Problem 1

Two point sources (a and b) of 555 nm wavelength light are located L_1=1.75 m in front of a lens with diameter 0.400 m. The sources are separated by a distance d. The lens produces images of the sources on a screen that is L_2=1.00 m behind the lens.

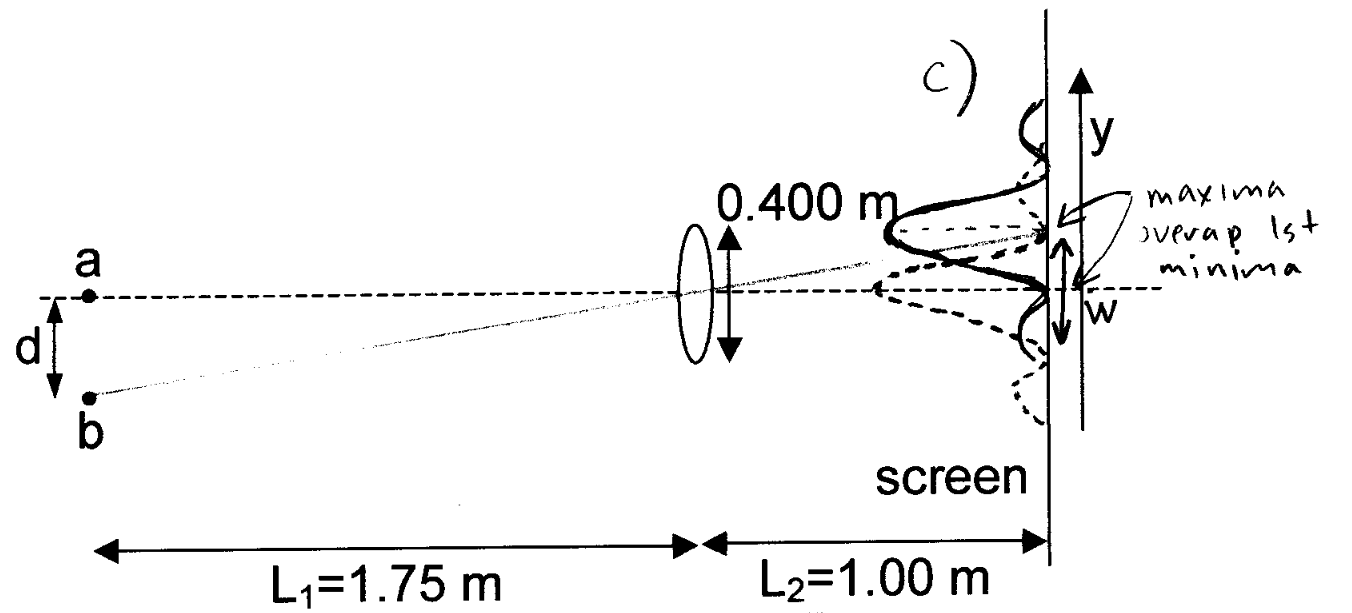
(a) What is the minimum angular separation θ_{min} between the two sources where the lens can just barely resolve them as separate sources? (5 points)

(b) What is the minimum distance d_{min} between the sources where the lens can still resolve the sources a separate? (5 points)

(c) Sketch on the figure below the image patterns (intensity vs. y-position) of the images for the two sources assuming that the d in the figure is d_{min}. Note that this may not actually be to scale! (5 points)

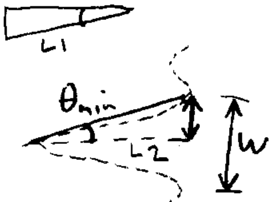
(d) What is the width w (distance between first minima on either side of central maximum) of the image for source a? (5 points)

(e) If source a and b were on the moon (3.83×10^8 m away from the lens), how close could they be (separated by d_{min}') and still be resolved by the lens? (5 points)



a) $\theta_{min} \cong \frac{\lambda}{D} = \frac{555\times10^{-9}\,m}{.40\,m} = \boxed{1.39\times10^{-6}\,rad = 7.95\times10^{-5}\,deg.}$

b) $d_{min} = L_1\,\theta_{min} = (1.75m)(1.39\times10^{-6}\,rad) = \boxed{2.43\times10^{-6}\,m = 2.43\,\mu m}$

d) 

$w = 2\,L_2\,\theta_{min} = 2(1.00m)(1.39\times10^{-6}\,rad) = \boxed{2.78\times10^{-6}\,m = 2.78\,\mu m}$

e) $d_{min}' = L_{moon}\,\theta_{min} = (3.83\times10^{8}\,m)(1.39\times10^{-6}\,rad) = \boxed{531\,m}$

Place your answers here:

Problem 2

A 5.00W beam of light with wavelength 454 nm is incident on a piece of cesium, which has a work function of 2.14 eV. Photoelectrons are emitted from the cesium at a rate R.

(a) What is the energy E of each photon in the beam? (4 points)

(b) How many photons per second are striking the cesium? (4 points)

(c) If an electron gets knocked out of the cesium by a photon, what is the maximum kinetic energy E_{max} that the electron can have? (4 points)

(d) What is the longest wavelength λ_{max} photon that could still knock an electron out of the cesium? (4 points)

What would happen to E_{max} and the rate of electron emission rate R if:

(e) the wavelength of the photons were halved (λ'=227 nm) while keeping the power at 5.00W? Explain. (4 points)

(f) the power of the beam were halved (P'=2.50 W) while keeping the wavelength at 454 nm? Explain. (4 points)

Leave answers in parts (e) and (f) in terms of the E_{max} and R.

a) $E = hf = hc/\lambda = 6.63\times10^{-34}\,J\cdot s\ \dfrac{3\times10^{8}\,m/s}{454\times10^{-9}\,m} = \boxed{4.38\times10^{-19}\,J = 2.74\,eV}$

b) $P = \dfrac{E_{tot}}{\Delta t} = \dfrac{n\,E_{1photon}}{\Delta t} = 5\,J/s = 5\,W \rightarrow n = \dfrac{E_{tot}}{E_{1photon}} = \dfrac{5\,J}{4.38\times10^{-19}\,J} = 1.14\times10^{19}$ photons

$\boxed{1.14\times10^{19}\ \text{photons/s}}$

c) $E_i = 2.74\,eV = E_f = E_{max} + W \longrightarrow E_{max} = E_i - W = 2.74\,eV - 2.14\,eV$

$= \boxed{0.60\,eV = 9.6\times10^{-20}\,J}$

d) $E_{i\,min} = E_{max\,min} + W = 0 + W = 2.14\,eV$

$= 3.42\times10^{-19}\,J$

$\lambda = \dfrac{hc}{E} = 6.63\times10^{-34}\ \dfrac{3\times10^{8}}{3.42\times10^{-19}} = \boxed{5.81\times10^{-7}\,m = 581\,nm}$

e) $\lambda' = \dfrac{\lambda}{2} \rightarrow E' = \dfrac{hc}{\lambda'} = \dfrac{hc}{\lambda/2} = 2\dfrac{hc}{\lambda} = 2E \rightarrow E_{max\,(e)} = E' - W = 2(2.74) - 2.14$

$= \boxed{3.34\,eV = 5.56\,E_{max} = 5.34\times10^{-19}\,J}$

Since photon energy is doubled but power is still the same, $P' = \dfrac{n'E'}{\Delta t} = \dfrac{n'2E}{\Delta t} = \dfrac{nE}{\Delta t} \rightarrow n' = \frac{1}{2}n$, the number of photons is halved $\rightarrow R_{(e)} = \boxed{\frac{1}{2}R}$

f) $\lambda' = \lambda \rightarrow E' = E \rightarrow E_{max\,(f)} = \boxed{E_{max}}$ photon energy the same $\rightarrow$ same photoelectron energy

$R_{(f)} = \boxed{\frac{1}{2}R}$ since half as many photons per second are arriving

Place your answers here:

(a) E = $4.38\times10^{-19}\,J$ or $2.74\,eV$ (4)

Problem 3

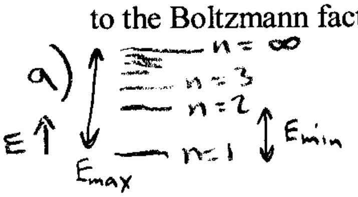
A gas of hydrogen atoms is heated and emits radiation. Recall that a photon carries one unit (positive or negative) of angular momentum and that total angular momentum is conserved in optical transitions (emission or absorption of photons).

(a) What are the shortest λ_{min} and longest λ_{max} wavelength photons that are emitted by the hydrogen gas in the Lyman series (transitions from excited states to the ground state n=1)? Assume that all the excited states are populated. (6 points)

(b) What is energy E_γ of the photon emitted when a H atom relaxes from the first excited state (n=2) down to the ground state (n=1)? What are the allowed value(s) for the initial and final angular momentum (ℓ_i and ℓ_f) of the states that allow this photon emission? Note that the final state depends on the initial state, so list the initial/final states in pairs, e.g., initial$_1$/final$_1$, initial$_2$/final$_2$. (6 points)

(c) What is energy E_γ' of the photon emitted when a H atom relaxes from the second excited state (n=3) down to the first excited state (n=2)? What are the allowed value(s) for the initial and final angular momentum (ℓ_i' and ℓ_f') of the states that allow this photon emission? Note that the final state depends on the initial state, so list the initial/final states in pairs, e.g., initial$_1$/final$_1$, initial$_2$/final$_2$. (6 points)

(d) Determine the ratio N_2/N_1 of the number of atoms with electrons in the first excited state (n=2) to the number of atoms in the ground state (n=1) at T=6.7x10^4 K? Hint: the number of electrons N_n in a state n is proportional to the probability P_n that the electron is in state n, which in turn is proportional to the Boltzmann factor. (7 points)

a) 

lowest energy transition from $n=2 \Rightarrow n=1$

$$\frac{1}{\lambda_a} = R_\infty\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = (1.097\times10^7 m^{-1})\left(\frac{3}{4}\right)$$

$$\lambda_{max} = \lambda_a = \boxed{121\ nm}$$

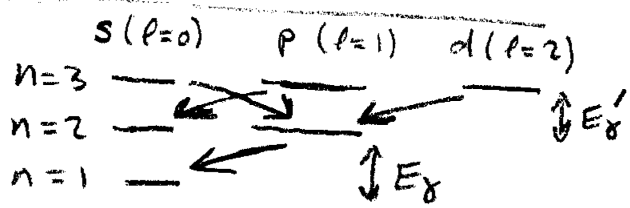
highest energy transition from $n=\infty \Rightarrow n=1$ $\quad \frac{1}{\lambda_b} = R_\infty\left(\frac{1}{1^2}-\cancel{\frac{1}{\infty^2}}^{\,0}\right)$

$$\lambda_{min} = \lambda_b = \boxed{91.1\ nm}$$

b) $E_\gamma = E_i - E_f = -21.8\times10^{-19}J\left(\frac{1}{n_i^2}-\frac{1}{n_f^2}\right) = -21.8\times10^{-19}\left(\frac{1}{2^2}-\frac{1}{1^2}\right)$

$= \boxed{1.64\times10^{-18}J = 10.2eV}$

$\boxed{\ell=1 \rightarrow \ell=0}$ only possibility



c) $E_\gamma' = E_i' - E_f' = -21.8\times10^{-19}J\left(\frac{1}{3^2}-\frac{1}{2^2}\right)$

$= \boxed{3.02\times10^{-19}J = 1.89eV}$

$\boxed{\ell=2 \rightarrow \ell=1}$, $\boxed{\ell=1 \rightarrow \ell=0}$, $\boxed{\ell=0 \rightarrow \ell=1}$ are possible

d) $P_1 \sim e^{-E_1/kT}$ $\quad P_2 \sim e^{-E_2/kT}$ $\quad$ from a) or b)

$$\frac{N_2}{N_1} = \frac{P_2}{P_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-(E_2-E_1)/kT} = e^{-(1.64\times10^{-18}J)/(1.38\times10^{-23}\frac{J}{K})(6.7\times10^4K)}$$

$$= \boxed{0.170 = 17.0\%}$$