In Problem 1d) in last year's final exam solutions the wavelength of the wave in the string λ_{string} was incorrectly equated with the wavelength of the sound wave in the air λ_{air}. These are different wavelengths as I tried to emphasize in the lecture!

Here's a simpler and correct way to solve the problem.

Even though the wave on the string is a standing wave with wavelength λ_{string} (which is determined by L), one can think of it as the superposition of two traveling waves with the same wavelength λ_{string} traveling in opposite directions with speed v_{string} (which is determined by the mass density and tension of the string). The frequency of this standing wave is then given by:

$$f_1 = \frac{v_{\text{string}}}{\lambda_{\text{string}}} \propto \frac{1}{L}$$

since $\lambda_{\text{string}} \propto L$ and assuming that v_{string} doesn't change with L (which it should not if the tension and mass density are unchanged) $f_1 \propto \frac{1}{L} = \frac{\text{Constant}}{L}$

This means that halving the string length doubles the frequency, which makes sense.

So how does the frequency change when L is changed?

$$\frac{f_1'}{f_1} = \frac{\dfrac{\cancel{\text{Constant}}}{L'}}{\dfrac{\cancel{\text{Constant}}}{L}} = \frac{L}{L'} \rightarrow L' = \frac{f_1}{f_1'} L = \frac{440\text{Hz}}{528\text{Hz}} 0.250m = 0.208\text{m}$$