Faculty In Focus

Dr. Y. C. Lee
Dear Doreen,
I have substantially revised my contribution to Faculty in Focus in the following, hopefully making it considerably easier to understand. Please let me know if this is acceptable.

Some Aspects in Condensed Matter Physics —— Summary of research

In this article I would like to review very briefly a selected few of the topics that are of particular interest to me. Only the essential physical features, perhaps aided by a physical picture or two, will be given. These might also serve as punctuation marks that divide broadly my research works in condensed matter physics into several areas.

1. Interaction of radiation with matter. A prominent example is the quantum theory of scattering of light by light in a plasma that acts as a mediator of the interaction.

As is well known, the classical theory of electromagnetism is a linear theory in which one electromagnetic wave does not interact with another. Quantized theory says otherwise. This is because, unlike in classical mechanics, the quantum mechanical vacuum is not a true vacuum. Electron-positron pairs could be created or destroyed by a photon in this "vacuum" (a form of "vacuum fluctuations"), as long as the violation of energy conservation is within the bounds of the uncertainty principle. Thus a photon could first be converted into an electron-positron pair which is later annihilated into another photon, the process appearing as photon-photon scattering via the mediation of the pair. However, the scattering cross-section is very small due to the large energy $2me^2$ of the pair (G. Breit and J. A. Wheeler, 1934; D. L. Burke et al, 1997). In a plasma, however, the role of virtual electron-positron pair in an electromagnetic vacuum is played, in a corresponding vacuum (filled with zero-point plasma wave fluctuations), by the plasmon — the elementary excitation of a plasma. Thus a photon can scatter off another photon by first creating and subsequently destroying a plasmon. Since the plasmon energy $h\omega_p$ is much smaller than that of an electron-positron pair, the cross-section of the nonlinear scattering of photons in a plasma was calculated to be much larger and indeed shown to be within the realm of experimental observation (H. Cheng and Y. C. Lee, 1965, 1966, 1967).

In an intrinsic semiconductor, there are the filled valence band and the empty conduction band. They are separated by a band gap ($\epsilon_{gap}$) of order $0.5eV$ or less. It is conceivable that a photon of energy somewhat less than the band gap may virtually create an electron-hole pair by exciting an electron from the valence band to the conduction band. This pair could even be bound into an exciton which decays later as the electron falls back to the valence band by emitting another photon. This is again a process of nonlinear photon-photon scattering, the band gap playing the role of the aforementioned
$2mc^2 \sim 1\text{MeV}$ in the Dirac theory. Since $\epsilon_{\text{gap}} \ll 2mc^2$ the scattering cross-section should be considerable.

2. Quantum Optics  I would like to cite as an example the study of the natural linewidth as enhanced by the coherence of the radiation emitted by an atomic gas (as compared to the natural linewidth of a single atom).

(a) When the interatomic distance $d$ of atoms is small compared with the spontaneous radiation wavelength $\lambda$, the photon emitted by one atom may be absorbed by other atoms in its neighborhood and may thus be trapped in the gas. The number of photons that are emitted is therefore reduced, as compared to the incoherent radiation of a gas of widely separated atoms. While it takes a much longer time for the release of the trapped photons it is correspondingly compensated by the coherently enhanced rate of the release of the rest of the photons. This photon trapping effect on the natural linewidth of the system as a whole is calculated and found to cause considerable enhancement of this radiative linewidth—a generalization of the Dicke superradiance to an extended system of randomly distributed atoms (Y. C. Lee and D. L. Lin, 1969; P. S. Lee and Y. C. Lee 1972; P. S. Lee, Y. C. Lee and C. T. Chang, 1973; P. S. Lee, Y. C. Lee, 1973; Y. C. Lee and P. S. Lee, 1974). Furthermore, a subsequent analysis shows that even the random thermal atomic motions would not generally destroy the above coherence effect (Y.C. Lee and D.L. Lin, 1969). The related phenomena of Resonance Fluorescence of thin crystal films, and crystalline solids are also studied (Y.C. Lee, C. Y. Chen and Y. Shan, 1978; 1979).

(b) A natural extension of the above Dicke superradiance in gaseous quantum optics to condensed matter physics is next in order. We are thus led to examine the theory of superradiance of excitons in thin crystal films. As we know, exciton (or polariton when combined with a photon into a composite) is an elementary excitation in a semiconductor which, like the phonon, has well defined momentum and energy. While the broken crystalline symmetry in the direction normal to the plane of the film renders the radiative decay of the exciton-polariton possible, the remaining crystal symmetry in the plane of the film leads to phase coherence between the exciton field and the photon field everywhere in the film-plane. Consequently a trading between the degrees of freedom of the radiation field and those of the atoms leads to a prediction of superradiative enhancement of the exciton decay rate by a factor of $(\lambda/d)^2 \gg 1$ in the optical region. (K. C. Liu, Y. C. Lee, and Y. Shan, 1975; Y. C. Lee, 1978; K. C. Liu and Y. C. Lee, 1980; Y. C. Lee and K. C. Liu, 1981).

Related studies on impurity states in a crystal slab (Y. C. Lee and D. L. Lin 1979); energy and structure of Wannier excitons in thin crystal films (Y. C. Lee and D. L. Lin 1979) are also carried out. At this stage it is natural to branch out into the study of the Disorder-caused Anderson localization such as the effect of finite field on Thouless’ maximum metallic resistance in thin wires (Y. C. Lee and K. C. Liu, 1982), effects of finite electric
field and inelastic scattering on Anderson localization in two dimensions (Y. C. Lee, C. S. Chu, and E. Castano, 1983). It is then not difficult to see that a more microscopic, quantum mechanical analysis of the above topics leads us further to the topic of the Hubbard model (W. N. Mei and Y. C. Lee, 1981).

(c) The study of the spin-related properties of ultra-thin magnetic films is of scientific and practical interest. It is hence important to acquire a thorough understanding of the spin dynamics and magnetic relaxation processes.

We extend our previous studies of superradiative decay of various elementary excitations (as in 2(b)) in confined geometries to the decay of magnetic excitations (magnons) in thin films into elastic waves (phonons) of the surrounding non-magnetic medium. For this purpose we must focus on the interaction between spin waves and the elastic deformation of the crystal lattice.

Not to let the nonessential mathematical and physical details cloud the main issue we first review and then adopt a simple generic approach to the magnon-phonon interaction (C. Kittel, 1958; 1949; 1963. S. W. Lovesey, 1972; Chikazumi, 1997). Although magnon-photon interaction would generally be present, it can be shown that it only leads to a much weaker decay rate by roughly a factor of $(c_{\text{light}}/c_{\text{sound}})$ and can hence be ignored. The physical reason is that a lattice displacement changes directly the exchange overlap integral between neighboring spins which basically involves the longitudinal, Coulombic interaction. On the other hand, an electromagnetic wave tends to affect the spin via the transverse, magnetic interaction (i.e., with light speed $c$ directly involved). This gives rise to the appearance of the factor $(c_{\text{light}}/c_{\text{sound}})$ in the ratio of the decay rates via the two different channels — phonons and photons.

As a consequence, the decay rate of a magnon into phonons in the surrounding elastic medium is much greater than that caused by magnon-photon coupling. Analogous to the exciton decay studied in 2(b) there is also the anticipated superradiative enhancement caused by the coherence between the magnetization wave field and the elastic wave field in the film.

Since the magnon energy, as expected physically, is an important energy scale that appears explicitly in the decay rate, an external applied magnetic field may serve as an experimental knob to tune the magnon energy and hence control its decay rate. (S. W. Nam, Y. C. Lee, and A. Sablauer, 2004)

(d) Radiative linewidth and radiative frequency shift are two faces of the same coin. Unlike the linewidth, a finite frequency shift of a collective elementary excitation — the exciton — can only be obtained in a manner similar to the famous Lamb shift in quantum-electrodynamics, namely, by devising a proper renormalization procedure that cures the ultraviolet divergences arising from virtual photons of large momenta. Furthermore, in calculating the frequency shift of the exciton in a thin film due to coherence effect alone, apparent infrared divergences of the logarithmic type turn up in the limit of zero $q$, the center-of-mass momentum of the exciton. In this work we show how the radiative fre-
quency shift of the exciton in a semiconductor film could be properly renormalized before a meaningful comparison between theoretical results and experimental observations can be made. However, this time the renormalization has to be properly generalized since it is for an interacting many-body system rather than for a single hydrogenic atom as in the case of the Lamb shift. The result shows, analogously, a finite, superradiatively enhanced excitonic frequency shift for a semiconductor film.

In the above calculation we first assumed, innocently, a two-band model for the semiconductor. As it turns out, all energy bands have to be invoked before a finite radiative energy shift can be extracted, serendipitously, from the two-band model. In other words, the familiar two-band model can only be justified, as a bonus of a proper renormalization of the semiconductor with a full set of energy bands.

As far as we know, except for our previous work on Renormalized Frequency Shift of coherent radiation from an atomic gas (Y. C. Lee and D. L. Lin, 1972) this is the first time that the concept of renormalization with respect to vacuum fluctuations is applied to condensed matter physics (Y. C. Lee, D. S. Chuu, and W. N. Mei, 1992).

3. **Physics of Broken symmetries in Confined Systems**

Pursuing further the effect of broken symmetry as in 2(b) we turn to the study of phenomena in some confined systems.

(a) New collective modes in reduced dimensions.

♦ (a1) Slender Acoustic Plasmon

A plasmon is the quantum of a collective oscillation mode in an electron gas due to the Coulomb interaction among the charge carriers. Since the Coulomb force acts differently in different dimensions, one can easily show that it gives rise to different dispersion relations of $\omega$ versus $k$, namely, $(\omega_P^{(d)})^2 \propto k^{3-d}$ where $d$ is the dimensionality and $k$ the wavenumber. This results, for example, in a plasma wave of the acoustic type in a strictly one dimensional system. But what happens "between" dimensions? In a slender or quantum metallic wire of small but finite cross-section, multiple non-Landau-damped acoustic modes are shown to exist. These slender acoustic plasmons (SAP) arise as a result of the collective, longitudinal oscillations of the electrons grouped in one of the discrete transverse-motion levels against those grouped in neighboring levels. The physics that gives rise to these collective longitudinal electronic oscillations with phase velocities less than $v_F$ is unique to such a system, although an analogy can be traced to the so-called Chock-Lee acoustic modes propagating along a quantizing magnetic field that was predicted earlier for bulk metals (D. Chock and Y. C. Lee, 1970a and 1970b). The fact that these modes could be immune to Landau damping owes its origin to the discreteness of the transverse levels for very thin wires, since then these levels become sufficiently far apart so that each level can hence accommodate a macroscopic number of electrons with different $k'$s, thereby guaranteeing the collective and organized nature
versus the possible single-particle-like nature and hence demise of these modes. They are also shown to persist even for a temperature comparable to the mode separations. An explicit example of these modes in GaAs shows that they should be observable.

(a2) Corrugation Phonon

In high temperature superconducting cuprates, there are copper oxide layers separated by the interlayer materials such as Ba, Y, or La, etc. It is generally agreed that the CuO layers play a major role in contributing to the superconductivity. It is noted that the CuO layers are by no means perfectly flat. There exist corrugations in these CuO planes in which the z-coordinates of O and Cu are slightly different, the z direction being normal to the planes. It is intriguing to note that in a parent nonsuperconducting compound such as LaCuO, the CuO planes are flat. In other words, the onset of superconductivity seems to be accompanied by the simultaneous appearance of the corrugation that gives rise to nonzero bond angles between successive bonds connecting neighboring atoms in the plane. The new phonon modes of small vibrations of the bond angles associated with a certain angular stiffness are investigated. Two distinct modes are obtained. While one of them has a linear (i.e., acoustic) dispersion relation, the other has a quadratic one. Of particular interest is the latter mode which would contribute to a linear temperature dependence of the heat capacity. This new phonon mode in a corrugated plane could thus be relevant to recent experimental findings. In view of the corrugating vibrations of the bond angles we might call this mode the "corrugation phonon" mode (H. Kang, J. P. Cheng, and Y. C. Lee, 1989).

(b) The "anharmonic phonon"— its physical momentum and broken translational symmetry

In most of the well-known solid-state text books such as Kittel's or Ashcroft-Mermin's it is stated and proved that phonons of any finite wave vector $\overrightarrow{q}$ in a crystal can only carry "crystal momentum" but not any true or physical momentum, except for the case of $\overrightarrow{q} = 0$. This is because a phonon coordinate of finite $\overrightarrow{q} \neq 0$ on a lattice involves only the relative coordinates of the atoms. As in a molecule, the relative internal motion of the constituent-atoms is not associated with any linear momentum of the molecule as a whole. Only an acoustic phonon of $\overrightarrow{q} = 0$ that corresponds to a uniform translation of all the atoms and hence to a translation of the center of mass can carry a physical momentum. On the other hand, anharmonic forces promote mode-mode coupling. Hence the possibility of the coupling of a $\overrightarrow{q} \neq 0$ phonon to a $\overrightarrow{q} = 0$ acoustic phonon cannot be ruled out. Then why shouldn't a $\overrightarrow{q} \neq 0$ phonon be allowed to couple anharmonically to a $\overrightarrow{q} = 0$ acoustic mate to form a composite of the same $\overrightarrow{q}$? If the answer is yes, then this composite would be enabled to carry finite physical momentum via the mate of $q = 0$. What is wrong with the proof in text books? The answer lies in the translational invariance that is usually taken for granted in conventional derivations. In the case of a crystal under no external forces, the translational invariance implies that the momentum of the
center of mass is always conserved. Hence the $\vec{q} \neq 0$ phonon modes of relative motions cannot be coupled to the $\vec{q} = 0$ acoustic mode of the motion of the center of mass, with or without anharmonic forces. However, when a crystal is heated, it is no longer isolated but rather placed in intimate contact with a thermal reservoir in its surrounding, thereby breaking the translational invariance. This removes the obstacle to acquiring a finite momentum for a $\vec{q} \neq 0$ phonon through anharmonic or Gruneisen coupling to its acoustic mate of $\vec{q} = 0$. Indeed, once the cloud is cleared, it is not difficult to show that the physical phonon momentum in an n-dimensional system is given by $\vec{q} \gamma = \hbar \gamma \vec{q}$, where the well-known Grüneisen constant $\gamma$ might call this momentum the "anharmonic momentum" which should be distinguished from the more familiar "crystal momentum", $\vec{p} = \hbar \vec{q}$. The physical difference between these two types of phonon momentum has been thoroughly discussed (Y. C. Lee and W. Z. Lee, 2006).

(c) Rate of thermal expansion

Unlike the study of thermal expansion as an accomplished act in statistical thermal equilibrium, the speed of thermal expansion in progress belongs to the more complex realm of statistical, non-equilibrium, nonlinear, thermo-dynamics. Numerical analysis even for a chain of ten or even fewer particles becomes a futile effort due to the two very different time-space scales involved. One is the very fast scale of microscopically small atomic vibrations, the other is the very slow but macroscopically measurable scale of thermal expansion. Trying to identify the slow orderly expansion by averaging over the extremely fast but tiny motions of the many randomly oscillating atoms upon heating amounts to looking for a needle in a haystack.

To look for a clue to this complex problem we first review the statistical mechanics of the thermal expansion coefficient. We start with the equation (the $P$-equation) for the ambient pressure $P$ of the system at temperature $T$. There are two contributions to this $P$. The first originates from the energy of the strained lattice in static equilibrium, including that of the zero-point fluctuations. The second term, $P_{ph}$, is the contribution by the phonons. Neglecting the small ambient atmospheric pressure we see that the equation for the total pressure simply expresses the mechanical balance of the (positive) outward pressure of $P_{ph}$ by the force of the stretched atomic springs that tend to restore the atoms to their original configuration. Although this equation is supposed to describe only the time independent state of total thermal equilibrium at temperature $T$ after the full thermal expansion has been attained, we may now turn the aforementioned troublesome disparity in the two time scales into an advantage by re-interpreting that equation as follows.

Based on the kinetic definition of temperature that relates it to the random kinetic energy of the atoms (i.e., of the phonons) the increase of $P_{ph}$ by $\Delta P_{ph}$ should be attained concurrently with the increase of temperature by $\Delta T$. Once these increases have been completed within the short time scale, the thermal expansion of the long thin bar, still
submerged in the large thermal reservoir at \( T + \Delta T \), would proceed at the much slower time scale isothermally. At every subsequent stage of the expansion process the lattice may be viewed as consisting of a quasi-static strained lattice plus the very fast lattice vibrations (phonons) about this quiescent lattice. They correspond, respectively, to the first and second term of the \( P \)-equation. At each such quasi-static stage, the thermodynamic variables \( T, P \) and \( V \) can thus be assumed to have well-defined values. However, unlike the total thermal equilibrium in which the two terms balance each other, the outward phonon forces would now be too large to be balanced by the restoring force of the strained atomic springs until the thermal expansion has finally been completed. This is of course why the system keeps expanding until thermo-mechanical equilibrium is reached.

Then, by exploiting the familiar concepts such as the heat capacity, the Gruneisen constant \( \gamma \), the thermal conductivity, the sound speed \( c_s \), etc., that are again valid in the long time scale of the evolving quasi-static stages discussed above, a detailed microscopic theory of the speed of thermal expansion for a long thin crystalline bar (Y. C. Lee, 2008) and also for a thin metallic or insulating disk of any shape (Y. C. Lee, 2009) have been recently developed. In this theory the all-important concept of anharmonic momentum (Y. C. Lee and W. Z. Lee, 2006) rather than that of the crystal momentum plays a key role. The physical reason is clear. Thermal expansion or, more specifically, its rate, is intimately tied to the pressure of the phonons pushing outward on the boundary of a specimen (e.g., the two ends of a long bar) that is in close contact with a heat reservoir. The presence of the latter breaks the translational invariance. Such a pressure owes its origin to, what else, but the physical momentum of the phonons! They bombard the boundary from the inside that results in an averaged, time-dependent, dynamic motion at temperature \( T \) of the specimen. In a crystal, these phonons must thus be the anharmonic phonons reviewed in 3(b)!

To our gratification this theory is being applied, for example, (K-J Kim and Y. Shvyd’ko and S. Reiche, 2008; K-J Kim and Y. Shvyd’ko, 2009) in the design of an x-ray free electron laser oscillator, for one of the main components is an x-ray optical cavity with thin crystal plates (0.05-0.1 mm) of diamond or sapphire used as Bragg mirrors for x-rays. We thank Dr. Y. Shvyd’ko of Argonne National Lab. for several helpful communications. Previous experimental works (D. W. Tang et al, Appl. Phys. Lett., 24, 3114 (1991) and references cited therein) could also be relevant.

Sample comments on the above works from the referees:

(1) The article ‘Speed of thermal expansion of a long, thin crystalline bar and the physical momentum of acoustic phonons’ by Y. C. Lee addresses the question about the time scale on which thermodynamic processes reach equilibrium on the example of thermal expansion. It is based on a recently established concept of the physical momentum versus the crystal momentum of acoustic phonons in a crystal lattice. Being an experimentalist who uses thermodynamic methods at a very high speed of temperature modulation, the article discusses question which I find of particular importance in a time in
which thermodynamic methods are minimized and used at very high speeds of temperature modulation to reach into the challenging field of nanophysics. I found the article very well written and develops the theoretical concepts in a way that even pure experimentalists can easily follow. The result that the sound speed is the quantity which determines the thermal expansion time is surprisingly simple and could have been guessed, but has never been theoretically shown before. I thus strongly suggest publishing this article in Journal of Physics: Condensed Matter, basically as it is. The only suggestion I have is that the author mentions in the introduction more clearly that this work is related to the physics of insulators (where he probably means when he talks about a crystalline bar), whereas the presence of free electrons will bring another time scale. He points towards a future work at the end of this article but it might be nice to write this in the beginning.

(2) To the best of my knowledge this is probably the first work on the theory of the rate of thermal expansion (as applied to a thin slab of any shape).

In experimental designs involving optical cavities with thin crystal plates used as diffraction mirrors, the temperature rise due to photoabsorption would cause thermal expansion of the crystal which, in turn, leads to a change in the lattice parameter. Such a change would be highly significant for the Bragg diffraction from the crystal. The time of expansion relative to the absorption time is hence crucial to the success of such experiments. Other experiments involving fast temperature modulations of thermodynamic objects would likewise be significantly affected by the rate of thermal expansion.

(d) Electron-electron correlation in a negative hydrogenic ion in a quantum wire—a three-body problem in confined geometry.

The problem of a hydrogenic ion involving the intimate correlation among a proton and two electrons is a long standing one since the very beginning of quantum mechanics. Because of the notorious difficulty of this three-body problem, there has been no exact or near-exact solution. Mostly we could only rely on variational or numerical methods. This is true regardless of the dimensionality, be it of one-dimensional or two or three dimensions. However, due to the hidden singularities or nonanalyticities in quasi-one dimension, these methods are no longer dependable because of the inherent wild fluctuations. Clearly a radically different approach is called for.

In a recent work, the nonanalytic behaviors of the binding energy $e_b^{2nd\ elec}$ of the second electron in a negative hydrogenic ion in one dimension, in a quantum wire (QWR), or in a nano-tube as a function of the Coulombic repulsive strength $\gamma$ and the wire radius $b$ are first identified. Rather than trying to avoid them they are tackled head-on, and then exploited, together with the recognition of the well-known near-infinite binding energy of the neutral hydrogenic atom, to set up an exact model that calculates the $e_b^{2nd\ elec}$ in a QWR directly, thereby avoiding the pitfalls of a variational approach to this problem. This $e_b^{2nd\ elec}$, found to be 0.4 Ryd, is an exact solution for a wire whose radius $b$ is nearly zero. For a small but finite radius $b$, it is shown that still $e_b^{2nd\ elec} \simeq 0.4$ Ryd, which is independent of $b$ as long as $0 < b < a_{Bohr}$, due to the inverse square nature of the
Coulomb force, $a_{Bohr}$ being the Bohr radius in the material medium. Thus this 3-body problem is solved exactly in one dimension and in quasi-one-dimension. It is also illuminating to compare this exact solution with the corresponding approximate results in two and three dimensions. (Y. C. Lee, J. F. Jan, and W. Z. Lee, 2005)

(e) Bose-Einstein condensation (BEC) of excitons in two dimensions

It is well-known that the BEC (Bose-Einstein Condensation or the condensation of a macroscopically large number of free bosons into a single-particle state of the lowest energy) originally proposed by Einstein for the free boson gas in three dimensions does not exist in two dimensions. Years later this concept of the BEC was extended and generalized by C. N. Yang (C. N. Yang, 1962) to that of the so-called "off-diagonal-long-range-order" (ODLRO) generally for interacting bosons. Subsequently it was rigorously proved by Hohenberg (P. C. Hohenberg, 1967) that ODLRO cannot exist in two dimensions even for an arbitrary system of interacting Bose particles.

It has now been shown that the above situation is changed drastically for noninteracting bosons in two dimensions in the presence of localized states, when these states have a discrete energy spectrum below the continuum of the extended states. This new kind of BEC, unlike that for three-dimensional Bose gases, possesses off-diagonal order only of finite range rather than the infinitely long range in ODLRO, and is thus not in violation of the Hohenberg theorem (J. F. Jan and Y. C. Lee, 1998). The observation of such a BEC in the transport properties of the bosonic excitons moving on the hetero-interface of a type-II semiconductor superlattice is suggested, bearing in mind the fact that the unavoidable random potential fluctuations on such surfaces would produce the required localized states (P. W. Anderson, 1958).

4. Possible SAP-mediated High $T_c$ Superconductivity

It is well known that the main ingredient of superconductivity is the Cooper pair, the binding of two electrons of opposite momenta and spins. The strength of the attractive coupling for the binding depends on how effective it is to mediate a force via the exchange of a phonon or other bosons between the two electrons (i.e., emission of a phonon by one electron followed by its absorption by the other electron). Two factors are essential. The first is the magnitude of the matrix element coupling the electron and the phonon, the second is whether the exchange process is far off the energy shell or nearly on the energy shell (i.e., nearly energy-conserving). We recall that the phonon originates from the collective vibrations of atomic ions which are heavier than an electron by several orders of magnitude. To emit a phonon requires an electron (a tiny mouse) to kick Coulombically a heavy ion (an elephant) into action. This scenario dramatizes the relative feebleness of the electron-phonon coupling on account of the unfavorable mass ratio. In comparison, if the phonon’s role could be played by a plasmon consisting of electron
oscillations, the elephant-like ions would be out of the picture while the relevant force between an electron and a plasmon remains basically the same — Coulombic type. Hence the unfavorable mass ratio is eliminated. However, with respect to the second factor that arises from the on-shell or off-shell consideration of the virtual boson exchange process we would much prefer the electron to emit a boson of smaller energy so as to minimize the energy violation in the intermediate state.

As we know, the acoustic phonon frequency is linearly proportional to the wave number $\overrightarrow{k}$, approaching zero in the long wavelength limit, independent of dimensionality. On the other hand, we already know from 3(a) that in three dimensions the plasma frequency $\omega_P$ is a constant independent of the wave number $\overrightarrow{k}$. This makes the idea of exploiting the high energy plasmon in lieu of the low energy acoustic phonon as the mediating boson unfavorable. However, in a one dimensional system the plasmon becomes acoustic-like, rendering it possible to compete on equal footing with the phonon as far as the second factor is concerned. Thus, when both factors are considered, a thin wire with the many branches of slender acoustic plasmon should compete favorably to be the mediating boson of the attractive force between the Cooper pair.

Based on the pairing interaction mediated by the quasi-one dimensional plasmons as well as the multiple branches of the so-called slender acoustic plasmons a possible mechanism of superconductivity in slender electronic systems is proposed. Numerical results on $T_c$ in various samples are presented, showing values in the $150 - 200 K$ range. The ratio $2\Delta_0/T_c$ differs generally from the BCS value due to the temperature dependence of mode damping. The associated coherence length is shown to be considerably small than the transverse dimension of the wires. This work is later extended to quasi-one-dimensional multiwire-structures, showing $T_c$ in the 200-K range. This is enhanced over the critical temperature of the single-wire system. This fact stems from the in-phase Coulomb wire-wire coupling. The numerical values are based on wires of cross section $200 \AA \times 100 \AA$. These predictions were first made prior to 1987 and continued along the same line later. (Y. C. Lee, B. Mendoza and D. S. Chuu, 1989; Y. C. Lee, B. S. Mendoza and S. Ulloa, 1989; Y. C. Lee and B. S. Mendoza, 1989; B. Mendoza and Y. C. Lee, 1989; Y. C. Lee, 1987; W. Zhu, H. Kang, and Y. C. Lee, 1994.)

After the discovery of the new crop of high-$T_c$ superconductors in 1987 there have been occasional rumors that some samples of thin wires had been found to exhibit superconductivity with surprisingly high critical temperatures. Although these rumors were communicated to us from reputable laboratories, none of them have appeared in print.


The concept of indistinguishability is the key element in quantum statistics. Indeed, quantum statistics was born when Bose rederived the Planck radiation law by a new way of counting that only asks how many particles (photons) are in each single-particle orbital without distinguishing which particle is which (S. Bose, 1924). Furthermore, it is these
or orbitals rather than the particles themselves that are treated as statistically independent. Bose also tacitly implied that the number of photons is not conserved (S. Bose, 1924; A. Pais, 1982). The extension to the case of a gas of a definite, conserved number of atoms by using this new way of counting was immediately made by Einstein (A. Einstein, 1924; 1925). The mystery that the particles then become statistically correlated in spite of the assumed absence of interaction was resolved by Dirac (Dirac, 1926), who pointed out that the intrinsic indistinguishability of identical particles in quantum mechanics is the origin of this correlation. The constraint of particle conservation first imposed by Einstein then leads to what is presently known as Bose-Einstein condensation or BEC (A. Einstein, 1924; 1925; A. Pais, 1982). The above short history of BEC brings out two things: one, the crucial role played by indistinguishability and, two, BEC and the accompanying statistical correlation should obtain even in the absence of any interaction among the particles. Point two has been brilliantly confirmed recently (M. H. Anderson et al., 1995; K. B. Davis, 1995). Testing point one, the role of indistinguishability, is another matter altogether. We are used to thinking of the spatial overlap of the wave functions of identical particles as a deciding factor for indistinguishability. However, such indistinguishability is only extrinsic, not associated with any possible difference with respect to the intrinsic, internal states of the particles themselves. But, are particles really either intrinsically indistinguishable or distinguishable? For a test we must look for a way to characterize the internal atomic state itself continuously. In other words, we want to find a parameter that can be used to tune the degree of intrinsic distinguishability in some controlled, continuous manner and see how it affects the behavior of a quantum gas, such as Bose-Einstein condensation (BEC)?

We have found just such a parameter in a gedanken experiment. It is generally complex with a definite phase that is connected with the spin orientations of the bosons. The intrinsic distinguishability can indeed be tuned at will by varying this parameter (Y. C. Lee and A. Sablauer, 2005; 2007).

We start with two boson gases from independent sources A and B, respectively. Initially one gas consisting purely of atoms of a given type, say, Rb$^{87}$, in the spin state $|F = 1, m_F = 1\rangle \equiv |1, 1\rangle$ while the other gas is also of Rb$^{87}$ atoms that are rather in another spin state $|1, 1'\rangle = a|1, 1\rangle + b|\alpha\rangle$, where $b|\alpha\rangle \equiv \sin^2 \frac{\theta}{2}|1, -1\rangle + \frac{\sin \theta}{\sqrt{2}}|1, 0\rangle$, so that $a = |\langle 1, 1|1, 1'\rangle = \cos^2 \frac{\theta}{2}, \langle 1, 1|\alpha\rangle = 0, \langle \alpha|\alpha\rangle = 1$. These spin polarizations can be achieved, for example, by subjecting the two gases separately to magnetic fields of two different directions, one along $\hat{z}$, the other along $\hat{z}'$ at angle $\theta$ with $\hat{z}$. When $\theta = 0$ or $|1, 1'\rangle = |1, 1\rangle$, the atoms in the two gases become intrinsically indistinguishable. As we manipulate the magnetic fields so that they begin to deviate directionally from each other, i.e., $\theta \gtrsim 0$, the two gases become distinguishable, the distinguishability increasing as $\theta$ deviates more and more from zero. In other words, the parameter $a \equiv |\langle 1, 1'|1, 1\rangle = \cos^2 \frac{\theta}{2}$(or $\theta$ itself) that represents the overlap between the two states $|1, 1\rangle$ and $|1, 1'\rangle$ in spin space serves as a tuning parameter for the intrinsic distinguishability (Y. C. Lee and A. Sablauer, 2007). Another more elaborate way of finding the tuning parameter, as was first suggested,
by the use of two sets of Stern-Gerlach apparatus (Y. C. Lee and A. Sablauer, 2005).
We now send the two polarized beams from the independent sources A and B into a common mixing chamber. After they are thoroughly and uniformly mixed (so that statistical averaging over the mixed gas makes sense), each atom in this chamber is described by an internal spin state $|\phi_{\text{spin}}\rangle$ which is associated with every momentum state $|\vec{k}\rangle$. This $|\phi_{\text{spin}}\rangle$ is a mixed state:

$$|\phi_{\text{spin}}\rangle = \sqrt{P_1} e^{i\alpha} |1, 1\rangle + \sqrt{P_1'} e^{i\alpha'} |1, 1'\rangle; \quad P_1 = \frac{N_1}{N}, \quad P_1' = \frac{N_1'}{N}, \quad N = N_1 + N_1'$$

(1)

$N_1$ denoting the number of atoms from source A, $N_1'$ denoting that from source B, and $N$ is the total number of atoms brought into the mixing chamber. The relative phase $\alpha - \alpha'$ is completely random since sources A and B are independent sources. This is what renders $|\phi_{\text{spin}}\rangle$ a mixed state; it immediately leads to $\langle\phi_{\text{spin}}|\phi_{\text{spin}}\rangle_{\text{phase-average}} = 1$ despite $\langle 1, 1'|1, 1\rangle = \cos^2 \frac{\theta}{2} \neq 0$. The integrity of the spin states is assumed intact even after entering the mixing chamber (Y. C. Lee and A. Sablauer, 2005).

Recognizing that the time scale for thermalizing among the translational degrees of freedom is much faster than that among the spin degrees of freedom we may factorize the orbital state $|l\rangle$ into the translational part and the spin part, i.e., $|l\rangle = |\vec{k}\rangle |\phi_{\text{spin}}\rangle$ (Lee and Sablauer, 2005). This is because while a purely spin-independent interatomic scattering may eventually change the atomic plane wave from $e^{i \vec{k} \cdot \vec{r}}$ to $e^{i \delta(\vec{k}, \vec{k}')} e^{i \vec{k} \cdot \vec{r}}$, yet the spin structure as expressed in the relative amplitudes of $a$ and $b$ in $|1, 1'\rangle$ would remain the same. An added phase factor such as $e^{i \delta(\vec{k}, \vec{k}')} \neq 0$ attached to $|1, 1'\rangle$ after the scatterings is not relevant to what happens later in the mixing chamber where the phase of one beam $|1, 1'\rangle$ relative to that of the other beam $|1, 1\rangle$ is assumed to be random anyway. With respect to spin-dependent scattering, we argue that the internal spin states $|1, 1\rangle$ and $|1, 1'\rangle$ in our gedanken experiment should remain essentially intact in a metastable manner. This is supported experimentally (A. Robert et al., 2001; F. Pereira Dos Santos et al., 2001), demonstrating thus the much shorter time scale for temperature equilibration for the translational motion than for the greatly suppressed spin thermalization among the metastable spin-polarized states.

To deal with mixed states it is natural to set up the corresponding density matrix. We may likewise factorize the density matrix $\rho_l$ for the single particle orbital $l$ into $\rho_l = \rho_{\vec{k}} \times \rho_{\text{spin}}$ for the non-interacting gas of bosons. The spin dependent part, given by

$$\rho_{\text{spin}} = \{|\phi_{\text{spin}}\rangle\langle\phi_{\text{spin}}|\}_{\text{phase-average}} = P_1 |1, 1\rangle\langle 1, 1| + P_1' |1, 1'\rangle\langle 1, 1'|$$

(2)

is obtained by averaging over the random relative phase $\alpha - \alpha'$ between the two component states $|1, 1\rangle$ and $|1, 1'\rangle$ in Eq. (1). In the orthonormal basis of $|1, 1\rangle$ and $|\alpha\rangle$
the above $\rho_{\text{spin}}$ becomes

$$\rho_{\text{spin}} = \rho_0 + \rho_{\text{coh}}$$  \hspace{1cm} (3)$$

$$\rho_0 = \begin{bmatrix} P_1 + P'_1|a|^2 & 0 \\ 0 & P'_1|b|^2 \end{bmatrix}$$  \hspace{1cm} (4)$$

and

$$\rho_{\text{coh}} = \begin{bmatrix} 0 & P'_1b^*a \\ P'_1ba^* & 0 \end{bmatrix}, \quad P_1 = \frac{N_1}{N}, \quad P'_1 = \frac{N'_1}{N}$$  \hspace{1cm} (5)$$

Note that if $|1,1\rangle$ were not a pure state the matrix elements in $\rho_{\text{coh}}$ would each become zero upon averaging the over the random relative phase between $a$ and $b$ in $|1,1\rangle = a|1,1\rangle + b|\alpha\rangle$. Thus $\rho_{\text{coh}}$ is originated purely from coherence.

Upon diagonalization we find

$$\rho_{\text{spin}} = P_+|\Psi_+\rangle\langle\Psi_+| + P_-|\Psi_-\rangle\langle\Psi_-|$$  \hspace{1cm} (6)$$

where $\langle\Psi_+|\Psi_+\rangle = 1 = \langle\Psi_-|\Psi_-\rangle$, $\langle\Psi_+|\Psi_-\rangle = 0$, $P_+ + P_- = 1$, we find

$$|\Psi_\pm\rangle = A_\pm|1,1\rangle + B_\pm|\alpha\rangle$$  \hspace{1cm} (7)$$

$$P_\pm = \frac{1}{2}[1 \pm \sqrt{1 - 4P_1P'_1|b|^2}]$$  \hspace{1cm} (8)$$

and

$$\frac{P_+ - P'_1 - P'_1|a|^2}{P'_1ab^*} = \frac{B_\pm}{A_\pm} = \frac{P'_1a^*b}{P_\pm - P'_1|b|^2}$$  \hspace{1cm} (9)$$

It is significant to note that, with $P_1 = \frac{N_1}{N_1 + N'_1}$, $P'_1 = \frac{N'_1}{N_1 + N'_1}$, it is only the ratio of the two populations, $\frac{N'_1}{N_1}$ (or the densities $\frac{n'_1}{n_1}$) rather than the individual $N_1$ and $N'_1$ that enters the constitution of the new states $|\Psi_\pm\rangle$ and their associated $P_\pm$. As will be shown later, this is highly important for that means they would remain unchanged when we lower the two densities $n_1$ and $n'_1$ as long as their ratio is kept fixed. In turn, the corresponding lowered densities $n_\pm = P_\pm(n_1 + n'_1)$ would affect the respective chemical potentials $\mu_\pm$ and the subsequent BEC’s, although their states $|\Psi_\pm\rangle$ and the other associated observable consequences would remain intact.

The above formulas constitute the major mathematical results of the present investiga-
tion, showing the two new mutually orthogonal spin eigenstates $\Psi_+$ and $\Psi_-$ together with the associated eigenvalues $P_+$ and $P_-$ for the atoms; they do not belong to any atomic Hamiltonian but rather to the quantum-statistical spin density matrix $\rho_{\text{spin}}$. We can re-classify the single-particle orbitals as the new set of $|\vec{k}; \Psi_{\pm}\rangle$ to replace the old set of non-orthogonal $|\vec{k}; 1, 1\rangle$ and $|\vec{k}; 1, 1'\rangle$. Thus the state $| \Psi_+ \rangle$ has the probability $P_+$ of being occupied by an atom in the mixing chamber, and likewise for the state $| \Psi_- \rangle$ and $P_-$. Correspondingly the statistically (and spatially) averaged density of atoms in $| \Psi_{\pm} \rangle$ is, respectively, $n_\pm = P_\pm n$ where $n = \frac{N}{V}$ is the overall atomic density in the mixing chamber. This stems from the previously stated fact that our $\rho_{\text{spin}}$, being a factor in $\rho_t = \rho_{\text{spin}}\rho_{\vec{p}}^{-1}$, is the same for every spatial state $|\vec{k}\rangle$. Since $| \Psi_+ \rangle$ and $| \Psi_- \rangle$ are orthogonal to each other, they behave effectively like two new, independent atomic states, each accommodating atoms of its own species that is statistically uncorrelated to the other species. Each species will then thermalize at $T$ as usual with respect to the translational energy. Correspondingly there will be two new chemical potentials $\mu_+$ and $\mu_-$. These two distinct species of Bose gas will undergo Bose-Einstein Condensation (BEC) independently at two different critical temperatures at which $\mu_+$ and $\mu_-$ separately approaches zero. These temperatures are given by the familiar expressions

$$k_B T_\pm = \left( \frac{2\pi \hbar^2}{m} \right) \left( \frac{P_\pm n}{2.612} \right)^{2/3}$$  \hspace{1cm} \text{(10)}$$

As a consequence, as the temperature is slowly lowered from above, the species of density $P_+ n$ with a higher $T_c = T_+$, say, in spin state $| \Psi_+ \rangle$, would undergo BEC first. Then the species of density $P_- n$ with the lower $T_c = T_-$ in spin state $| \Psi_- \rangle$ would follow later, i.e., the BEC’s occur at two successive stages of the cooling process. At each of these stages is a coexistence or condensation region which consists of a mixture of two thermodynamic phases. One is the condensed phase composed purely of particles with $\vec{p} = 0$, the other is the normal phase composed of particles with $\vec{p} \neq 0$. For an ideal Bose gas in general, the coexistence region at a given $T \leq T_c$ is represented by a $T$-isotherm in a $P$-$v$ plot that extends from $v = 0$ for the condensed phase to a critical value $v_c(T) > 0$ for the normal gas phase, $v$ being the specific volume of the particles. These two phases can be likened to the liquid phase and the vapor phase of an ordinary liquid at the liquid-vapor phase transition where the condensed (liquid) phase with a smaller specific volume $v$ or higher density mixes with the normal (rarified gas) phase of a much greater $v$ or lower density. If the particles of the ideal Bose gas are placed in a gravitational field, then in the condensation region there would be an observable spatial separation of the two phases, just as in a gas-liquid condensation.

A more direct, elegant and dramatic way to observe the BEC is illustrated in the figure below (due Dallin Durfee and Wolfgang Ketterle, Massachusetts Institute of Technology): Bose-Einstein condensation of sodium atoms, as observed by absorption of laser light.
The images show a cloud cooled to just above the transition point (left), a cloud just after the condensate appeared (middle), and a cloud after further cooling left an almost pure condensate (right).

For our present case of the two distinct species of bosons in $|\Psi_+\rangle$ and $|\Psi_-\rangle$ respectively, we may likewise watch the atomic velocity distribution say, of the $|\Psi_+\rangle$-particles first. As the temperature is lowered to below $T_+$, we would see the emergence of a dramatic cone-shaped distribution pattern. This cone corresponds to the density of particles in the condensation region that includes the condensed phase, hence roughly proportional to $[1 - (\frac{T}{T_+})^{3/2}]$; it should thus become sharper and higher as $T$ gets lower, signifying unmistakably the appearance of BEC of the first stage ($T \leq T_+$) in our present context. Then, upon further decreasing the temperature until $T_-$, the second stage of BEC, i.e., that of the $|\Psi_-\rangle$-species, will start and the same episode will repeat. Indeed, the two cones representing the populations of the two species appearing successively in their respective condensations would superpose on one another. They might be conceivably identifiable experimentally, perhaps, by separately measuring the averaged magnetic moments of the atoms in the two stages, each with a definite, distinct spin structure. Indeed, in accordance with $\langle \Psi_- | \Psi_+ \rangle = 0$ the magnetic moments should be mutually perpendicular to each other.