The ability to mold the flow of light?

Discrete nonlinear Schrödinger equation with cubic-quintic nonlinearity

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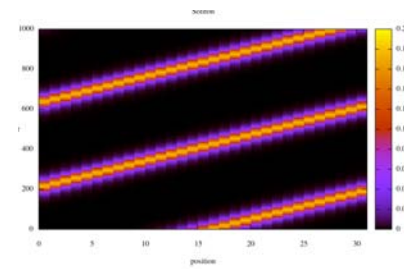
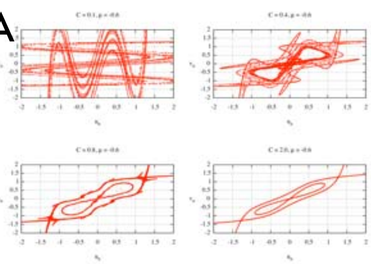


A


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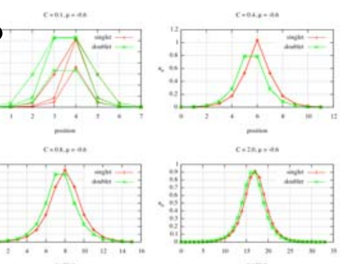

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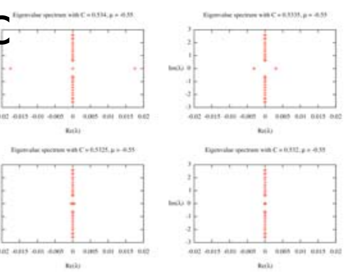
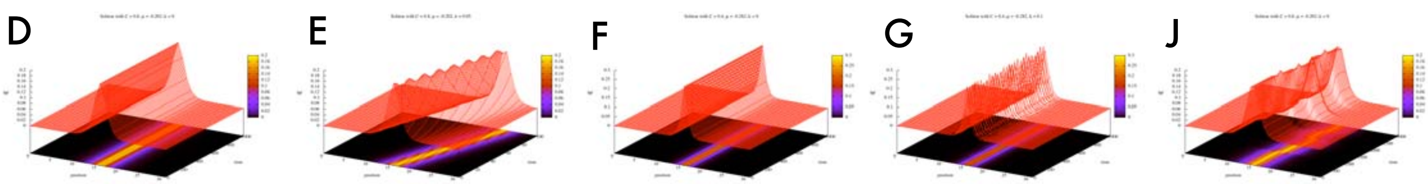
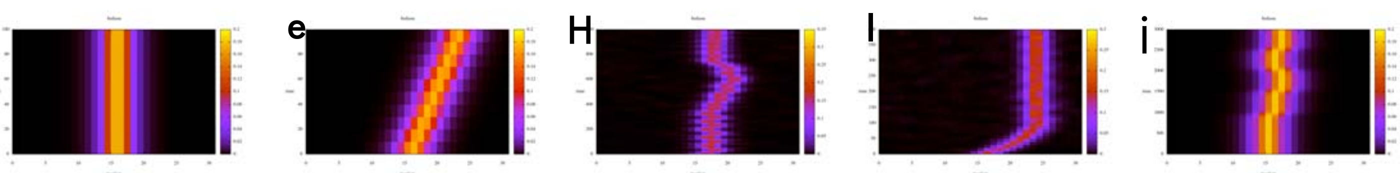


A. Homoclinic orbits
B. Initial solitons
C. Eigenvalue spectrum
D. Stable singlet soliton without the kick
E. The same initial setting as D with the kick
F. Unstable singlet soliton without the kick
G. Same as F with the kick
H. Unstable phenomena
I. Moving soliton comes to halt
J. Unstable doublet soliton without the kick

D E F G J


d e H I j


The study in nonlinear discrete system has attracted growing attention in recent years. One of the major theories – Discrete Non-Linear Schrödinger equation (DNLS) has received great interest. Including of Bose - Einstein condensates, solid-state physics, optical physics, even biology are applied to this active research. Before explaining the phenomena happened in the nature, construct the model and investigate the physical meaning of the equation are necessary for comprehensive understanding.

In continuous model, nonlinear Schrödinger equation $i\partial_t \psi = -\psi_{xx} + f(|\psi|)\psi$ is a nonlinear pattern of Schrödinger equation. General discrete model with cubic-quintic nonlinearity $i\partial_t \psi_n = -C(\psi_{n+1} + \psi_{n-1} - 2\psi_n) - B|\psi_n|^2\psi_n + Q|\psi_n|^4\psi_n$ where $\psi_n = u_n e^{-i\mu t}$ will be used in my work. By the study of discrete dynamical system, homoclinic orbits with fixed points at origin correspond to bright soliton solutions. Using the value obeyed the initial condition of homoclinic orbits, one could obtain the numerically exact solution by Newton's iterating method. While C(dispersion or diffraction) coefficient is small, the homoclinic tangles are plentiful. All of these numerically exact soliton might be stable or unstable. By introducing the perturbation analysis with the form $\psi_n = (u_n + \epsilon u_n)e^{-i\mu t}$ where index n means lattice site, one have to solve 2N-by-2N (N is total number of grid points.) eigenvalue problem. The corresponding eigenvalue spectra could be use to explain the (in)stability. With the fully understand of stationary soliton, moving solitons are introduced for extending usage on research and application. By applying the appropriate kick to a stable stationary solution $\psi_n = u_n e^{ikn}$ where *k* represents the kick, which is presented by adding the energy, one can achieve to move the soliton. Even the soliton starts moving, the moving soliton solution might meet different solution with a higher energy and looks like get stuck at the barrier, which is called Peierls-Nabarro(PN) barrier. To control and to investigate the moving solitons is my further step.

Reference

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