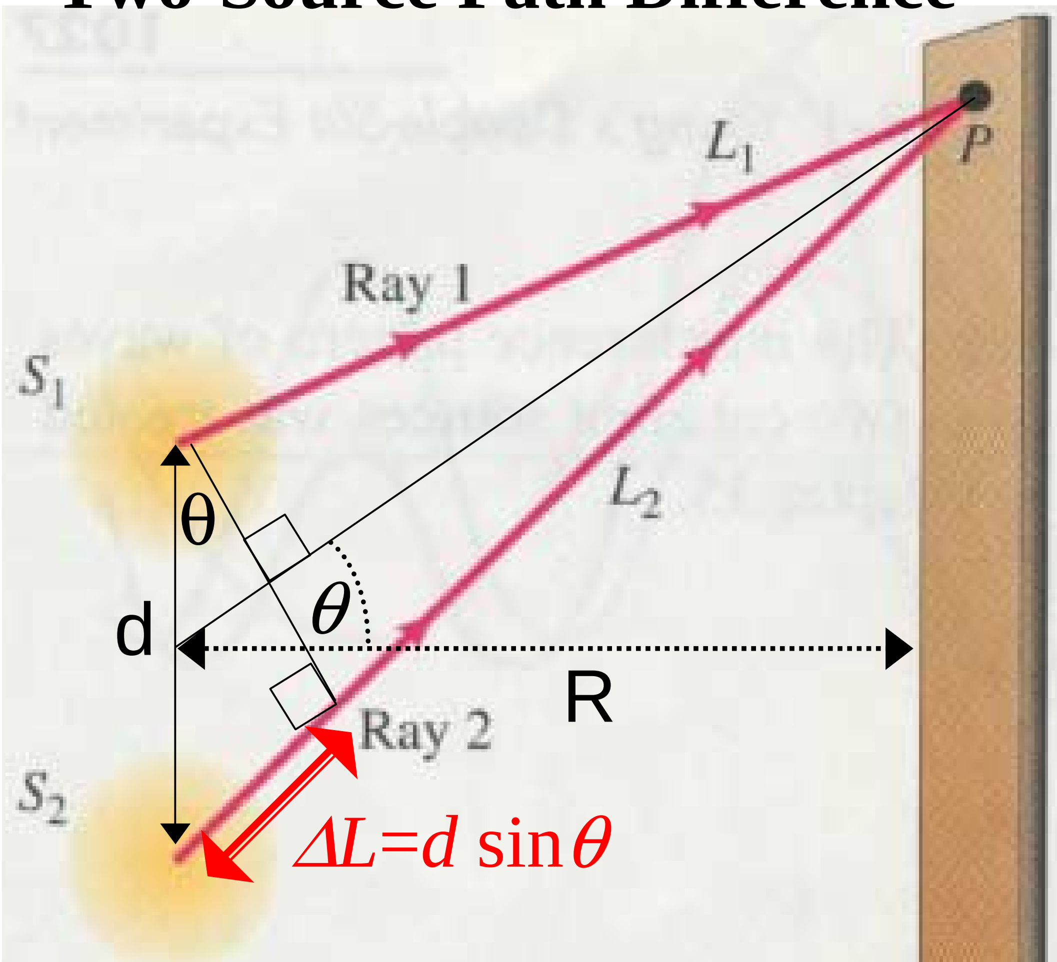
Double slit interference



If beam is already coherent,
slit S is not necessary

(a)

Two-Source Path Difference



$\Delta L = d\sin\theta$ (when is this true?)

If both sources are in phase:

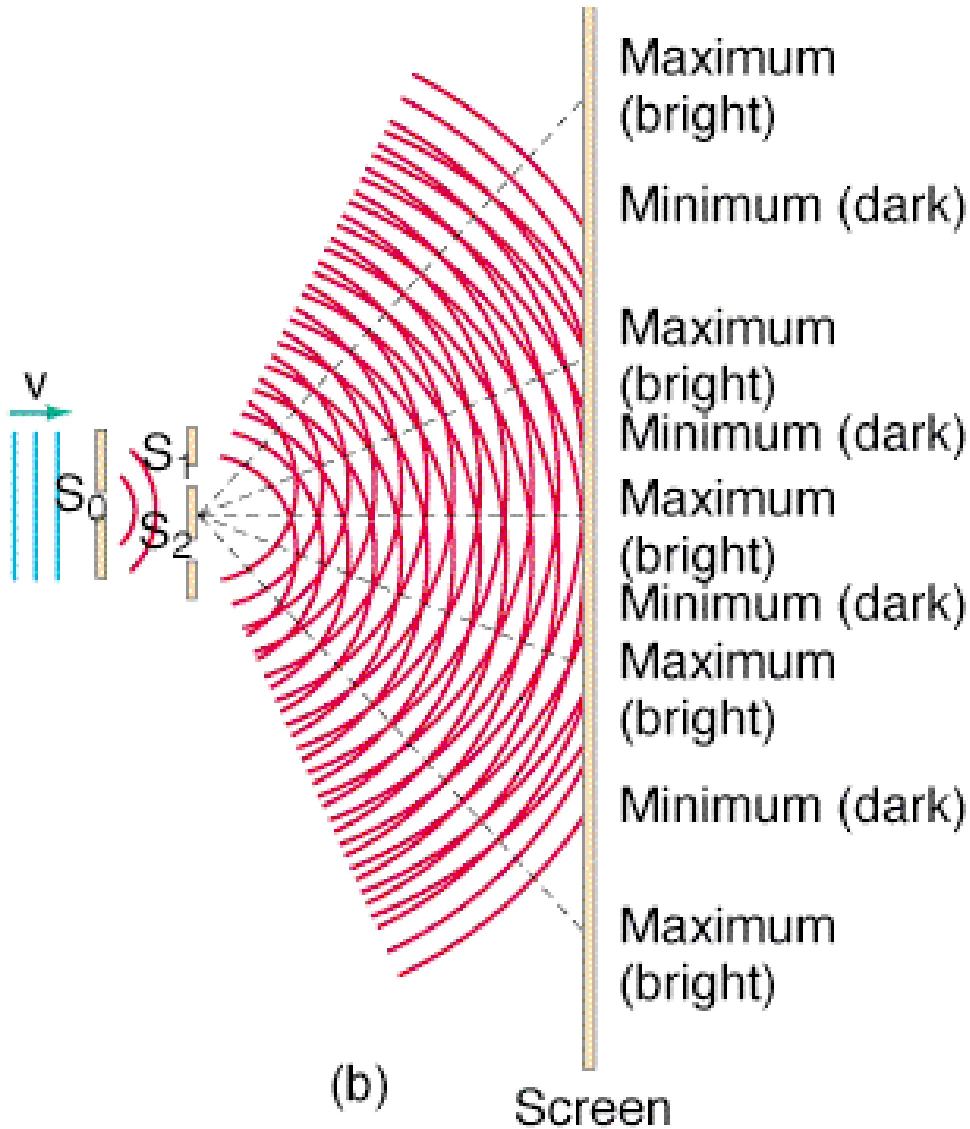
$\Delta L = n\lambda = d\sin\theta$ ⇨ $\sin\theta = \dfrac{n\lambda}{d}$ $\quad n = 0, \pm1, \pm2, \ldots$

constructive interference

$\Delta L = \left(n + \dfrac{1}{2}\right)\lambda$ ⇨ $\sin\theta = \left(n + \dfrac{1}{2}\right)\dfrac{\lambda}{d}$ $\quad n = 0, \pm1, \pm2, \ldots$

destructive interference

same as in Ch. 14 !



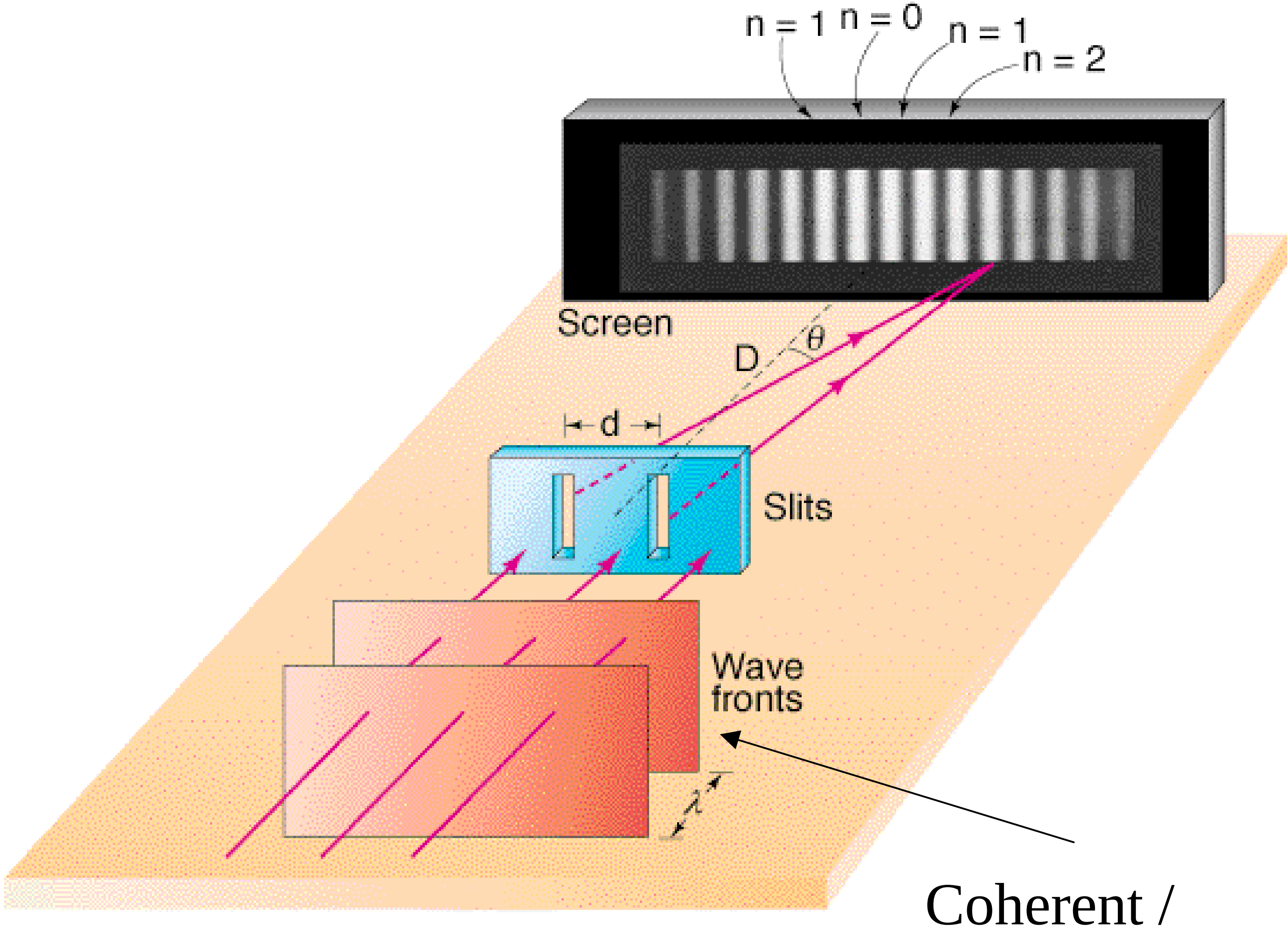
for constructive interference

$$\Delta L = n\lambda \qquad n=0, \pm 1, \pm 2, \ldots$$

for destructive interference

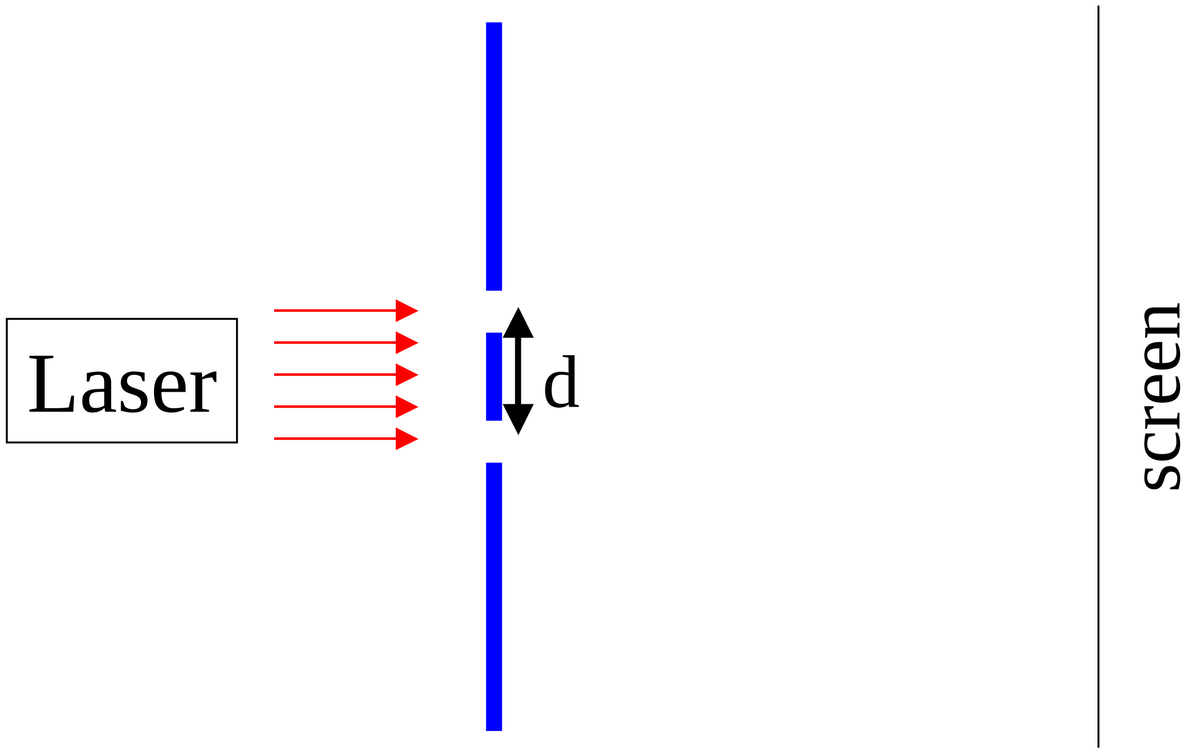
$$\Delta L = \left(n + \frac{1}{2}\right)\lambda \qquad n=0, \pm 1, \pm 2, \ldots$$

Two-Source Interference Pattern: cannot be explained using geometrical optics!



Coherent /
Incoherent?

Demo 37-1: Young's 2-slit experiment using a laser



What happens to spacing $\Delta\theta$ of maxima when d changes?

d increases $\rightarrow$

d decreases $\rightarrow$

relationship between $\Delta\theta$ and d?

Intensity in the double slit experiment

Adding two ***incoherent*** waves: Intensities add

$$I_{\text{total}} = I_1 + I_2 \propto \left\langle E_1{}^2 \right\rangle + \left\langle E_2{}^2 \right\rangle$$

Adding two ***coherent*** waves: ***E*** fields add

$$I_{\text{total}} = \left\langle E_{\text{total}}^2 \right\rangle \propto \left\langle \left(E_1 + E_2\right)^2 \right\rangle$$

$$= \left\langle E_1^2 \right\rangle + \left\langle E_2^2 \right\rangle + 2\left\langle \mathbf{E}_1 \cdot \mathbf{E}_2 \right\rangle$$

$2\left\langle \mathbf{E}_1 \cdot \mathbf{E}_2 \right\rangle = 0$ for ***incoherent*** waves

For ***coherent*** waves (and $I_1 = I_2 = I_0$)

$I_{\text{total}} = 0$ for max. destructive interference $\left(\mathbf{E}_1 = -\mathbf{E}_2\right)$

$I_{\text{total}} = 4I_0$ for max. constructive interference $\left(\mathbf{E}_1 = +\mathbf{E}_2\right)$

Intensity pattern for two coherent sources

$$E_1 = E_0 \sin(\omega t)$$ Wave 1

$$E_2 = E_0 \sin(\omega t + \phi)$$ Wave 2

note that $I_1 = I_2 = I_0 \propto \left\langle E_1^2 \right\rangle = E_0^2 \left\langle \sin^2(\omega t) \right\rangle = \frac{E_0^2}{2}$

The ratio between the phase difference and 2π
Is equal to the ratio between L and λ :

$$\frac{\phi}{2\pi} = \frac{\Delta L}{\lambda} \quad \Rightarrow \quad \phi = 2\pi \frac{\Delta L}{\lambda} = \frac{2\pi}{\lambda} d \sin\theta$$

$$E_{\text{net}} = E_1 + E_2 = E_0 \left(\sin(\omega t) + \sin(\omega t + \phi) \right)$$

Use identity:

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$$\Rightarrow \quad E_{\text{net}} = 2E_0 \cos\left(\frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$$

$$S_{net} \propto E_{net}^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)\sin^2\left(\omega t + \frac{\theta}{2}\right)$$

$$I_{net} = \langle S_{net} \rangle \propto \langle E_{net}^2 \rangle$$

$$= 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right)\right\rangle$$

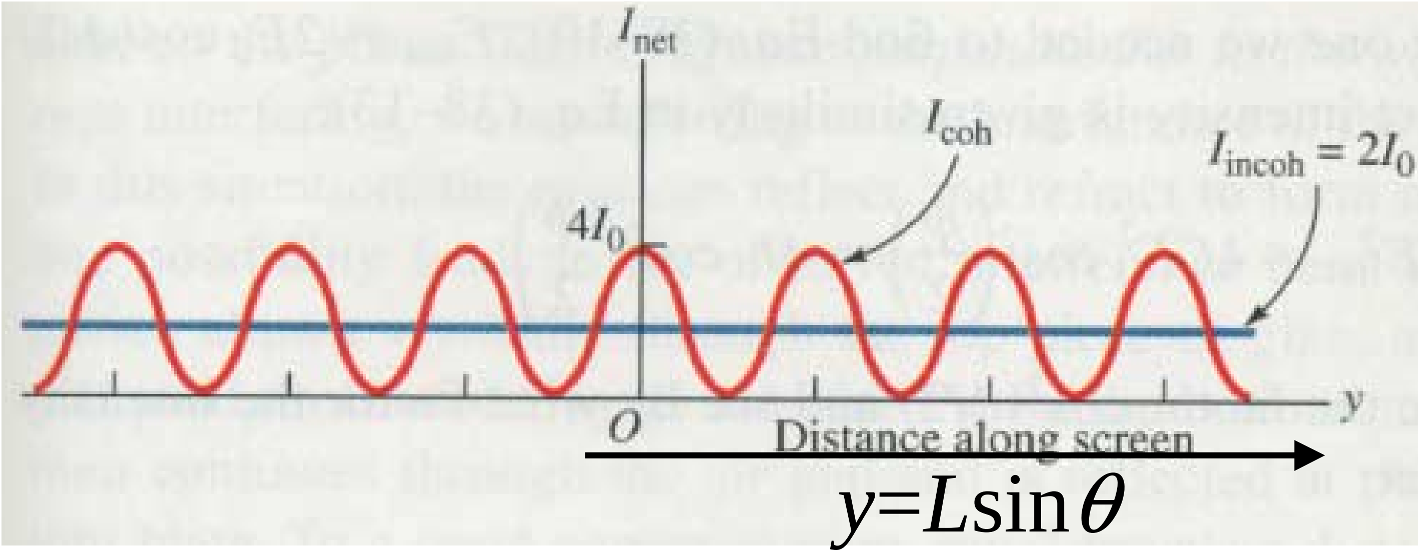
$$= 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)\cdot\left(\frac{1}{2}\right) = 2E_0^2 \cos^2\left(\frac{\phi}{2}\right)$$

$$I_{net} = 4I_0 \cos^2\left(\frac{\phi}{2}\right) = 4I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

where we have used the fact that $I_0 \propto \frac{E_0^2}{2}$

$$I_{net} = 4I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

$$\frac{I_{\theta=0^\circ}}{I_0} = 4$$



$y = L\sin\theta$

$$I_{net} = 4I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

For small θ: $\sin\theta \to \theta$

$$\Rightarrow \quad I_{net} \approx 4I_0 \cos^2\left(\frac{\pi d}{\lambda}\theta\right) \propto \cos^2\theta$$

Relationship between peak spacing $\Delta\theta$ and d and λ?

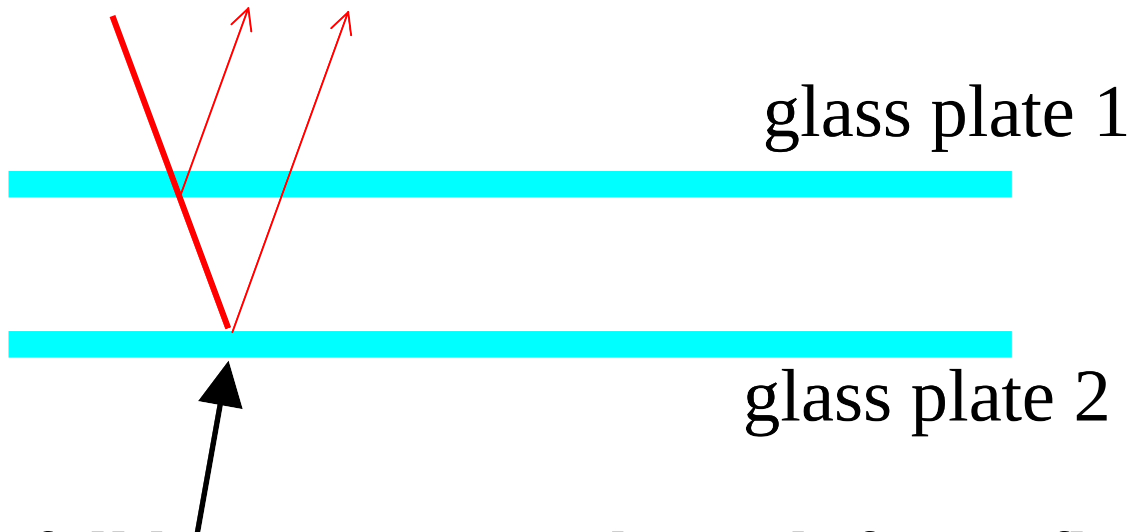
Intensity peaks when $\frac{\pi d}{\lambda}\theta = n \cdot 2\pi$

$$\frac{\pi d}{\lambda}\Delta\theta = 2\pi \to \Delta\theta = \frac{2\lambda}{d}$$

<u>Interference from reflection</u>

So far: interference between waves *from two different* sources (slits / antennae)

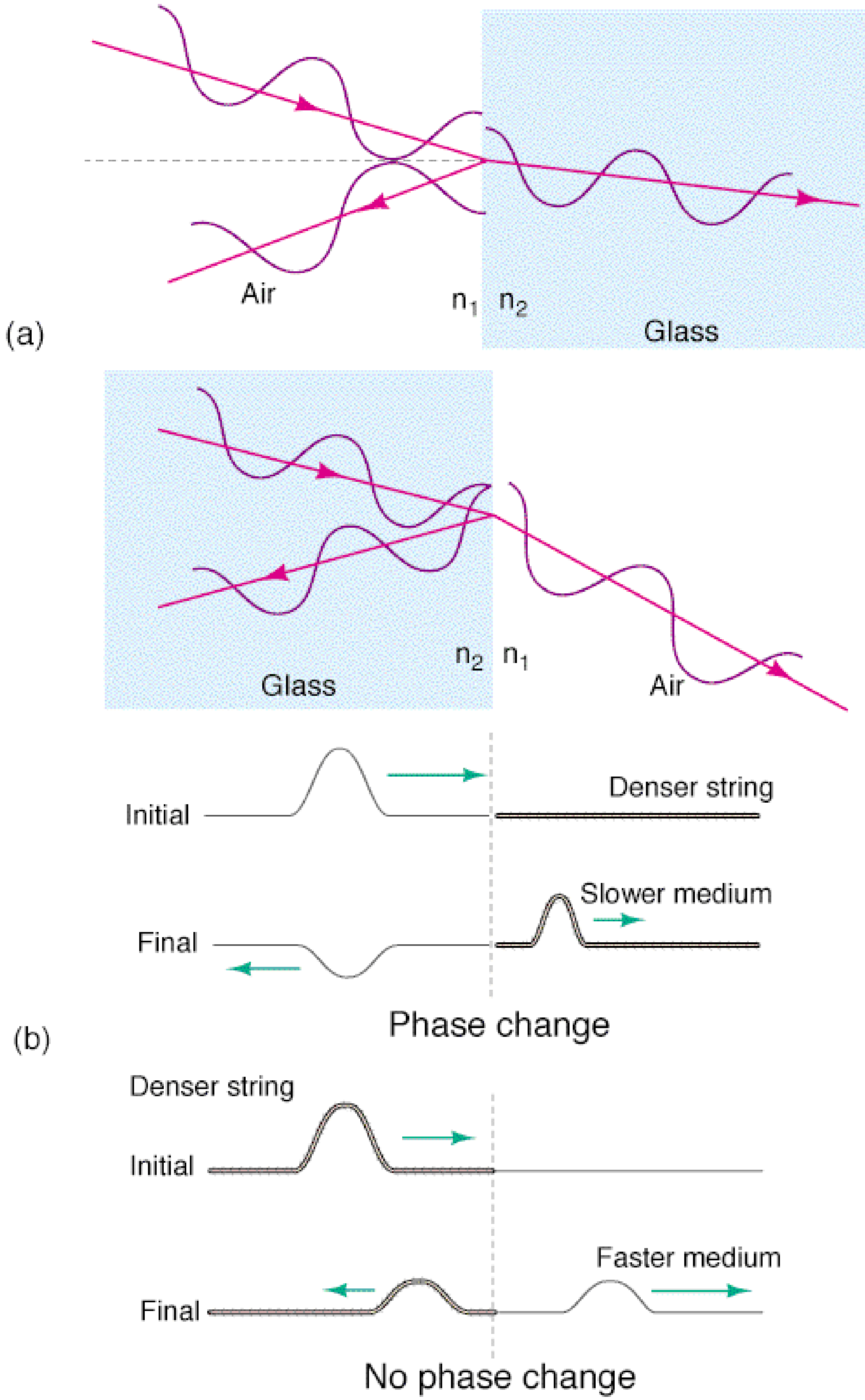
Now: *create two waves from one* by partial reflection and transmission and let those two waves interfere



carefull here: may get phase shift at reflections

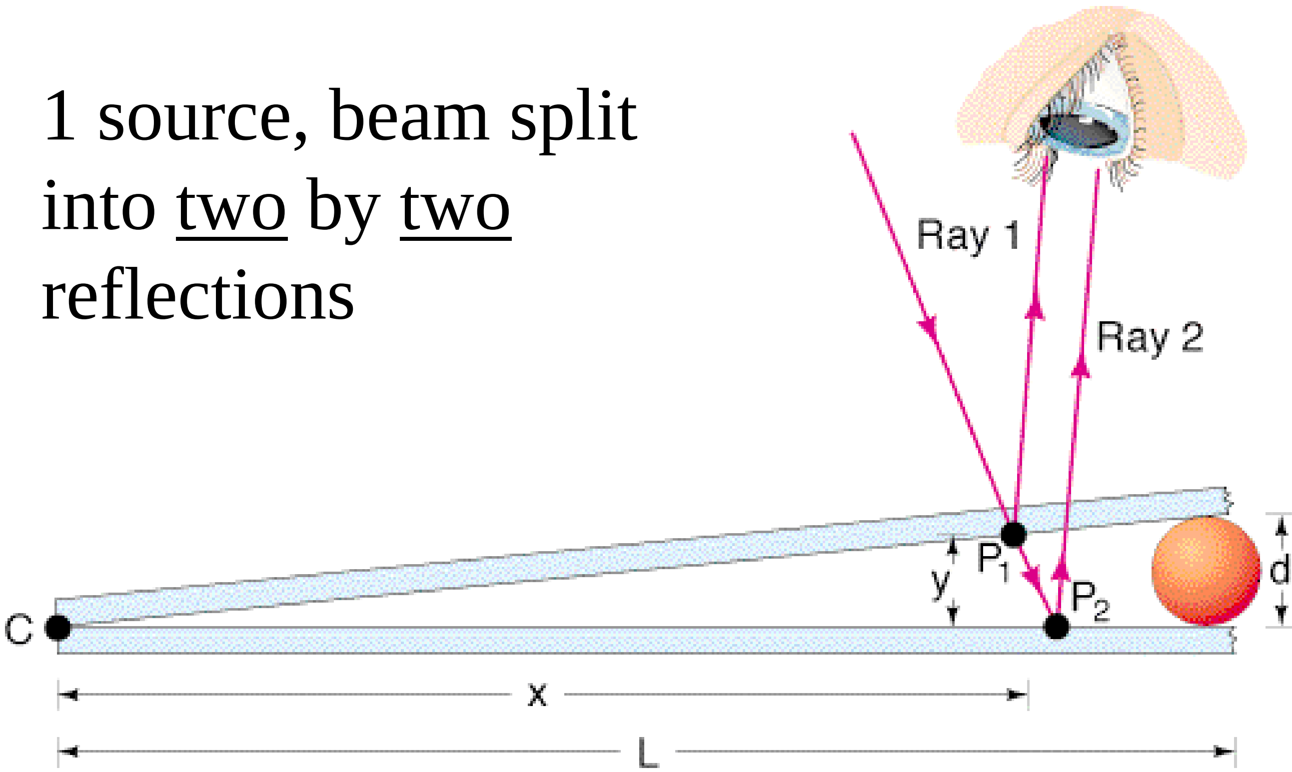
Interference from reflection explains the colors on the wings of butterflies, the colors of oil films and of soap bubbles, and many other phaenomena

Phase change from reflection



Interference from reflection

1 source, beam split into <u>two</u> by <u>two</u> reflections



Path difference of beam: $\Delta L = 2P_1P_2 = 2y$

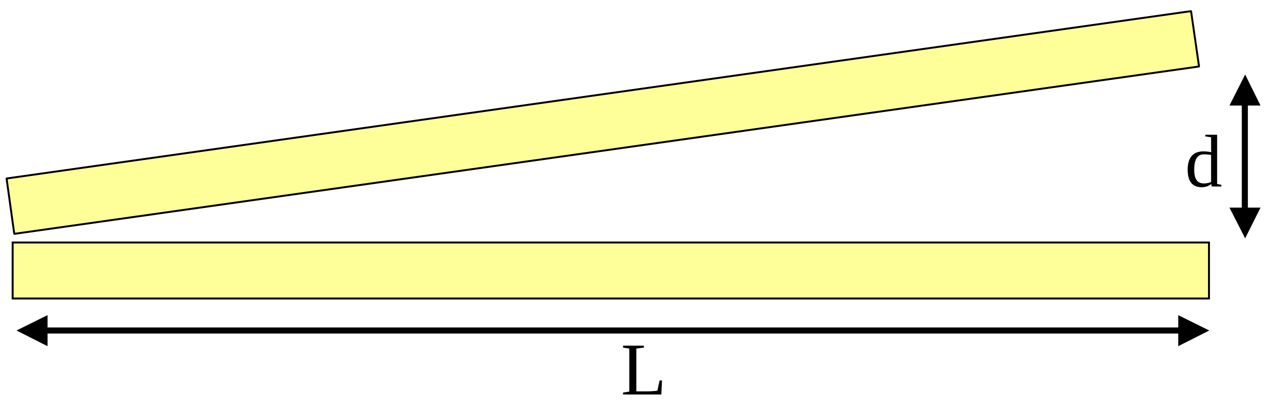
At P_2 , the reflected beam has a phase shift of π (=> this is as if the wave jumped ahead by $\lambda/2$)

⇒Condition for max. constructive interference:

$$\Delta L = 2P_1P_2 = \left(m + \frac{1}{2}\right)\lambda \ , \quad m = 0, \pm 1, \pm 2, \ldots$$

Ex. 37-5 (see previous slide)

L=10 cm, d=0.01mm, l=420 nm
distance between adjacent maxima?



Condition for constructive interference:

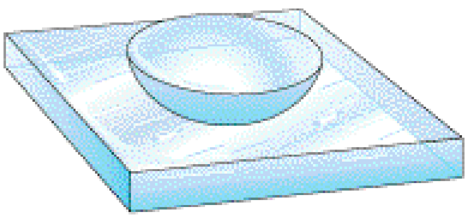
$$2y = \left(m + \frac{1}{2}\right)\lambda \ , \quad m = 0, \pm 1, \pm 2, \ldots$$

The gap size y is related to the distance x to the corner:

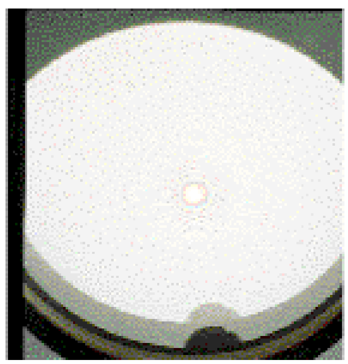
$$\frac{y}{x} = \frac{d}{L} \quad \Rightarrow \quad x = \frac{L}{d} y = \frac{1}{2}\frac{L}{d}\lambda\left(m + \frac{1}{2}\right)$$

$$\Delta x = \frac{L}{d}\frac{\lambda}{2} \cong 2\,\mathrm{mm}$$

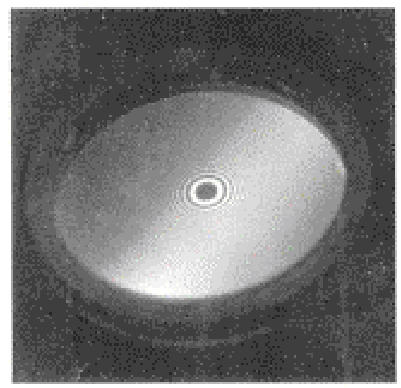
Newton's rings



(a)

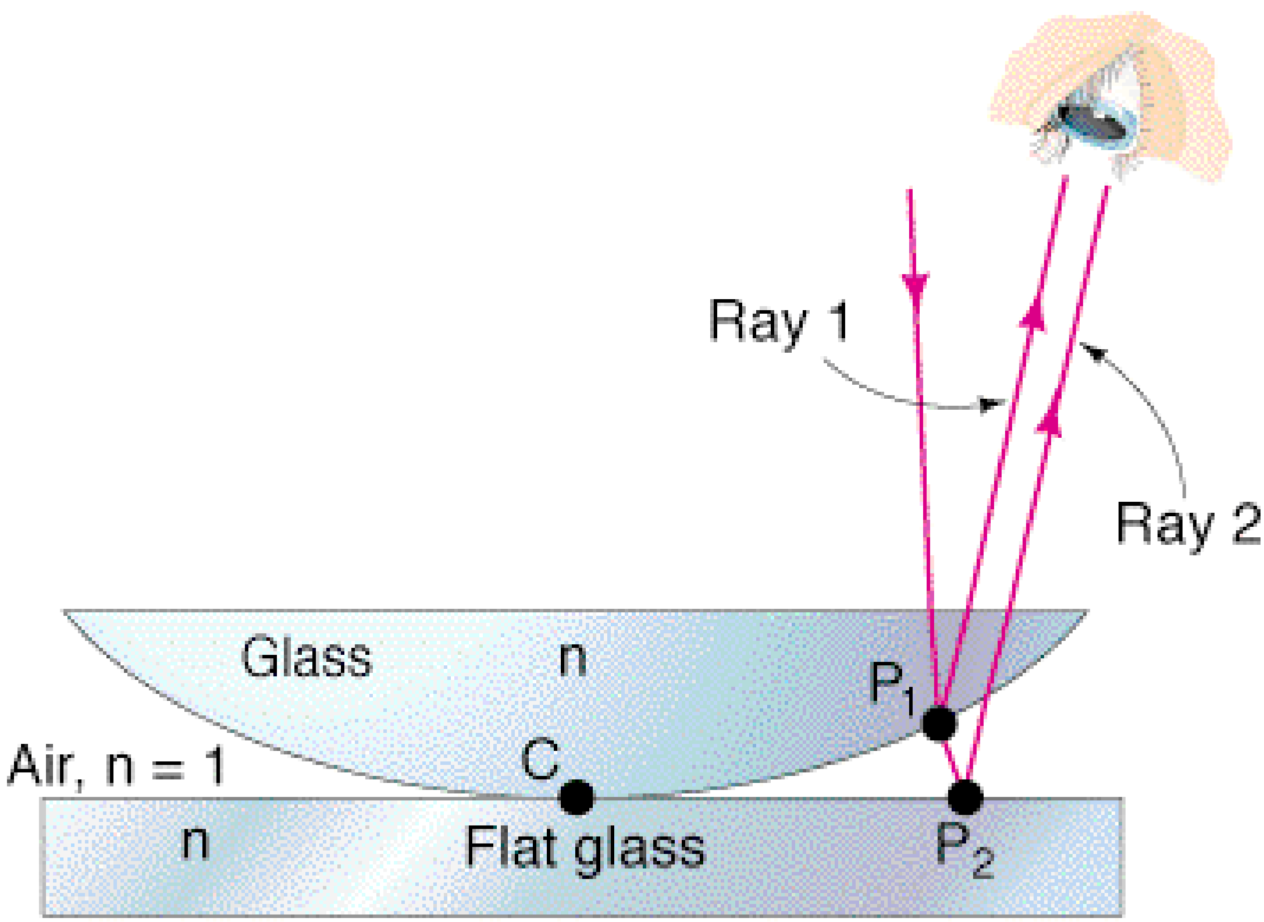


(b)

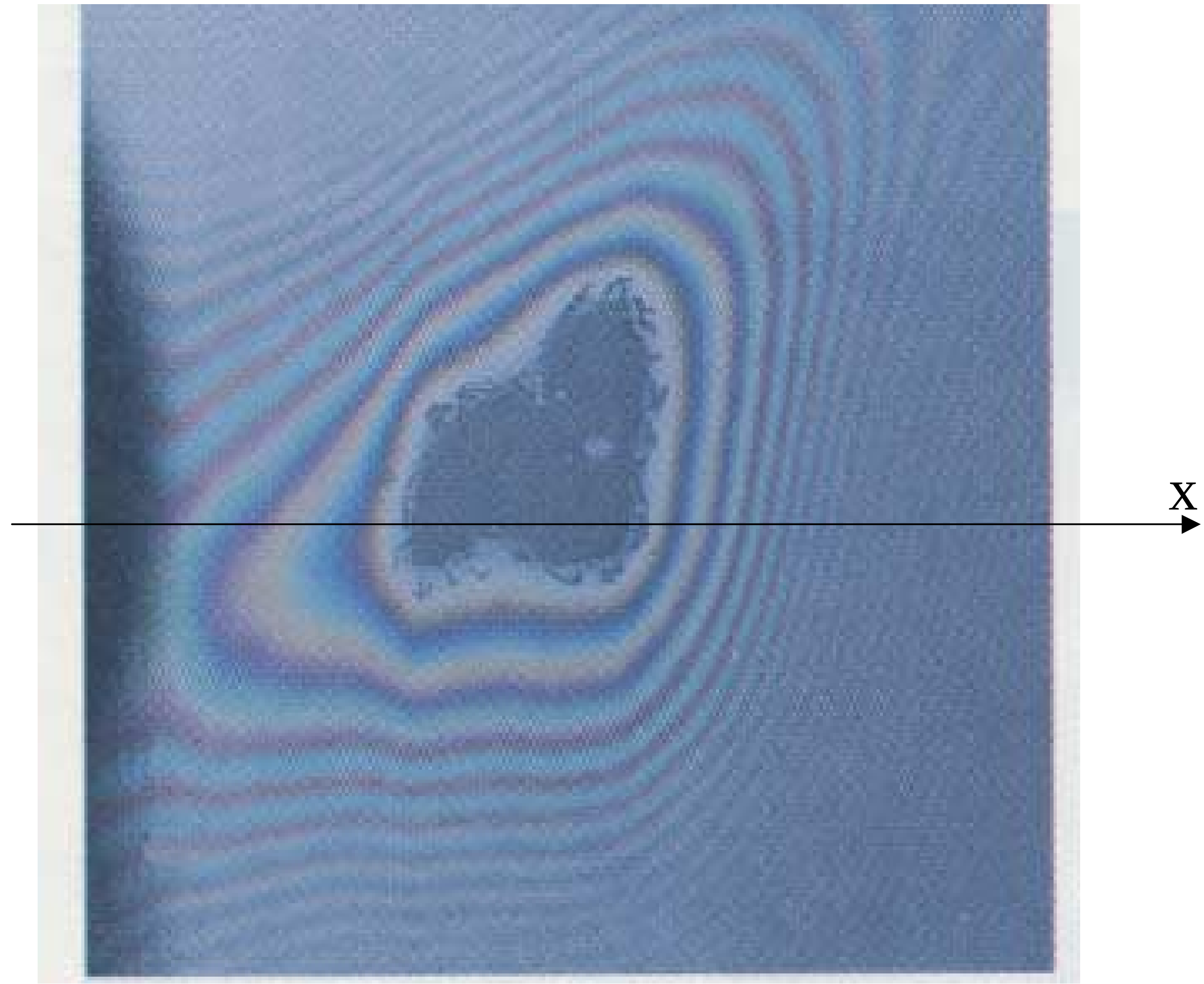


(c)

$$\Delta\phi_{\text{total}} = \Delta\phi_{\substack{\text{path}\\ \text{difference}}} + \Delta\phi_{\substack{\text{reflection}\\ \text{difference}}}$$

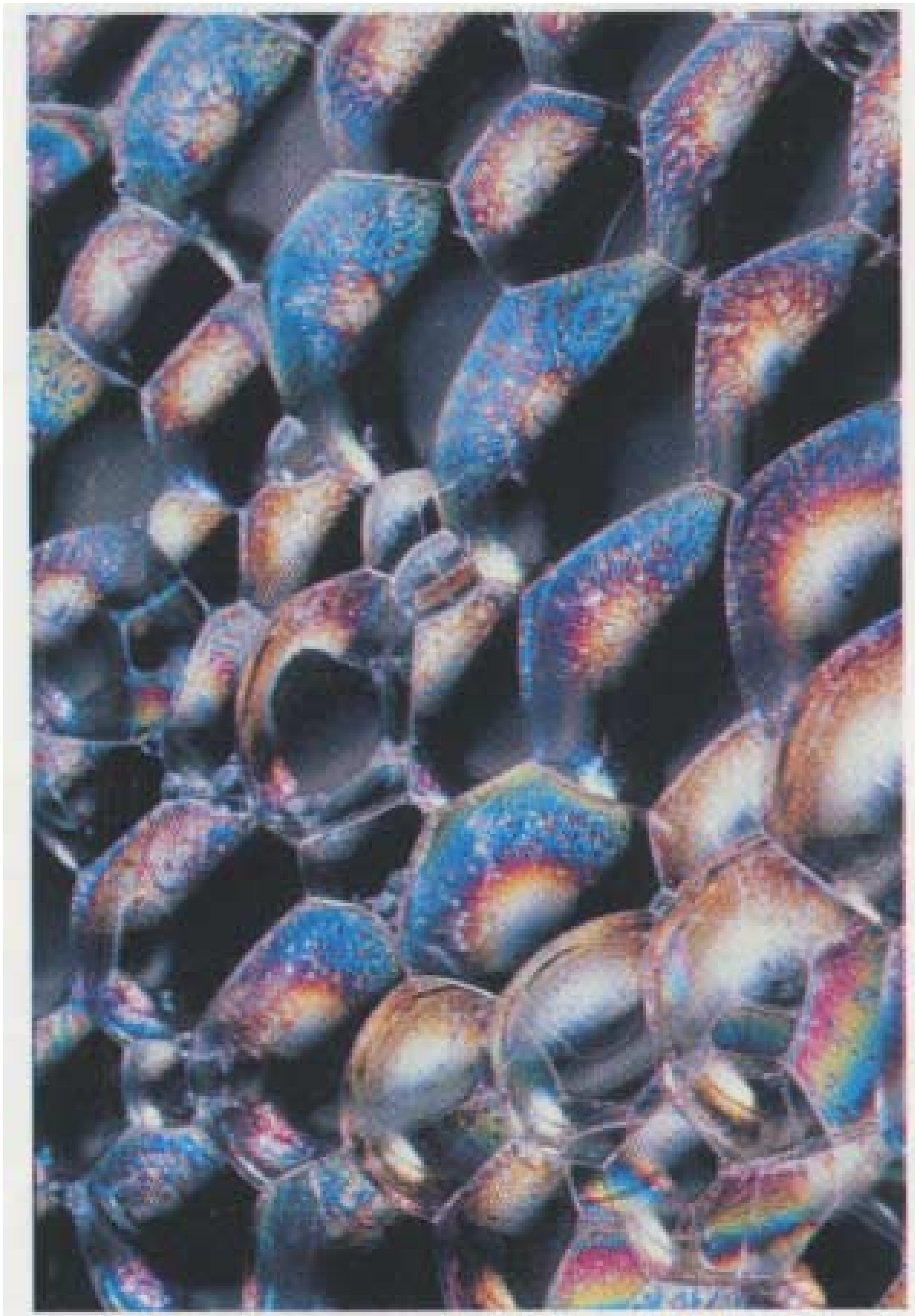


Optical flatness test using interference



What does the cross section look like?

Interference due to reflections in soap bubbles => *Thin-Film Interference*

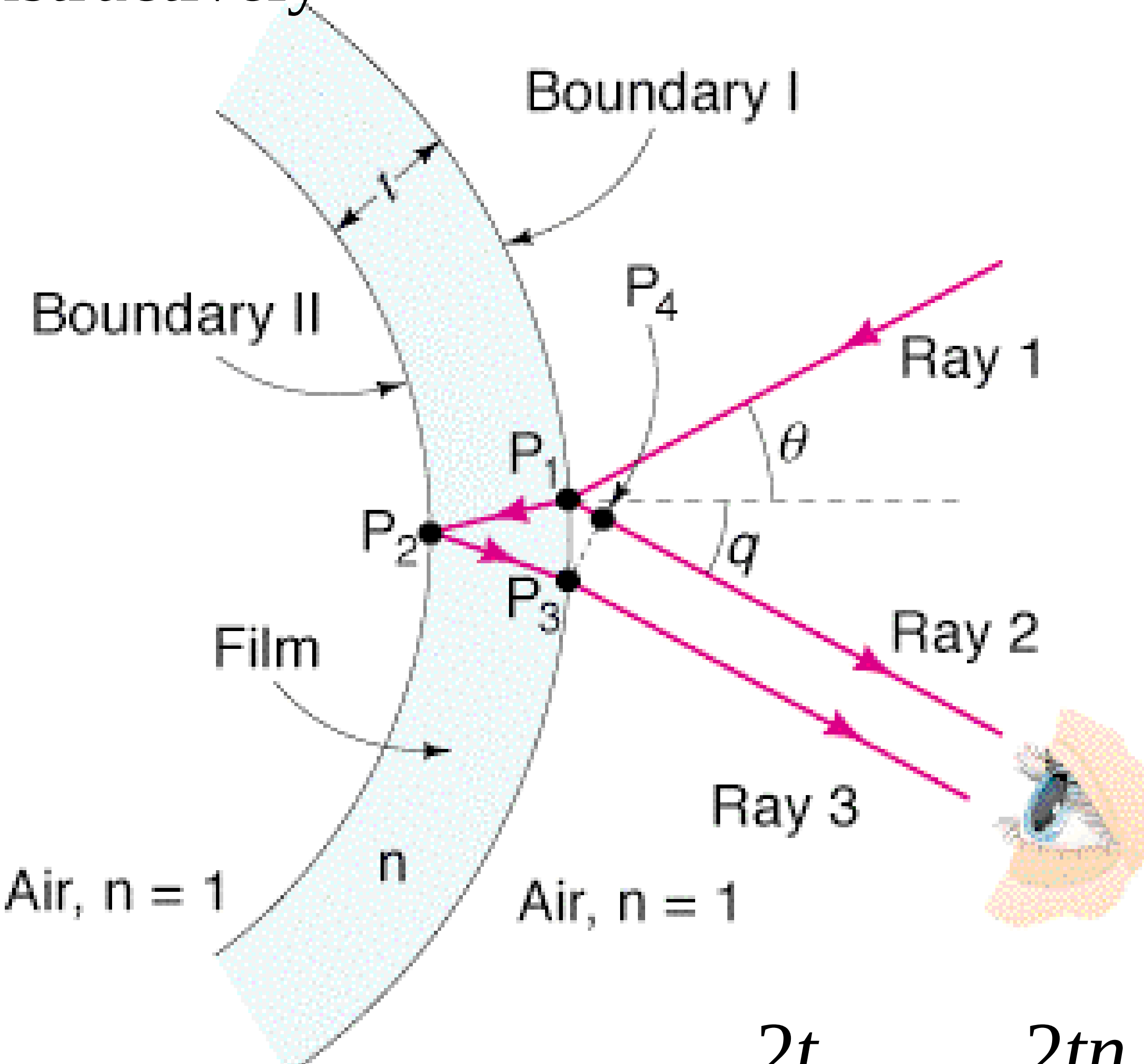


Ex. 37-6:

Soap bubble, thickness t, n, λ

a) Condition for constructive interference?

b) t = 400nm, n =1.3: which color interferes constructively



Phase shift of ray 3: $$\phi_{\Delta L} = \frac{2t}{\lambda_n} 2\pi = \frac{2tn}{\lambda} 2\pi$$

Phase shift of ray 2: π

Phase difference between ray 2 & 3:

$$\phi = \pi + \phi_{\Delta L} = \pi + \frac{4\pi t n}{\lambda}$$

Constructive interference when:

$$\phi = \pi + \frac{4\pi t n}{\lambda} = m \cdot 2\pi$$

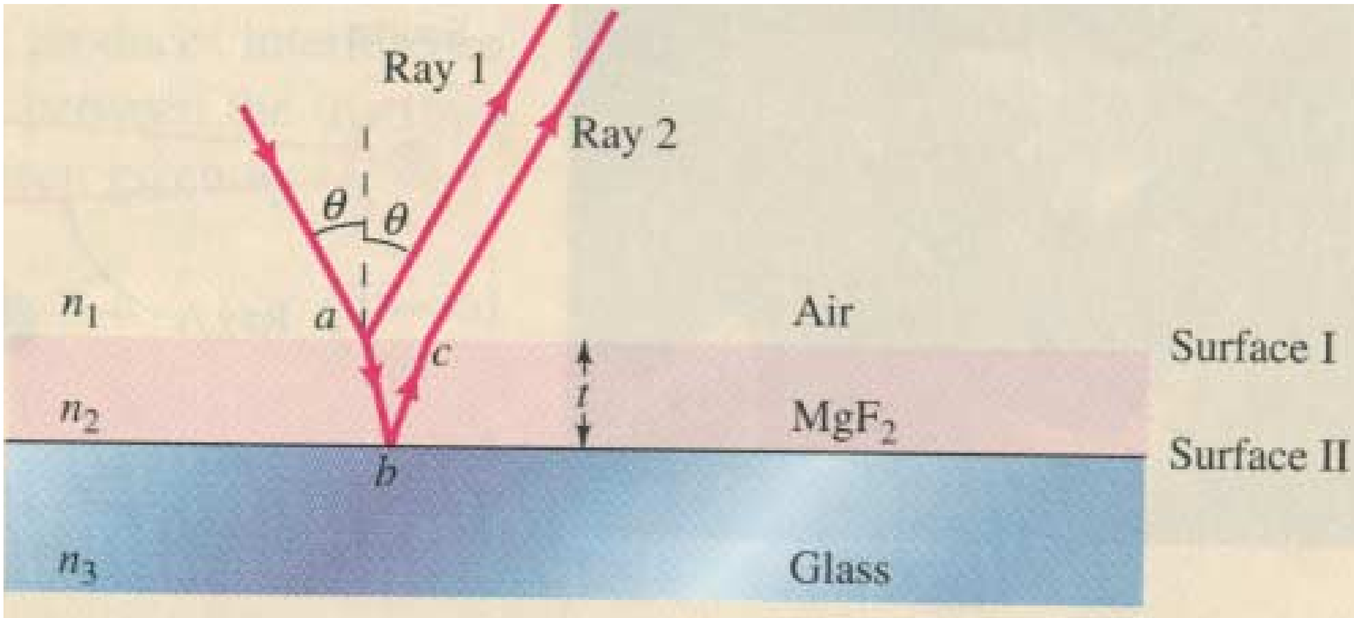
$$4tn = (2m-1)\lambda \ , \ m = 1,2,3,\ldots$$

b. Which color interferes constructively?

$$\lambda = \frac{4tn}{(2m-1)} = \frac{4 \cdot 4\times10^{-7}\,\text{m} \cdot 1.3}{(2m-1)}$$

$$\lambda = 2100\text{nm},\ 700\text{ nm},\ 420\text{ nm},\ \cdots$$

Antireflective coating



How do we minimize reflection?

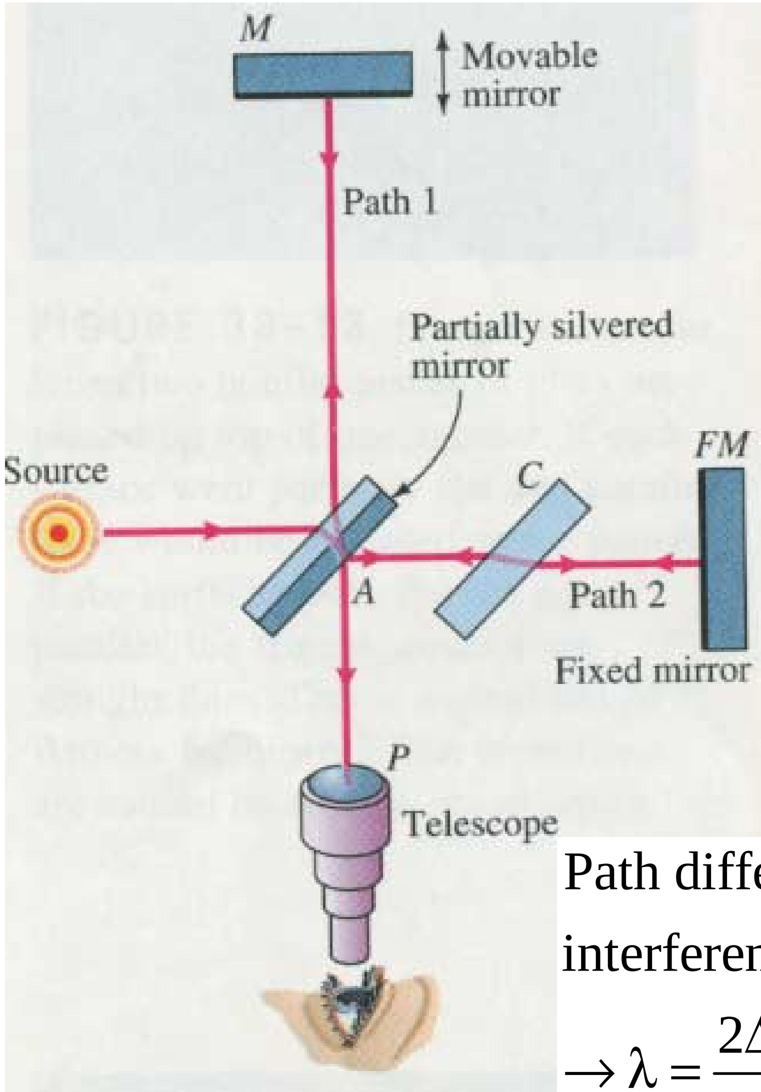
Here: have phase shift of π in both a and b

=> total phase shift is $\phi = \dfrac{4\pi t n_2}{\lambda}$

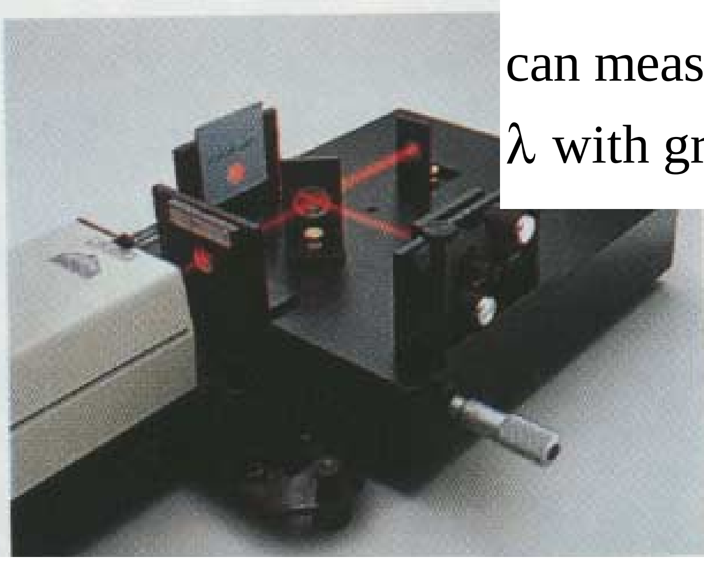
Condition for *destructive* interference:

$$\phi = \frac{4\pi t n_2}{\lambda} = (m + \frac{1}{2})2\pi$$

$$t = \frac{\lambda}{2n_2}\left(m + \frac{1}{2}\right)$$



(a)



Interferometers:

a. Michelson

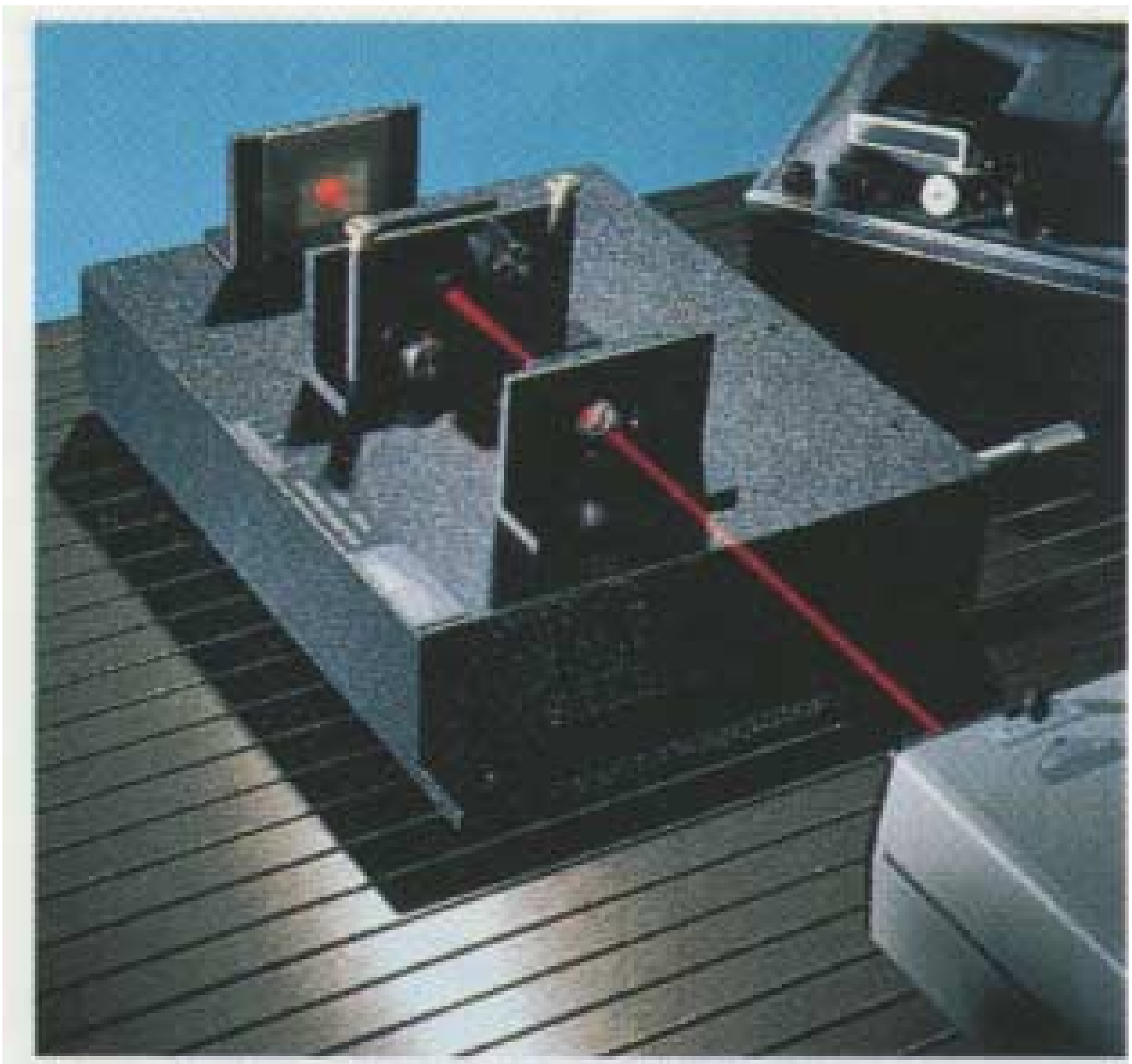
Path difference=$2\Delta L = N\lambda$ for constr. interference (maxima)

$$\rightarrow \lambda = \frac{2\Delta L}{N}$$ since N is easy to count

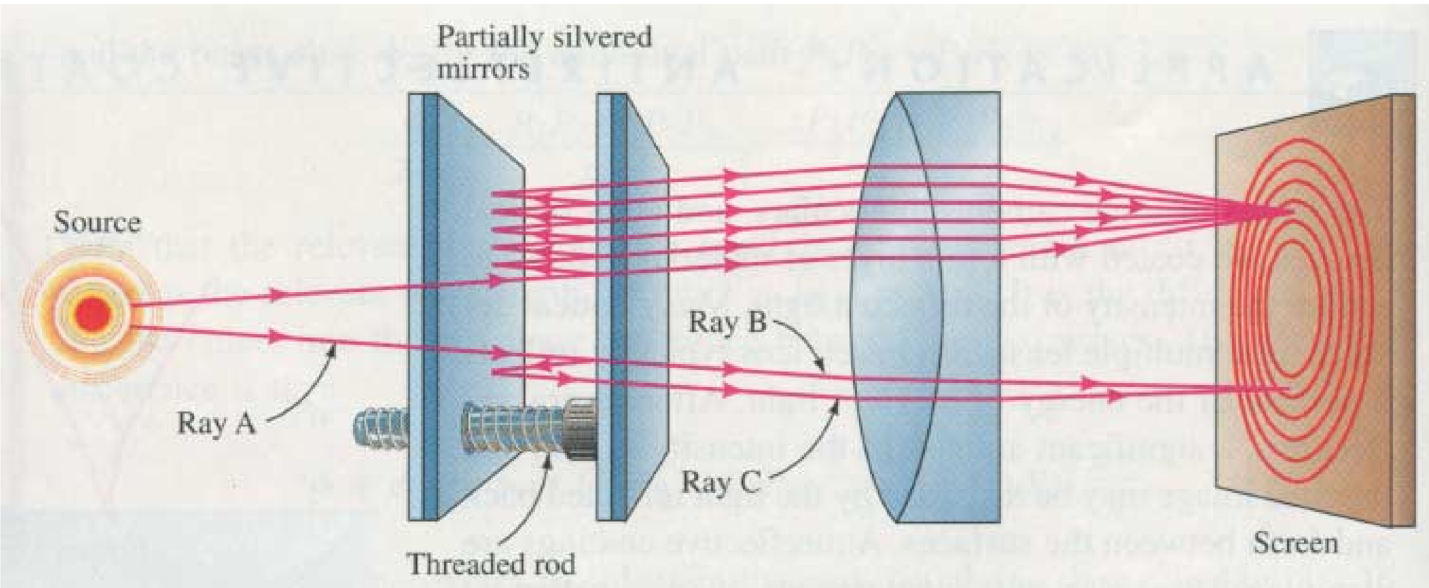
and is easy to measure (i.e., large), one can measure the very short wavelength λ with great accuracy

Interferometers:

b. Fabry-Perot (in-line system)

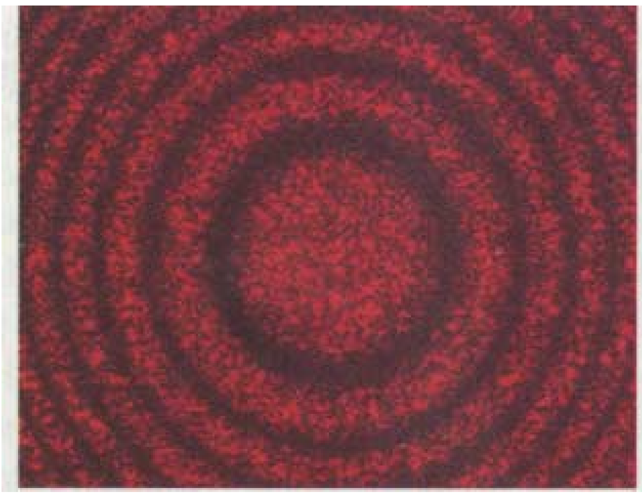


Interferometers: b. Fabry-Perot (in-line system)

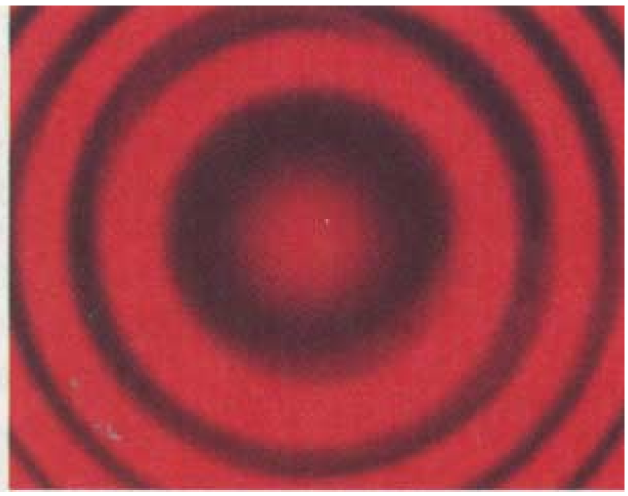


(b)

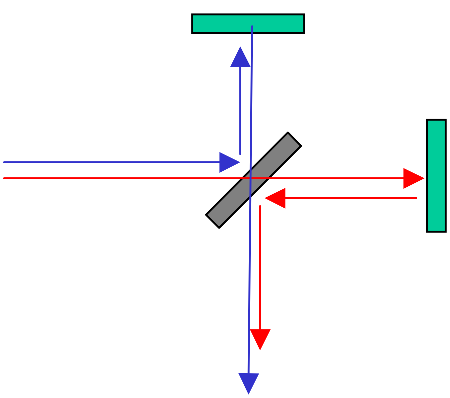
Interference patterns from interferometers



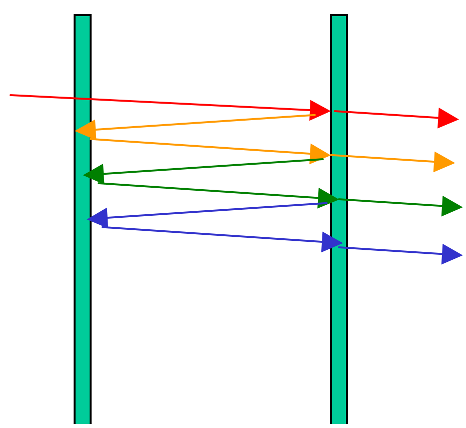
a) Michelson



b) Fabry-Perot



Only two beams



Multiple reflections
(n beams)
See Ch 39 for n-slit