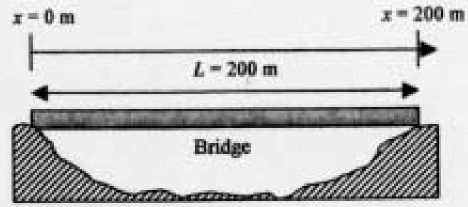
Final Exam

Phy 207 Final Spring '05

Problem 1 (25 points)

Bridges can have standing waves, just as string between would between two fixed ends. Assume the bridge has a 200 m free span and the ends of the bridge are fixed. A bridge can even be destroyed by its resonance. That is why marching bands are trained not to march in lockstep across a bridge, but rather in breakstep.

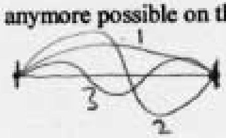


(a) What are the wavelengths of the three first modes *n* of the standing waves on the bridge? (6 points)

(b) If the speed of the wave on the bridge is 100 m/s, what are the frequencies of the first three modes of the standing waves? (6 points)

(c) If the amplitude of the oscillation is 10 cm, and if the bridge is flat at t=0s, what is the equation $A(x,t)$ that describes the displacement of the bridge for the 1[st] (n=1) mode? (6 points)

(d) A pillar has been added as a support in the middle ($x = 100\,\text{m}$) of the bridge. For the standing waves, this means that there is now a node in the middle of the bridge. Which of the modes *n* are not anymore possible on the bridge with the added pillar? (7 points)

a) 

$\frac{1}{2}\lambda_1 = L \rightarrow \lambda_1 = 2L = \boxed{400\,m}$

$\lambda_2 = L = \boxed{200\,m}$ $\quad \frac{3}{2}\lambda_3 = L \rightarrow \lambda_3 = \frac{2}{3}L = \boxed{133\,m}$

b) $\lambda f = v \rightarrow f = \frac{v}{\lambda}$ $\quad f_1 = \frac{100\,m/s}{400\,m} = \boxed{0.250\,Hz}$

$f_2 = \frac{100\,m/s}{200\,m} = \boxed{0.500\,Hz}$ $\quad f_3 = \frac{100\,m/s}{133\,m} = \boxed{0.750\,Hz}$

c) $A_1(x,t) = A_0 \sin(k_1 x + \delta)\cos(\omega_1 t + \phi)$ $\quad \boxed{A_0 = 10\,cm}$

$A(0,t) = A(200,t) = 0 \rightarrow \boxed{\delta = 0}$ $\quad k_1 = 2\pi/\lambda_1 = \frac{2\pi}{2L} = \frac{\pi}{L} \rightarrow \sin\left(\frac{\pi}{L}L\right) = 0$ ✓

$A(x,0) = 0 \rightarrow \boxed{\phi = \pi/2}$ $\quad \omega_1 = 2\pi f_1 = \frac{\pi}{2}$

$$\boxed{A_1(x,t) = (10\,cm)\sin\left(\frac{\pi}{L}x\right)\cos\left(\frac{\pi}{2}t \pm \frac{\pi}{2}\right) = (10\,cm)\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{2}t\right)}$$

d) 

n=1 not allowed | n=2 allowed | n=3 not allowed | n=4 allowed

n=5 not allowed | n=6 allowed

only even numbered modes are allowed.

n=1,3,5 have <u>antinodes</u> at $x = L/2$

$\boxed{\text{all odd numbered modes } n = 1, 3, 5 \ldots \text{ are forbidden}}$

Place your answers here

(a) λ_1 = ________ m, $\quad\lambda_2$ = ________ m, $\quad\lambda_3$ = ________ m (6)

(b) f_1 = ________ Hz $\quad f_2$ = ________ Hz $\quad f_3$ = ________ Hz (6)

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Problem 2 (25 points)

A convex thin **lens** made of ZnSe has an index of refraction of 2.45 and is used to image an upright arrow that is placed 7.00 cm in front of it (see figure). The magnitudes of the radii of curvature for the front and back surfaces of the lens are 11.0 cm and 12.0 cm, respectively. The height of the arrow is 3.0 cm.

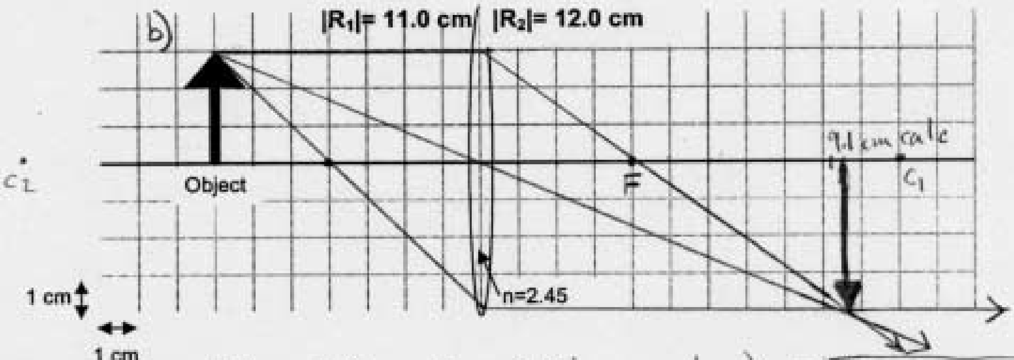
(a) What is the focal length f (magnitude and sign) of this lens? Draw the focal point F on the figure. Use the 1 cm spaced grid lines to accurately locate this point. (5 points)

(b) Draw **three** principal rays to show the location of the image. Use the 1 cm spaced grid lines to accurately locate this point. (6 points)

(c) Calculate the image distance d from the mirror. (6 points)

(d) What is the magnification M? What is the image height H? (4 points)

(e) Is the image real or virtual? Is the image upright or inverted? (4 points)



a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.45)\left(\frac{1}{+11\text{cm}} - \frac{1}{-12\text{cm}}\right) \rightarrow \boxed{f = +3.96\text{ cm}}$

c) $\frac{1}{i} + \frac{1}{s} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{s} = \frac{1}{+3.96\text{cm}} - \frac{1}{+7\text{cm}} \rightarrow i = d = \boxed{9.12\text{ cm}}$ ✓

close to location determined by principal rays

d) $M = \frac{-i}{s} = -\frac{9.12\text{cm}}{7.00\text{cm}} = \boxed{-1.30}$

$H = M H_i = (-1.30)(3\text{cm}) = \boxed{-3.91\text{cm}}$

e) Real, inverted

Place your answers here

(a) f= ______________ (5)

(b) draw on figure (6)

(c) d= ______________ (6)

Problem 3 (25 points)

A beam of white light (wavelengths ranging from 400 to 750 nm) shines on a piece of barium.

(a) What are the energies (E_{min} and E_{max}) of the lowest and highest energy photons in the white light beam. (6 points)

(b) Only wavelengths shorter than 501 nm are observed to eject electrons from the barium. What is the work function W of the barium? (6 points)

(c) What are the kinetic energies (K_{min} and K_{max}) of the lowest and highest energy electrons emitted by the white light beam? (6 points)

(d) What is the stopping voltage V that is required to stop the electrons with the highest kinetic energy? (7 points)

a) $E_{min} = hf_{min} = h\frac{c}{\lambda_{max}} = 6.63\times10^{-34}\,\frac{3\times10^{8}}{750\times10^{-9}} = \boxed{2.65\times10^{-19}\,J}$

$E_{max} = h f_{max} = h\frac{c}{\lambda_{min}} = \boxed{4.97\times10^{-19}\,J}$

b) $hf = h\frac{c}{501\times10^{-9}m} = 3.97\times10^{-19}\,J$

minimum energy photon just barely gets electrons out

$\frac{1}{2}mv^2 = 0 = hf - W \rightarrow W = hf = \boxed{3.97\times10^{-19}\,J}$

c) $K_{min} = \frac{1}{2}mv_{min}^2 = hf_{min} - W = 2.65\times10^{-19}J - 3.97\times10^{-19}J < 0$ X

$K_{min} = \boxed{0}$ when $\lambda_{min} = 501\,nm$

$K_{max} = hf_{max} - W = 4.97\times10^{-19}J - 3.97\times10^{-19}J = \boxed{1.00\times10^{-19}\,J}$

when $\lambda = 400\,nm$

d) $K_{max} = eV_{max} \rightarrow V_{max} = \frac{K_{max}}{e} = \frac{1\times10^{-19}J}{1.6\times10^{-19}C} = \boxed{0.625\,V}$

kinetic energy K_{max} is converted into potential energy eV, when $K_{max} = eV_{max}$, electrons stops

Place your answers here

(a) E_{min} = ____________________ E_{max} = ________________________ (6)

Short Problem S1 (15 points)

A sound source that emits acoustic waves in all directions has a sound level of 120 dB at a distance of 5.0 m from the source.

(a) What is the intensity I of the sound at 5.0 m? (5 points)

(b) What is the power P of the source? (5 points)

(c) At what distance D is the sound level 90 dB? (5 points)

a) $\beta = 10 \log_{10} \frac{I}{I_0} \rightarrow I = I_0 10^{\beta/10} = 10^{-12}\, 10^{120/10} = \boxed{1.00\ W/m^2}$

b) $P = I\ Area = \left(1 \frac{W}{\cancel{m^2}}\right)\left(4\pi (5.0\,\cancel{m})^2\right) = \boxed{314\ W}$

c) $I_2 = \frac{P}{A_2} = \frac{P}{4\pi R_2^2} = I_0 10^{\beta_2/10} = 10^{-12}\, 10^{90/10} = 1\times10^{-3}\ W/m^2$

$R_2 = \sqrt{\frac{P}{4\pi I_2}} = \sqrt{\frac{314\ W}{4\pi (1\times10^{-3}\ W/m^2)}} = \boxed{158\ m}$

Place your answers here

(a) $I =$ __________ (5)

(b) $P =$ __________ (5)

(c) $D =$ __________ (5)

Short Problem S2 (15 points)

NASA sends radio signals to its probes Voyager1 (beyond Pluto) and Cassini (orbiting Saturn). The distance between Earth and Voyager is 94AU (astronomical units), while the distance between Earth and Cassini is 10AU. ($1AU = 1.496\times10^{11}$ m).

(a) How long does it take the radio signal to go from Earth to Cassini? (7 points)

(b) If Cassini receives an intensity of $1.0 \frac{mW}{m^2}$ signal, what intensity does Voyager receive? (8 points)

a) $v t_c = R_c \rightarrow t_c = \frac{R_c}{v} = \frac{(10)(1.496\times10^{11}\ \cancel{m})}{3\times10^{8}\ \cancel{m}/s} = \boxed{4.99\times10^{3}\ s}$

$v = c$

b) $P = I_c A_c = I_v A_v$

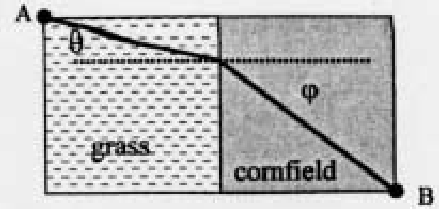
$I_c (4\pi R_c^2) = I_v (4\pi R_v^2)$

$I_v = I_c \frac{R_c^2}{R_v^2} = \left(1\times10^{-3} \frac{W}{m^2}\right) \frac{(10\,AU)^2}{(94\,AU)^2} = \boxed{1.13\times10^{-5}\ W/m^2}$

Place your answers here

Short Problem S3 (15 points)

A person stands at point A and wants to reach point B in the shortest time possible. To reach Point B the person has to run over grass and through a cornfield. The maximum running speed on grass is 6.0 m/s and the maximum running speed in the cornfield is 4.0 m/s.



(a) In analogy to the speeds of light in different media, what is (for running speeds) the "index of refraction" n_{CF} of the cornfield, if 6.0 m/s is the fastest a human can run on any surface? (7 points)

(b) Assume that you have to aim at the cornfield at an angle of $\theta = 20°$ to reach point B in the shortest time. What is the angle φ? (8 points)

similar to $v_n = \frac{c}{n}$

a) $v_{CF} = \frac{v_{max}}{n_{CF}} \to n_{CF} = \frac{v_{max}}{v_{CF}} = \frac{6.0\,m/s}{4.0\,m/s} = 1.50$

b) $v_{grass} = \frac{v_{max}}{n_{grass}} = \frac{v_{max}}{1}$

Use Snell's Law $n_{grass}\sin\theta = n_{CF}\sin\varphi \to \frac{1\sin 20^\circ}{1.5} = \sin\varphi$ $\boxed{\varphi = 13.2^\circ}$

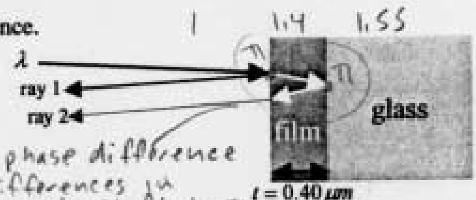
Place your answers here

(a) Index of refraction for running speeds in the cornfield: n_{CF} = ________ (7)

(b) For $\theta = 20°$, the angle φ = ____________ (8)

Short Problem S4 (15 points)

Some lenses of eyeglasses have anti-reflection coating. These glasses are coated with a very thin film of material to reduce the reflected light of certain wavelengths. The material of the film has an index of refraction of $n_F = 1.4$ while the glass of the lens has an index of refraction of $n_G = 1.55$. The thin film has a thickness of $t = 0.40\,\mu m$. Assume normal incidence.



no phase difference due to differences in reflection, both shift by π

(a) What is the phase difference between ray 1 and ray 2 as a function of λ and t? (8 points)

(b) Which visible wavelength ($400nm \le \lambda \le 700nm$) interferes destructively when reflected on the thin film (assume normal incidence)? (7 points)

a) $\Delta\varphi = \underbrace{\Delta\varphi_{refl}}_{0} + \Delta\varphi_{path} = \Delta\varphi_{path} = 2\pi\frac{\Delta L}{\lambda_{film}} = 2\pi\frac{2t}{\lambda/n_{film}} = \boxed{\frac{4\pi t n_{film}}{\lambda}}$

b) $\Delta\varphi_{dest} = (m+\frac{1}{2})2\pi = \frac{4\pi t n_{film}}{\lambda} \to \lambda = \frac{2tn_{film}}{m+1/2} = \frac{1.12\times10^{-6}}{m+1/2} = \boxed{448\,nm}$ ✓ if m=2

$m = 0, 1, 2, \ldots$

= 747 nm ✗

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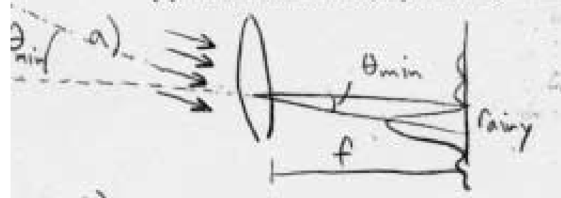
Short Problem S5 (15 points)

A lens of diameter $D = 10$ cm and focal length $f = 5$cm is used to focus sunlight on a piece of paper (at the distance $i = f$ from the lens). The intensity of sunlight is $I_0 = 1000 \frac{W}{m^2}$. Due to diffraction, the sunlight cannot be focused into one infinitely small point. Instead the light is focused into a so-called Airy disk.

(a) Assume the average wavelength of sunlight is $\lambda = 500$ nm. What is the diameter d of the Airy disk on the paper? (7 points)

(b) What is the intensity I_p of the light on the paper? (8 points)

a)



$$r_{airy} = f\,\theta_{min} = f\,\frac{\lambda}{D}$$

$$= (0.05\,m)\left(\frac{500\times10^{-9}\,m}{0.10\,m}\right) = 2.5\times10^{-7}\,m$$

$$d_{airy} = 2r_{airy} = \boxed{5.0\times10^{-7}\,m}$$

b)

$$P = I_0 A_0 = I_p A_p$$

$$I_p = I_0 \frac{A_0}{A_p} = I_0 \frac{\pi(0.05\,m)^2}{\pi(2.5\times10^{-7}\,m)^2} = \boxed{4.0\times10^{13}\ W/m^2}$$

Place your answers here

(a) Diameter of Airy disk: $d =$ __________ (7)

(b) $I_p =$ __________ (8)

Short Problem S6 (15 points)

A proton is trapped in the x-direction by a one-dimensional box with width $W = 1.0\times10^{-15}$ m (approximately the size of a nucleus).

(a) Calculate the energies E_1, E_2, and E_3 for the lowest three energy states. (9 points)

(b) The proton relaxes from the n=2 energy state to the n=1 energy state by emitting a photon. What is the wavelength λ of the emitted photon? (6 points)

a)

$\lambda_1 = 2W \quad \lambda_2 = W \quad \lambda_3 = \frac{2}{3}W$ Standing wave

$p = \frac{h}{\lambda} \quad E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ $\boxed{E = pc \text{ also ok since } v \approx c}$

$$E_1 = \frac{h^2}{2m_p(2W)^2} = \boxed{3.3\times10^{-11}\,J},\quad E_2 = \boxed{1.3\times10^{-10}\,J}$$

$$E_3 = \boxed{3.0\times10^{-10}\,J}$$

b) $\Delta E = E_2 - E_1 = 1.3\times10^{-10} - 3.3\times10^{-11} = hf = h\frac{c}{\lambda}$

$$\lambda = \frac{(6.63\times10^{-34})(3\times10^{8})}{9.7\times10^{-11}} = \boxed{2.1\times10^{-15}\,m}$$