Problem 1

A stringed instrument is constructed by stretching a wire between two attachment Points (A and B) that are fixed on walls (see figure). The walls are 2.50 m apart and the wire is stretched to a tension of 60.0 N. The linear mass density of the wire is 2.00×10^{-3} kg/m. Calculate the following:

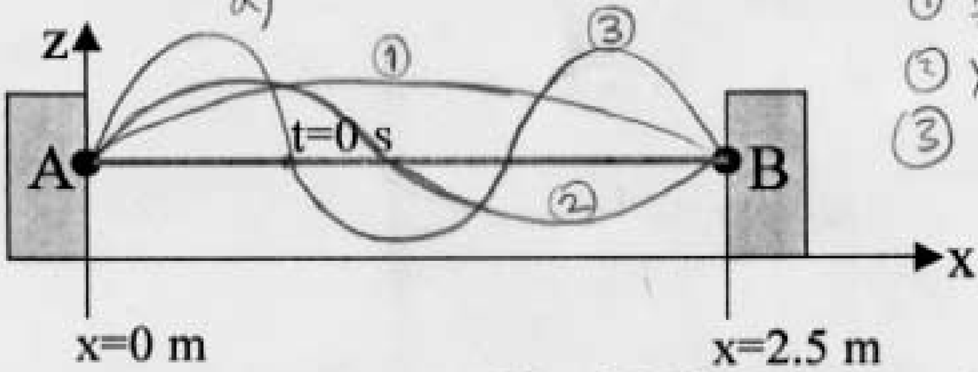
(a) The longest three wavelengths λ_1, λ_2, and λ_3 of waves produced by this instrument. Sketch the modes (the spatial pattern that the vibrating string makes) that correspond to these three wavelengths. (9 points)

(b) The lowest three frequencies f_1, f_2, and f_3 produced by the instrument. (6 points)

(c) The wavenumber k_1 and **angular** frequency ω_1 for the longest wavelength mode produced by the instrument. (6 points)

(d) The string is set to vibrate in its lowest frequency mode with a displacement amplitude of 5.00 cm. At time t=0, the string has **zero displacement** along its entire length (as shown in figure below, which is a snapshot of the string at t=0). What is the equation $Z_1(x, t)$ representing the string displacement for the longest wavelength mode of the string. (4 points)

a)



1 $\frac{\lambda_1}{2} = L \rightarrow \lambda_1 = 2L = \boxed{5.00\,m}$

2 $\lambda_2 = L \rightarrow \lambda_2 = \boxed{2.50\,m}$

3 $\frac{3}{2}\lambda_3 = L \rightarrow L_3 = \frac{2}{3}L = \boxed{1.67\,m}$

b) $f = \frac{v}{\lambda}$ $\quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60.0\,N}{2.00\times10^{-3}\,kg/m}} = 173\,m/s$

$f_1 = \frac{173\,m/s}{5.00\,m} = 34.6\,s^{-1} = \boxed{34.6\,Hz}$

$f_2 = \frac{173}{2.50} = \boxed{69.3\,Hz}$ $\quad f_3 = \frac{173}{1.67} = \boxed{104\,Hz}$

c) $k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi}{5.00} = \boxed{1.26\,rad/m}$ $\quad \omega_1 = 2\pi f_1 = \boxed{217\,rad/s}$

d) Standing wave $\rightarrow Z_1(x,t) = Z_0 \sin(k_1 x + \delta)\cos(\omega_1 t + \phi)$

$Z_0 = 5.00\times10^{-3}\,m$

$Z_1(0,t) = 0 = Z_0 \sin(\delta)\cos(\omega_1 t + \phi) \rightarrow \sin(\delta) = 0 \rightarrow \delta = 0$ (or $\pi, 2\pi$ etc)

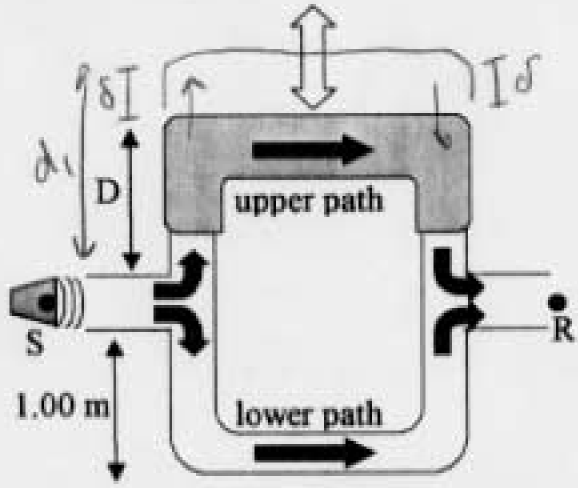
$Z_1(2.5, t) = Z_0 \sin\left(\frac{2\pi}{2L}L\right)\cos(\omega_1 t + \phi) = 0$ since $\sin(\pi) = 0$

$Z_1(x,0) = 0 = Z_0 \sin(k_1 x)\cos(\phi) = 0 \rightarrow \cos(\phi) = 0 \rightarrow \phi = \frac{\pi}{2}$

$$\boxed{Z_1(x,t) = (5.00\times10^{-2}\,m)\sin\left(1.26\,\tfrac{rad}{m}\,x\right)\cos\left(217\,\tfrac{rad}{s}\,t + \tfrac{\pi}{2}\right)}$$ (or $\frac{3}{2}\pi, \frac{5}{2}\pi$ etc)

Problem 2

A single speaker S emits sound waves with a frequency of 660 Hz. The sound is split into two equal parts and then recombined through a system of tubes shown below. The sound is detected by a receiver at point R. The upper path length can be varied by sliding the upper section. You should neglect any cavity effects, i.e., treat the sound waves as traveling waves.



$\Delta L = 2\delta$
goes extra distance δ going up and extra δ going back down

(a) What is the wavelength of the sound that the speaker produces? (4 points)

Initially, the upper sliding tube section is set at D= 1.00m

(b) What is the path difference between the sound that travels through the upper path and the sound that travels through the lower path? Is the interference at point R constructive, destructive, or something in between? (4 points)

Starting At D= 1.00 m, the upper section is slid upward until it reaches a position of D=d_1 where no sound is heard at R.

(c) What is the path difference between the upper and lower sound waves?. (4 points)

(d) What is the phase shift in radians between the two sound waves? (4 points)

(e) Calculate the distance d_1. (4 points)

(f) Make a qualitative plot (show maxima and minima) showing the intensity of sound at R as a function of slide position D, as D is increased from 1.00m to 2.00m. (5 points)

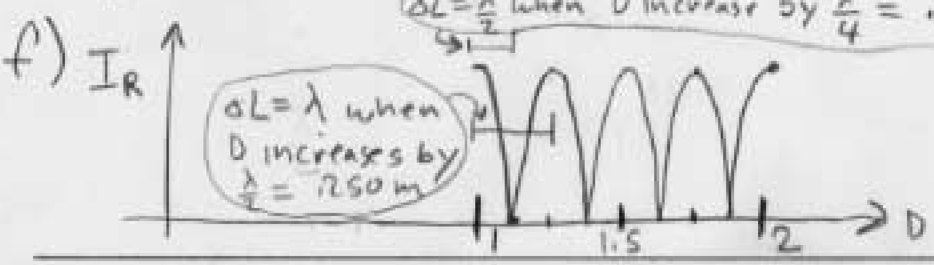
a) $\lambda = \frac{v}{f} = \frac{330 \text{ m/s}}{660 \text{ 1/s}} = \boxed{0.500 \text{ m}}$ b) $\boxed{\Delta L = 0} \rightarrow$ constructive

c) first destructive interference condition occurs at $\Delta L = \frac{\lambda}{2} = \boxed{0.250 \text{ m}}$

d) $\frac{\lambda}{2} \rightarrow \boxed{\pi}$ e) $\Delta L = 2\delta = 2(d_1 - D) = \frac{\lambda}{2} \rightarrow d_1 = \frac{\lambda}{4} + 1 = \boxed{1.13 \text{ m}}$

$\Delta L = \frac{\lambda}{2}$ when D increase by $\frac{\lambda}{4} = .125$ m

f)



Place your answers here:

Problem 3

A dipole antenna is located at the origin with its axis along the z-axis. As electric current oscillates up and down the antenna, linearly polarized electromagnetic (EM) radiation travels away from the antenna along the positive y-axis. At Point A on the y-axis, the intensity of this radiation is I_A=13.0 W/m^2.

(a) Determine the **amplitude** (i.e., the maximum value) and **direction** of the Poynting vector $\mathbf{S}_A$ at Point A. (4 points)

(b) Is the antenna emitting radiation equally in all directions? Briefly explain your answer. (2 points)

(c) A snapshot is taken of the EM wave when the electric field is a *positive maximum* at Point A, calculate the **magnitudes** and **directions** of the electric E_A and magnetic B_A fields at Point A at that time. (6 points)

(d) The radiation the encounters a linear polarizer P immediately after passing Point A. The polarizer is normal to the beam (i.e., the polarizer surface is parallel to the x-z plane) with a polarization axis that is rotated by 25.0° from vertical (z-axis). After the light passes through the polarizer, what is the intensity I_B, at Point B? Since Points A and B are very close to each other compared to their distance from the origin, you may assume that the only change in the intensity of the radiation is due to the polarizer. (6 points)

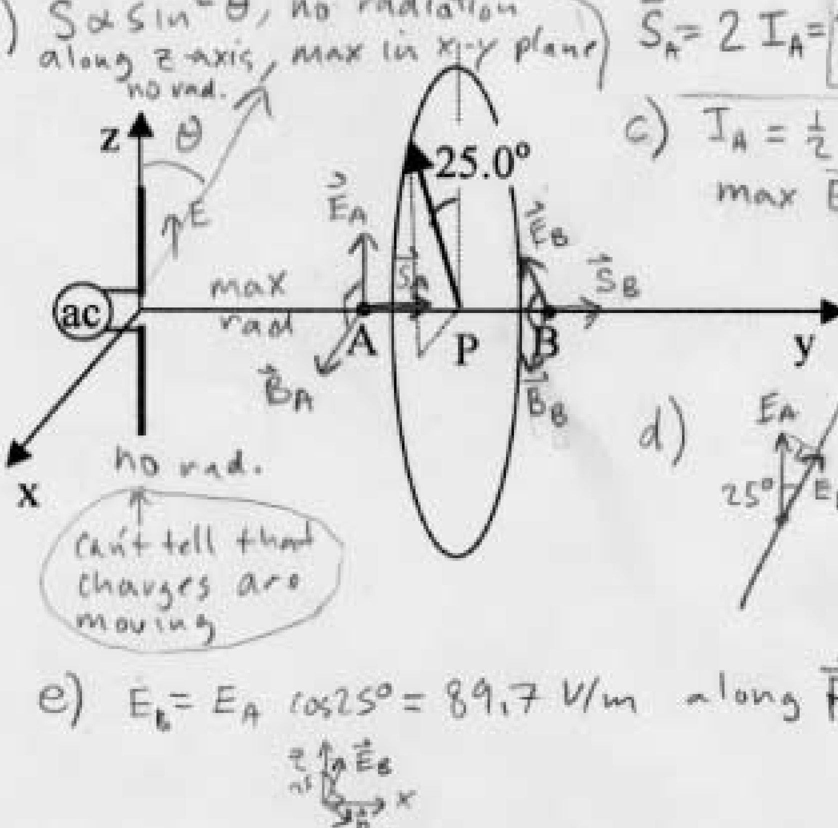
(e) A snapshot is taken of the EM wave when the electric field is a *positive maximum* at Point B, calculate the **magnitudes** and **directions** of the electric E_B and magnetic B_B fields at Point B at that time. (7 points)

a) $I = \langle S \rangle = \frac{1}{2} S$, since $S \propto \cos^2(kx-\omega t)$

$\langle S \rangle = S \langle \cos^2(kx-\omega t) \rangle = S\,\frac{1}{2}$

$\vec{S}_A = 2 I_A = \boxed{26.0 \frac{W}{m^2}\,\hat{j}}$ along y

b) $S \propto \sin^2\theta$, no radiation along z-axis, max in x-y plane



can't tell that charges are moving

c) $I_A = \frac{1}{2} c \epsilon_0 E_A^2$

max $\vec{E} = \vec{E}_A = \sqrt{\frac{2 I_A}{c \epsilon_0}} = \boxed{99.0\ V/m\ \hat{k}}$ up

$\vec{B}_A = \frac{E_A}{c}\hat{\imath} = \boxed{3.30 \times 10^{-7}\ T\ \hat{\imath}}$ along x direction

d)

$E_B = E_A \cos 25°$

$I_B = \frac{1}{2} c\epsilon_0 E_B^2 = \left(\frac{1}{2} c\epsilon_0 E_A^2\right)\cos^2 25°$

$= I_A \cos^2 25° = \boxed{10.7\ W/m^2}$

e) $E_B = E_A \cos 25° = 89.7$ V/m along $\vec{P}$

$\vec{E}_B = E_B(\sin 25°\,\hat{\imath} + \cos 25°\,\hat{k})$

$= \boxed{37.9\frac{V}{m}\,\hat{\imath} + 81.3\frac{V}{m}\,\hat{k}}$

$B_B = \frac{E_B}{c} = 2.99 \times 10^{-7}$ T $\perp$ to E_B and so $\vec{B}_B = B_B(\cos 25°\,\hat{\imath} - \sin 25°\,\hat{k})$

$= \boxed{2.71 \times 10^{-7}\,T\,\hat{\imath} - 1.26 \times 10^{-7}\,T\,\hat{k}}$