Exam 1 Spring '04

Problem 1

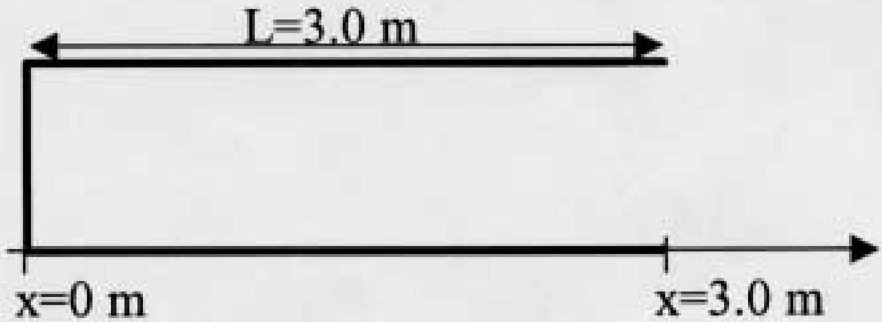
A pipe organ consists of a tube with length L= 3.0 m, with the left end closed and right end open (see figure).

(a) Sketch graphs showing the air displacement $Z_i(x)$ as a function of position along the tube for the longest three wavelengths λ_1, λ_2, and λ_3 of waves produced by the pipe. Calculate λ_1, λ_2, and λ_3. (9 points)

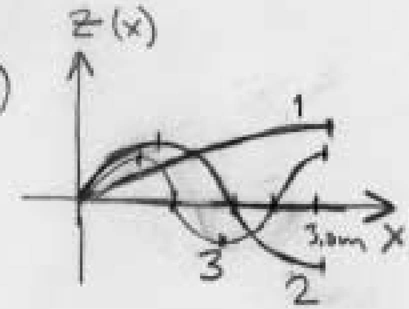
(b) Calculate lowest three frequencies f_1, f_2, and f_3 produced by the pipe. (3 points)

(c) Calculate the wavenumber k_1 and **angular** frequency ω_1 for the longest wavelength mode produced by the pipe. (4 points)

(d) In the lowest frequency mode the displacement amplitude is 2.0 cm. The closed end of the pipe is at x=0.0 m and the open end is at x=3.0 m. At time t=0, displacement along its entire length of the pipe is **zero**. What is the equation $Z_1(x, t)$ representing the air displacement for the longest wavelength mode of the ~~string~~ sound wave. Along which axis (along or perpendicular to the tube axis) are the air molecules moving? (9 points)



a)



① $\frac{\lambda_1}{4} = L \rightarrow \lambda_1 = 4L = 12m$

② $\frac{3}{4}\lambda_2 = L \rightarrow \lambda_2 = \frac{4}{3}L = 4.0m$

③ $\frac{5}{4}\lambda_3 = L \rightarrow \lambda_3 = \frac{4}{5}L = 2.5m$

b) $f\lambda = v \Rightarrow f = \frac{v}{\lambda}$

$f_1 = \frac{v}{\lambda_1} = \frac{330 m/s}{12m} = 28 Hz$

$f_2 = \frac{v}{\lambda_2} = \frac{330 m/s}{4m} = 82 Hz$

$f_3 = \frac{v}{\lambda_3} = \frac{330 m/s}{2.5m} = 132 Hz$

c) $k_1 = \frac{2\pi}{\lambda_1} = .52 m^{-1}$ $\qquad \omega_1 = 2\pi f_1 = 173$ rad/s

d) standing wave $Z_1(x,t) = Z_0 \sin(k_1 x + \delta)\cos(\omega_1 t + \phi)$

$Z_1(x=0, t) = 0 \Rightarrow \sin(0+\delta) = 0 \Rightarrow \delta = 0, \pi, 2\pi$ etc.

$Z_1(x=L, t)$ = maximum $\sin(\frac{2\pi}{4L}L + 0) = \sin(\frac{\pi}{2}) = 1$ ✓

Problem 2

A series of upright pulses are sent to the right along a lighter rope with linear mass density μ_1 of 112 g/m (see figure). The pulses are separated by a distance S_1=15.0 cm and travel at a speed of v_1=20.0 m/s. These pulses encounter at point A a heavier rope with linear mass density μ_2 of 224 g/m.

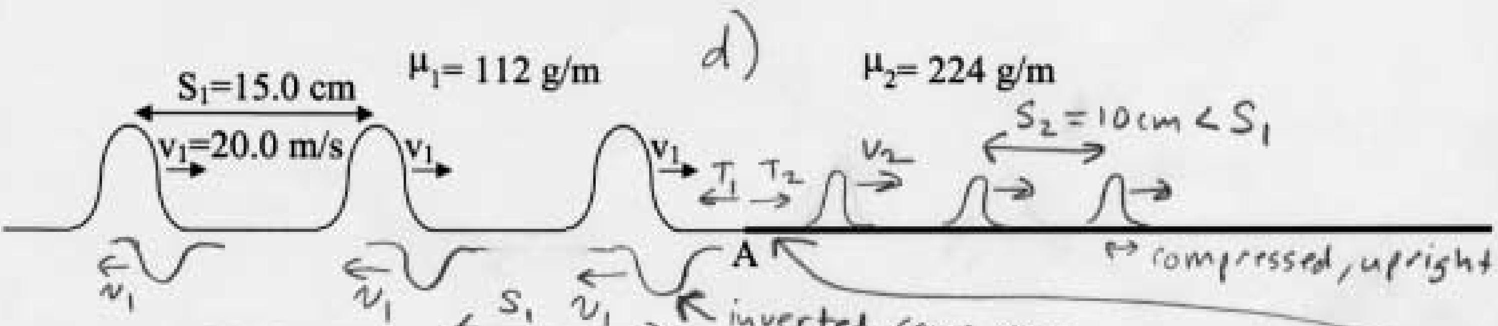
(a) What is the tension T_1 in the light rope? What is the tension T_2 in the heavier rope? (4 points)

(b) At what frequency f_1 do the pulses arrive at point A? At what frequency f_2 do the pulses leave point A as they go into the heavier rope? (4 points)

(c) What is the speed v_2 of the pulses in the heavier rope? What is the distance S_2 separating the pulses in the heavier rope? (4 points)

(d) Sketch as accurately as possible in the figure below the series of transmitted and reflected pulses. (6 points)

(e) The amplitude of the incident pulses is 5.00 cm while the amplitude of the transmitted pulses is 3.00 cm. What is the amplitude of the reflected pulses? (7 points)



a) $v_1 = \sqrt{\frac{T_1}{\mu_1}} \rightarrow T_1 = \mu_1 v_1^2 = (.112 \frac{kg}{m})(20 m/s)^2 = 44.8 \underbrace{kg\,m/s}_{N}$

$T_2 = T_1 = 44.8\ N$ otherwise rope at pt. A would start accelerating to the left or right

b) $f\lambda = v \rightarrow f_1 = \frac{v_1}{\lambda_1} = \frac{20 m/s}{.15 m} = 133\ Hz$ $f_2 = f_1 = 133\ Hz$ otherwise pulses would start building up or appearing out of nowhere.

c) $v_2 = \sqrt{\frac{T_2}{\mu_2}} = \sqrt{\frac{T_1}{2\mu_1}} = \frac{1}{\sqrt{2}}\sqrt{\frac{T_1}{\mu_1}} = \frac{1}{\sqrt{2}} v_1 = 14.1\ m/s$

$f_2\lambda_2 = v_2 \rightarrow \lambda_2 = \frac{v_2}{f_2} = \frac{14.1 m/s}{133} = 0.106\ m = S_2$

e) $\mu_1 \frac{\cancel{\omega_i^2} A_i^2}{\cancel{2}} v_i = \mu_1 \frac{\cancel{\omega_r^2} A_r^2}{\cancel{2}} v_r + \overset{2\mu_1}{\cancel{\mu_2}} \frac{\cancel{\omega_t^2} A_t^2}{\cancel{2}} \overset{v_1/\sqrt{2}}{\cancel{v_t^2}}$

$\cancel{\mu_1} A_i^2 \cancel{v_1} - 2\cancel{\mu_1} A_t^2 \frac{\cancel{v_1}}{\sqrt{2}} = \cancel{\mu_1} A_r^2 \cancel{v_1} \rightarrow A_r = \sqrt{A_i^2 - \ldots}$

Problem 3

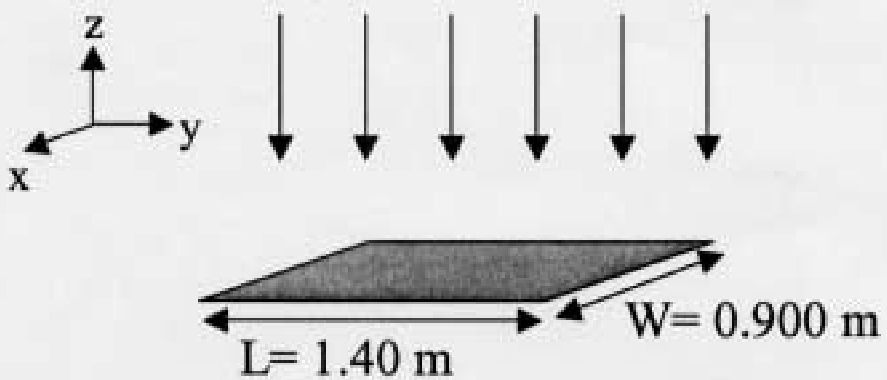
Sunlight is coming straight down (-z direction) on a solar panel (length L=1.40 m and width W=0.900 m) belonging to the Spirit Mars rover (see figure). The amplitude of the electric field in the solar radiation is 673 V/m and is uniform (the radiation has the same amplitude everywhere).

(a) Calculate the amplitude of the magnetic field in the solar radiation. If at a given time and position the electric field of the solar radiation is pointing in the +x direction, what is the direction of the magnetic field at that same time and position? (5 points)

(b) Determine the **amplitude** (i.e., the maximum value) and **direction** of the energy flux (Poynting vector) of the solar radiation. (5 points)

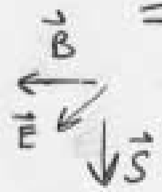
(c) Calculate the intensity of solar radiation striking the solar panel. (7 points)

(d) What is the average power of sunlight that strikes the panel? If the solar panel has an efficiency of 18.0 percent in converting solar radiation into electrical power, how much average power can the panel generate from the solar radiation? (8 points)



a) $B_o = \frac{E_o}{c} = \frac{673\ V/m}{3\times10^8\ m/s}$

$= 2.24\times10^{-6}\,T$



B along $-y$ axis $(-\hat{j})$

b) $S = \frac{1}{\mu_0} E_o \times B_o = \frac{1}{4\pi\times10^{-7}}(673)(2.24\times10^{-6}) = 1.20\times10^3\ J/m^2$

$-z$-direction, $-\hat{k}$

c) $I = \langle S \rangle = \frac{1}{2} S = 600\ W/m^2$

or

$I = \frac{1}{2} c\,\epsilon_o E_o^2 = \frac{1}{2}(3\times10^8)(8.85\times10^{-12})(673)^2 = 601\ W/m^2$

d) $\langle P_{radiation} \rangle = IA = (600\ \frac{W}{m^2})(1.4\,m)(.9\,m) = 756\,W$

$\langle P_{elec.} \rangle = (.18)\langle P_{radiation} \rangle = (.18)(756W) = 136\,W$

a little more than two