Problem 1

A 4.0 cm wide diffraction grating with 100 grooves per cm is used to resolve two closely spaced lines (a doublet) in a spectrum. The doublet consists of two wavelengths, λ_a=630.2 nm and λ_b=630.8 nm.

(a) What resolution R is required to resolve the doublet? (4 points)

(b) Determine the first order resolution R_1 of the grating. Assume that the entire grating is illuminated. (3 points)

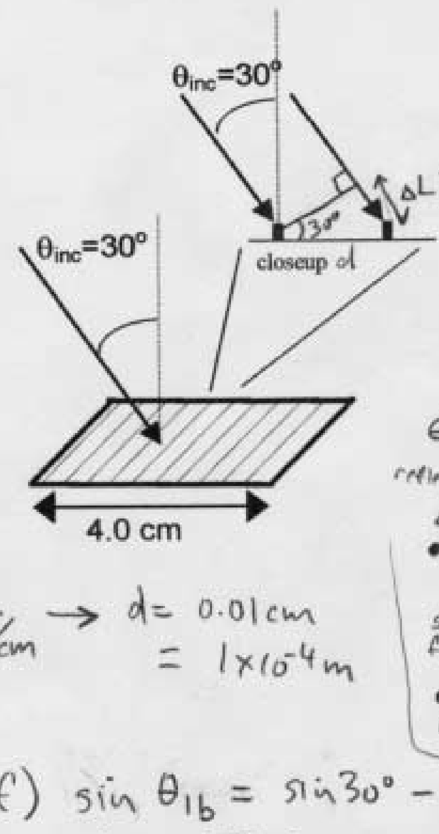
(c) Determine the second order resolution R_2 of the grating. Assume that the entire grating is illuminated. (3 points)

(d) To which order in diffraction must one go to be able to completely resolve the doublet? (3 points)

(e) Determine the angles θ_{1a} and $\theta_{\substack{2a\\10}}$ with respect to the normal to the surface for first order diffraction (rays from neighboring lines are separated by one wavelength in path difference) and second order diffraction (two wavelength path difference from neighboring lines) beams for λ_a=630.2 nm. Hint: keep track of the path differences of two rays going to neighboring lines before and after the grating as shown in closeup of figure below. (6 points)

(f) Determine the angles θ_{1b} and θ_{2b} with respect to the normal to the surface for first order and second order diffraction beams for λ_b=630.8 nm. (6 points)

$R = \lambda/\Delta\lambda$



a) $R = \frac{630.2\text{ nm}}{0.6\text{ nm}} = \boxed{1050}$ or $R = \frac{630.8\text{nm}}{0.6\text{nm}} = \boxed{1051}$

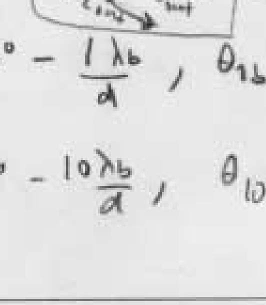
$\Delta\lambda = 630.8\text{nm} - 630.2\text{nm}$

b) $R_1 = 1N = 1(4.0\text{cm} \times 100 \frac{\text{lines}}{\text{cm}}) = \boxed{400}$

c) $R_2 = 2N = \boxed{800} < 1050$ not good enough

d) $R_3 = 3N = 1200 > 1050$, $\boxed{\text{3rd order}}$ is required

e) $\Delta L = \Delta L_{in} - \Delta L_{out} = d\sin 30° - d\sin\theta_{out}$



$\sin\theta_{out} = \sin 30° - \frac{\Delta L}{d}$

$\sin\theta_{1a} = \sin 30° - \frac{1\cdot\lambda_a}{d}$, $\theta_{1a} = 29.583°$

$\lambda_a = 630.2\text{nm}$

$\sin\theta_{2a} = \sin 30° - 10\frac{\lambda_a}{d}$, $\theta_{10a} = \boxed{25.899°}$

100gr/cm → $d = 0.01\text{cm} = 1\times10^{-4}\text{m}$

f) $\sin\theta_{1b} = \sin 30° - \frac{1\lambda_b}{d}$, $\theta_{1b} = 29.582°$

$\lambda_b = 632.8\text{nm}$

$\sin\theta_{2b} = \sin 30° - \frac{10\lambda_b}{d}$, $\theta_{10b} = \boxed{25.895°}$

① can separate the 10th order diffraction spots better

② make sense since $R = mN$

$R_{10} = 10 R_1$

Place your answers here:

(a) R = ____________________ (4)

(b) R_1 = ____________________ (3)

(c) R_2 = ____________________ (3)

Problem 2

Consider a proton in a tin nucleus that is known to lie within a sphere whose diameter is about 1.4×10^{-14} m. Calculate the following:

(a) What is the uncertainty in the proton's momentum? (5 points)

(b) What is the uncertainty in the proton's kinetic energy? (5 points)

Consider a proton that can move completely freely (not bound to or localized by anything) that has a de Broglie wavelength of 1.4×10^{-14} m. Calculate the following:

(c) What is the momentum of this proton? (5 points)

(d) What is the uncertainty in this proton's position? (5 points)

(e) What is the uncertainty in this proton's momentum? (5 points)

a) $\Delta p_x \Delta x > \hbar \quad \Delta p_x > \frac{\hbar}{\Delta x} = \frac{\hbar}{1.4 \times 10^{-14}\,m} = \frac{6.63 \times 10^{-34}/2\pi}{1.4 \times 10^{-14}} = \boxed{7.54 \times 10^{-21}\ kg\,m/s}$

$.7 \times 10^{-14}$ ok or 1.51×10^{-20} kg m/s

b) $E = \frac{p^2}{2m} \rightarrow \Delta E = \frac{(\Delta p)^2}{2m_p} = \frac{(7.54 \times 10^{-21})^2}{2(1.67 \times 10^{-27})} = \boxed{1.70 \times 10^{-14}\ J}$

$= \boxed{1.06 \times 10^{5}\ eV}$

high energy!

6.8×10^{-14} J also ok

c) $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.4 \times 10^{-14}} = \boxed{4.74 \times 10^{-20}\ J \quad 0.296\ eV}$

d) $\Delta p_x = 0$, λ exactly defined, wave goes forever

$\Delta x > \frac{\hbar}{\Delta p_x} = \frac{\hbar}{0} = \boxed{\infty}$ proton could be anywhere in the universe

or since the proton is free and could be anywhere $\Delta x > \boxed{\infty}$ $\boxed{\Delta x_{min} \approx \infty}$

e) $\Delta p_x = 0$

or $p_x > \frac{\hbar}{\Delta x} = \frac{\hbar}{\infty} = 0$ $\boxed{\Delta p_{x\,min} \approx 0}$

Place your answers here:

(a) Uncertainty in momentum for bound proton = ________________________ (5)

(b) Uncertainty in kinetic energy for bound proton = (5)

Problem 3

Consider a hydrogen atom in its ground state.

(a) What is the wavelength of the lowest energy photon that can be absorbed by the hydrogen atom? Indicate the quantum numbers n and ℓ for the initial and final states (recall that angular momentum is conserved and that a photon carries one unit of angular momentum). Note that for hydrogen the energy is independent of ℓ. (6 points)

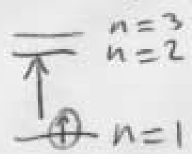
A neutral chlorine (P) atom consists 15 protons in the nucleus surrounded by 15 electrons. Determine the following:

(b) Make a sketch to show the pattern of the electron energy-level occupation for P at T=0 K (absolute zero). Assume that energy increases with angular momentum, e.g., the state (n=2, ℓ=1) has higher energy than the state (n=2, ℓ=0). (5 points)

(c) What are the quantum numbers n_w and ℓ_w for the most weakly bound electron in a P atom at T=0 K? (5 points)

(d) Consider the **lowest energy** transition involving the absorption of a **single** photon by an electron in the n=1 state of a P atom at T=0 K. Indicate the quantum numbers n and ℓ for the possible final state for this transition (note that the Pauli exclusion principle as well as conservation of angular momentum must be satisfied by this transition). (5 points)

(e) Does one expect photons to be emitted from a P atom at T=0 K? Explain (4 points)

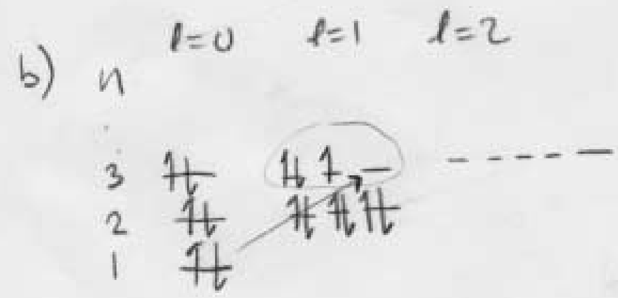
a) 

$\Delta E_{min} = E_1 \to E_2 = 13.6\,eV \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 10.2\,eV$

$E = hf = h\frac{c}{\lambda} \to \lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\ 3\times10^{8}}{1.63\times10^{-18}} = \boxed{122\,nm}$ $= 1.63\times10^{-19}\,J$

initial $\boxed{n=1,\ \ell=0}$ $\to$ final $\boxed{n=2,\ \ell=1}$ $\Delta\ell=1$

$n=2,\ \ell=0$ not allowed state $\Delta\ell=0$

b) 

c) most weakly bound → highest energy electrons

$n_w=3,\ \ell_w=1$

d) final state $\boxed{n=3\quad \ell=1}$, $\Delta\ell=1$

e) No. Electrons are fully relaxed, there's no place for higher energy electron to relax down to

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Place your answers here:

(a) λ =__________ initial state(s) =__________ final state(s) =__________ (6)

(b) sketch =__________ (5)