Problem 1

A violin string is L=25.0 cm long and is **fixed at both ends** as shown in Fig. A. The string sounds an A note (frequency f_1=440. Hz) when played. In this case the longest wavelength mode for the transverse wave with wavelength λ_1 is dominating to provide this frequency f_1.

(a) What is the wavelength λ_{air} of the A-note **sound wave**? (~~6~~) 5

(b) What are the wavelengths λ_1, λ_2, and λ_3 for the three longest wavelengths of standing waves that can be supported by the string (L=35.0 cm)? Sketch the displacement as a function of position for these three waves in Fig A. Note that these are the wavelengths in the string and not the wavelengths of sound in the air! (7 points)

(c) If the amplitude of the displacement is 1.50 mm and the string is initially flat across its entire length at t=0 s, what is the equation A(x,t) that describes the transverse displacement of the string as a function of position for the longest wavelength mode with wavelength λ_1? Note that this function must satisfy the boundary conditions at the ends of the string. (~~4~~ 7 points)

(d) One can effectively shorten the string (L'<25.0 cm) by pinching the string against the violin neck as shown in Fig. B. Where must one pinch the string (reducing the effective length to L') to play a C (f_1'=528 Hz)? Again assume that the longest wavelength mode with wavelength λ_1' is dominant. Note that the neither the tension nor the density of the string changes. (6 points)

Fig. A

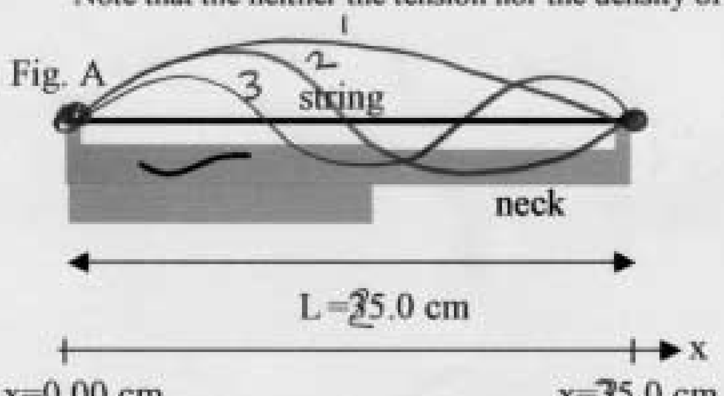
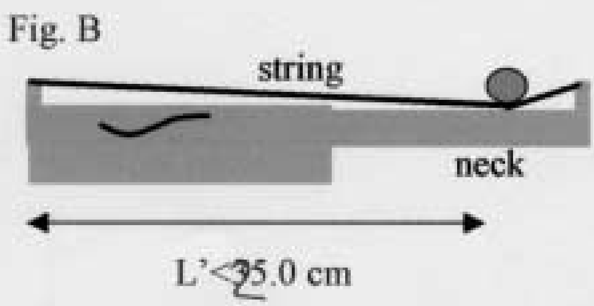


Fig. B



a) $f_1 \lambda_{air} = v_{sound} = 330\,m/s \quad \lambda_{air} = \frac{330\,m/s}{f_1} = \frac{330\,m/s}{440\,1/s} = \boxed{.75\,m}$

b) $\frac{\lambda_1}{2} = L \rightarrow \lambda_1 = 2L = 50.0\,cm = \boxed{.500\,m}$

$1\lambda_2 = L \rightarrow \lambda_2 = L = 25.0\,cm = \boxed{.250\,m}$

$\frac{3}{2}\lambda_3 = L \rightarrow \lambda_3 = \frac{2}{3}L = 16.7\,cm = \boxed{.167\,m}$

c) $\boxed{A(x,t) = 1.50\,mm \sin(k_1 x)\sin(\omega_1 t)}$ standing wave

$k_1 = \frac{2\pi}{\lambda_1} = 12.6\,m^{-1} \quad \omega_1 = 2\pi f_1 = 2.76 \times 10^3\,rad/s$

$A(0,t) = A(L,t) = 0$ ✓ $\quad A(x,0) = 0$ ✓

d) $\lambda_1' = 2L' \quad \lambda_1 f_1 = \lambda_1' f_1' = v =$ constant since μ & T unchanged

$\lambda_1' = 2L' = \frac{\lambda_1 f_1}{f_1'} \rightarrow L' = \frac{1}{2}\lambda_1 \frac{f_1}{f_1'} = \frac{1}{2}(.50)\frac{440}{528} = \boxed{.208\,m}$

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Problem 2

f 2.5

A spherical **mirror** with radius of curvature R=5.0 cm is used to image an upright arrow that is placed 8.0 cm in front of it (see figure). The height of the arrow is 4.0 cm.

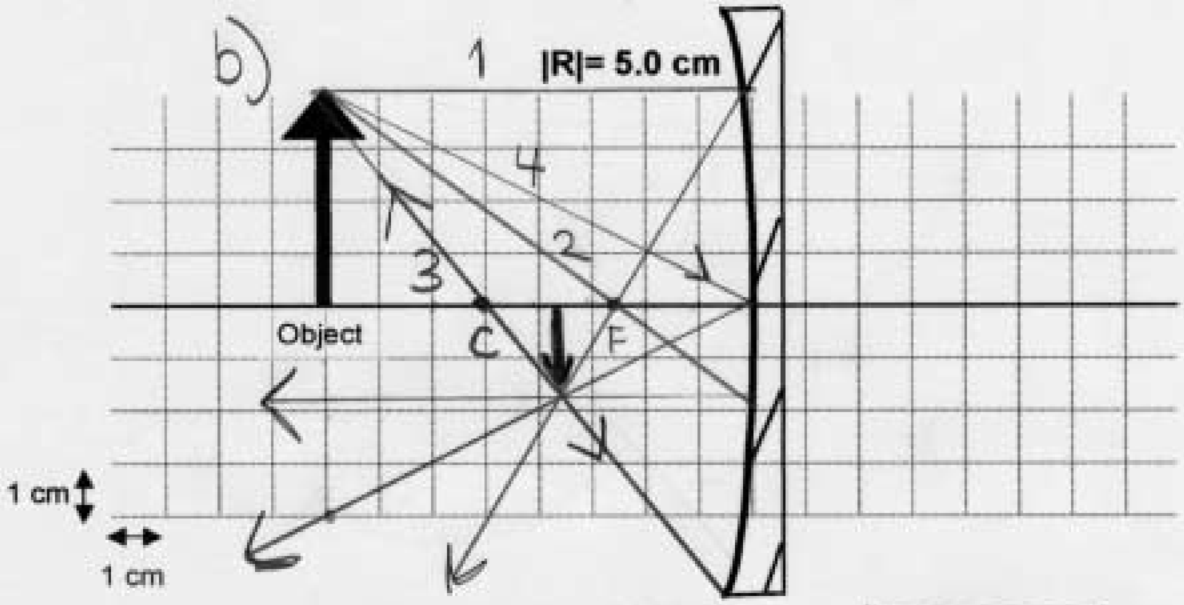
(a) What is the focal length *f* (magnitude and sign) of this mirror? Draw the focal point on the figure. Use the 1 cm spaced grid lines to accurately locate this point. (5 points)

(b) Draw **three** principle rays to show the location of the image. Use the 1 cm spaced grid lines to accurately locate this point. (6 points)

(c) Calculate the image distance *d* from the front of the mirror. (6 points)

(d) What is the magnification *M*? What is the image height *H*? (4 points)

(e) Is the image real or virtual? Is the image upright or inverted? (4 points)



a) $f = \frac{R}{2} \rightarrow R = 2f = 2(+2.5) = \boxed{+5.0\,cm}$

c) $\frac{1}{s} + \frac{1}{i} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{s} = \frac{1}{+2.5\,cm} - \frac{1}{8.0\,cm} \rightarrow \boxed{i = +3.6\,cm = d}$

✓ agrees with sketch

d) $M = -\frac{i}{s} = -\frac{3.6}{4.0} = -.45$

$H = M H_0 = (-.45)(4.0\,cm) = \boxed{-1.8\,cm}$

e) $\boxed{\text{real}}$, $\boxed{\text{inverted}}$

Problem 3

A 6.0 cm wide diffraction grating with N=150 grooves per cm is used to resolve two closely spaced lines (a doublet) in a spectrum. The doublet consists of two wavelengths, λ_1=630.2 nm and λ_2=630.8 nm.

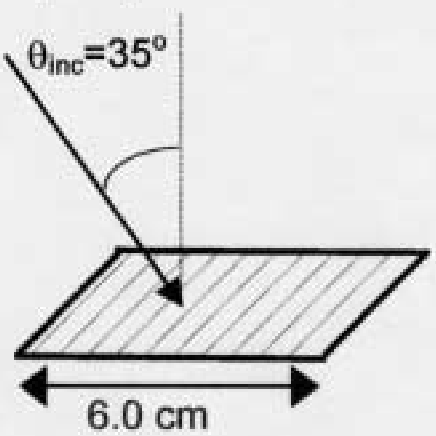
(a) What resolution R is required to resolve the doublet? (5 points)

(b) Determine the first order resolution R_1 of the grating. Assume that the entire grating is illuminated. (5 points)

(c) Determine the second order resolution R_2 of the grating. Assume that the entire grating is illuminated. (5 points)

(d) To which order ϑ in diffraction must one go to be able to resolve the doublet? (5 points)

(e) How many grooves N' on the grating are required to resolve the doublet in first order diffraction? (5 points)



a) $R = \frac{\lambda}{\delta\lambda} = \frac{630\,nm}{.6\,nm} = \boxed{1050}$

b) $R_1 = mN = (1)\,900 = \boxed{900}$

$N = n \times w = \left(150\,\frac{gr}{cm}\right)(6\,cm) = 900\,gr$

c) $R_2 = (2)(900) = \boxed{1800}$

d) $R_1 < R$ but $R_2 > R$, so one needs 2nd order

e) $R' = (1)\,N' = R = 1050 \rightarrow N' = 1050$

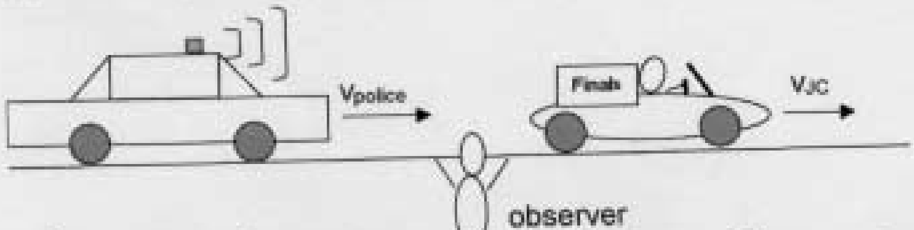
$n' = \frac{N'}{w} = \frac{1050\text{ grooves}}{6\,cm} = \boxed{175\text{ grooves/cm}}$

Short Problem S1

Dr. Cerne is late for the Physics 207 final exam. He is racing towards UB at v_{jc}=65 miles per hour (29.1 m/s) and is followed by a police car that is travelling at v_{police}=75 miles per hour (33.5 m/s). The siren on the closing police car is sounding at 715 Hz. (Please note that this is just an example, Dr. Cerne does not condone speeding!)

(a) What frequency f does a stationary observer at the side of the road hear? (7 points)

(b) What frequency f' does a Dr. Cerne hear? (8 points)



a) $f = \left(\frac{v - v_r}{v - v_s}\right) f_0 = \left(\frac{330 - 0}{330 - 33.5}\right) 715 = \boxed{796 \text{ Hz}}$

b) $f' = \left(\frac{v - v_r}{v - v_s}\right) f_0 = \left(\frac{330 - 29.1}{330 - 33.5}\right) 715 = \boxed{726 \text{ Hz}}$

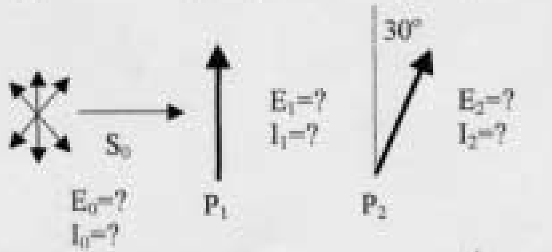
Short Problem S2

An unpolarized collimated laser beam (non-diverging with circular cross section) with average power 20 mW and beam radius of 1.0 mm is incident on a series of two polarizers. The polarizer axis of polarizer P_1 is in the vertical direction, while P_2 is rotated by 30° from vertical.

(a) What is the intensity I_0 of the laser beam before it goes through any polarizers? What is the initial electric field E_0? (5 points)

(b) After the light passes through P_1, what are the intensity I_1 and electric field E_1? (5 points)

(c) After the light passes through P_2, what are the intensity I_2 and electric field E_2? (5 points)



a) $A = \pi r^2$ $\quad I_0 = \frac{\langle P \rangle}{A} = \frac{20 \times 10^{-3} \text{ W}}{\pi (1 \times 10^{-3} \text{ m})^2} = \boxed{6370 \text{ W/m}^2}$

$I_0 = \frac{1}{2} c \epsilon_0 E_0^2 \rightarrow E_0 = \sqrt{\frac{2 I_0}{c \epsilon_0}} = \sqrt{\frac{2(6370)}{(3\times10^8)(8.85\times10^{-12})}} = \boxed{2.19 \times 10^3 \text{ V/m}}$

b) $I_1 = \frac{1}{2} I_0 = 3.19 \times 10^3 \frac{\text{W}}{\text{m}^2}$ $\quad E_1 = k\sqrt{I_1} = k\sqrt{\frac{I_0}{2}} = \frac{1}{\sqrt{2}} \underbrace{k\sqrt{I_0}}_{E_0} = \frac{1}{\sqrt{2}} E_0 = 1.55 \times 10^3 \text{ V/m}$ $\quad k = \sqrt{\frac{2}{c\epsilon_0}}$

c) E_2 E_1

$E_2 = E_1 \cos 30° = \boxed{1.34 \times 10^3 \text{ V/m}}$ $\quad I_2 = I_1 \cos^2\theta = \boxed{2.39 \times 10^3 \text{ W/m}^2}$

Place your answers here:

(a) I_0=________________ E_0=________________ (5)

(b) I_1=________________ E_1=________________ (5)

(c) I_2=________________ E_2=________________ (5)

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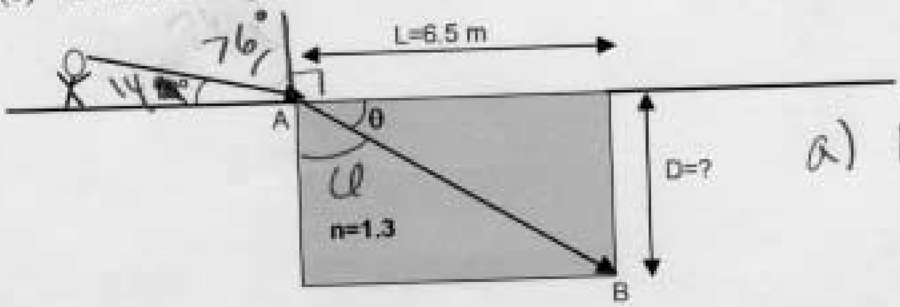
Short Problem S3

In order to measure the depth of a pool, I.M. Snell aligns his line of sight so that the closest edge of the pool (pt. A) is aligned with the far edge at the bottom of the pool (pt. B) as shown in the figure.

(a) If the ray going from I.M. Snell's eye to point A is 14° with respect to the **horizontal**, what is the angle θ (with respect to the **horizontal**) of the ray that continues from point A going to point B? (7 points)

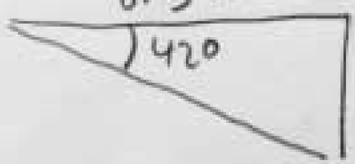
(b) Given that the pool is 6.5 m long, what is the depth D of the pool? (8 points)



a) $1 \sin(76) = 1.3 \sin \varphi$

$\varphi = 48° \rightarrow \boxed{\theta = 42°}$

b)



$\tan 42° = \frac{D}{6.5} \rightarrow D = 6.5 \tan 42°$

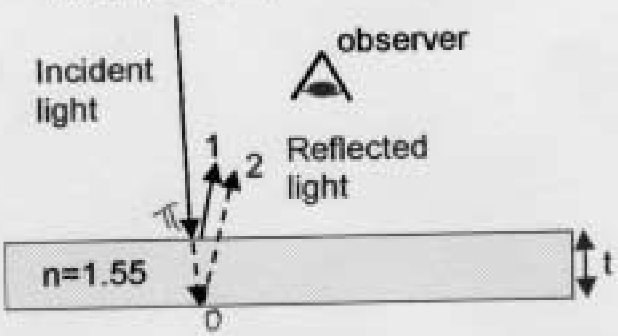
$\boxed{D = 5.8\,m}$

Short Problem S4

Light is reflected from a transparent film of thickness t and refractive index n=1.55. The radiation is at normal incidence (the rays in the figure are separated only for clarity). Monochromatic light with a wavelength λ=633 nm (in air) illuminates the film.

(a) What is the wavelength λ_n of the light in the film? (5 points)

(b) Determine the **minimum** thickness $t_{min\text{-}destr}$ that will results in **destructive** interference between rays 1 and 2. (10 points)



a) $f_{air} = \frac{V_{air}}{\lambda_{air}} = f_n = \frac{V_n}{\lambda_n}$

$\frac{V_{air}}{\lambda_{air}} = \frac{c}{\lambda_{air}} = \frac{v_n}{\lambda_n} = \frac{c/n}{\lambda_n}$

$\rightarrow \lambda_n = \frac{\lambda_{air}}{n} = \frac{633nm}{1.55} = \boxed{408nm}$

b) $4n\,t = 2(m-1)\lambda$

$t = \frac{2}{4n}(m-1)\lambda = \frac{1}{2n}(1)\lambda = \boxed{204nm}$

m=1 → t=0 X (m=2)

Since $\frac{\lambda}{2}$ shift due to reflection, need λ shift due to path to get $\frac{3}{2}\lambda$ shift → destructive interf.

$\Delta L = 2t = \lambda_n = \frac{\lambda}{n}$

$t = \frac{1}{2}\frac{\lambda}{n} = 204nm$

Place your answers here:

(5)

Short Problem S5

An electron is localized inside a cubic region of sides 0.20 nm (recall that 1nm $=10^{-9}$ m).

(a) What is the uncertainty in the x, y, and z components of the electron's momentum, Δp_x, Δp_y, and Δp_z respectively? (7 points)

(b) What is the uncertainty in the electron's kinetic energy ΔE_K? You may treat the electron as non-relativistic. (6 points)

a) $\Delta x \, \Delta p_x > \hbar \qquad \Delta p_x > \frac{\hbar}{\Delta x} = \frac{1.05 \times 10^{-34}}{.20 \times 10^{-9}} = 5.27 \times 10^{-25}$ kg m/s

$\Delta p_{x\,min} = \Delta p_{y\,min} = \Delta p_{z\,min} = \boxed{5.27 \times 10^{-25} \text{ kg m/s}}$

b) $E = \frac{p^2}{2m}$

$\Delta E \gtrsim \frac{|\Delta p|^2}{2m} = \frac{\Delta p_x^2}{2m} + \frac{\Delta p_y^2}{2m} + \frac{\Delta p_z^2}{2m} = 4.58 \times 10^{-19}$ J

$= \boxed{2.86 \text{ eV}}$

$\frac{\Delta E}{\Delta p} = \frac{p}{m} \qquad \Delta E = \frac{p \, \Delta p}{m} = 5.73$ eV also ok

$p \sim \Delta p \sim \Delta p$ Place your answers here:

Short Problem S6

Consider a gas of hot hydrogen atoms

(a) Calculate the energies E_1, E_2, and E_3 for the lowest three energy states. (9 points)

(b) An electron from the n=2 energy state relaxes to the n=1 energy state by emitting a photon. What is the wavelength λ of the emitted photon? (6 points)

a) $E_1 = \frac{-13.6 \text{ eV}}{1^2} = \boxed{-13.6 \text{ eV}} \qquad E_2 = \frac{-13.6 \text{ eV}}{2^2} = \boxed{-3.40 \text{ eV}}$

$E_3 = \frac{-13.6 \text{ eV}}{3^2} = \boxed{-1.51 \text{ eV}}$

b) $\Delta E = E_2 - E_1 = 10.2 \text{ eV} = 1.63 \times 10^{-18} \text{ J} = hf = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{1.63 \times 10^{-18}} = \boxed{122 \text{ nm}}$